Structural Dynamics Toolbox Primer

SDTools

REVISED FOR SDT 6.2
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Finite element modeling

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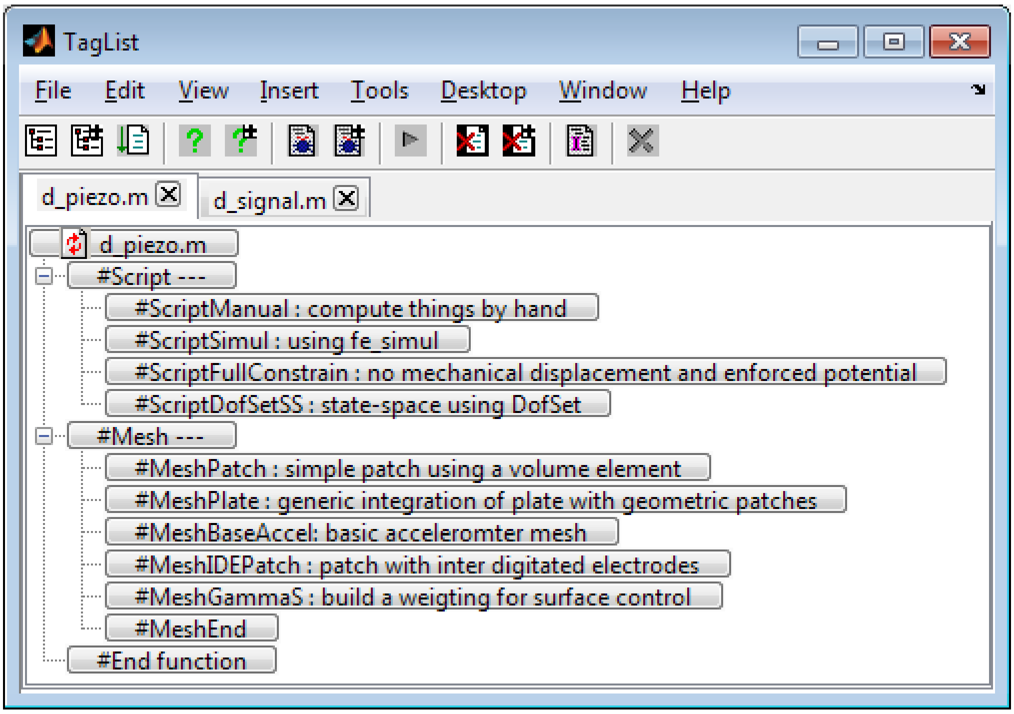
1.1 SDT Basics and concepts

1.1.1 Functions and commands

The SDT capabilities are grouped in a relatively small number of functions. Many of these functions implement a large number of capabilities which are accessed trough string commands and a variable number of arguments `function('Command',arg1,...);`

The basic structure of commands is seen through `sdtweb('_taglist','function')`. For example



`sdtweb` is used to access SDT documentation (which is properly integrated with the MATLAB documentation but not the `doc` command) which does not support external applications. The HTML (or PDF) documentation is most complete and can be opened with a call of the form `sdtweb('feutil')`. Calls to open base documentation pages are shown throughout this documentation.

1.1.2 Conventions in manual

The following typesetting conventions are used in SDTools manuals

`courier`	blue monospace font : Matlab function names, variables
`feplot`	light blue monospace font: SDT function names
`command`	pink : strings and SDT commands
`var`	italic pink: part of command strings that have to be replaced by their value
`% comment`	green: comments in script examples
Italics	MATLAB Toolbox names, mathematical notations, and new terms when they are defined
Bold	key names, menu names and items
Small print	comments

1.1.3 Data structures and stacks

All data in SDT are stored in Matlab data structures. The basic structures are

- `model` FEM model, test wire frame, ... see `sdtweb model`
- `def` deformations (typical FEM output) see `sdtweb def`
- `curve` responses and general datasets ... see `sdtweb curve`

When extensible and possibly large lists of mixed data are needed, SDT uses `.Stack` fields which are N by 3 cell arrays with each row of the form `{'type','name',val}`. The purpose of these cell arrays is to deal with unordered sets of data entries which can be classified by type and name.

`stack_get`, `stack_set` and `stack_rm` commands are low level commands used to get/set/remove single or multiple entries from stacks. `iiplot` and `feplot` implement easier named based indexing, see `sdtweb diiplot#CurveStack`.

1.1.4 SDT pointer strategies

SDT pointer objects are

- `sdth` pointers to data contained in figures.Will be illustrated in section 1.2.2. In particular, these are used to ease script base modification of `feplot` and `iiplot` figures.
- `v_handle` implement pointers. Fundamental for handling of large data sets stored elsewhere (out-of-core file reading for example).

1.2 FEM model

1.2.1 FEM model data structure

The mesh generated during construction of an FE model is a mathematical representation of the structure. FE packages allow definition of a geometry in the form of nodes, lines, surfaces which are used as a guide during meshing. There is a distinct difference between model geometry and the model mesh.

The geometry of a square beam structure can be described as four nodes (vertices) or four lines. Both descriptions are identical - the square is defined uniquely.

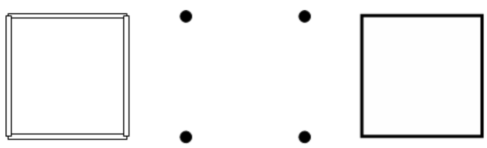


Figure 1.1: Square beam structure and geometry definition

The mesh is defined to approximate the behavior of the true structure. In this case beam elements would be the most suitable, and would be defined between the nodes or along the lines defined by the geometry. There is, however, an element of choice and therefore engineering judgment in this process and the solution is not unique. The main trade off during meshing is between the accuracy required and the computational expense.

Once meshed, material, element properties, and boundary conditions are defined to simulate the model interactions with outside influences - such as a load or a rigid attachment - and are stored as a case. The case along with the geometry and mesh make up the model. The model can then be solved, and the desired output extracted.

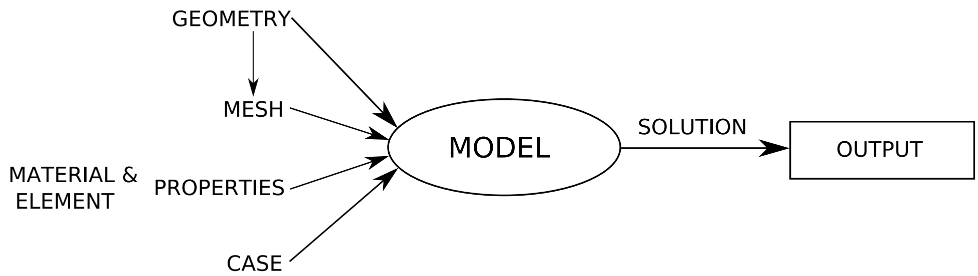


Figure 1.2: Model flow chart

All data in SDT are stored in Matlab data structures. A FE model is a Matlab data structure containing fields (see `sdtweb model`)

- `Node` matrix describing the model nodes (see `sdtweb node`)
- `Elt` matrix of the element connectivity (see `sdtweb elt`)
- `pl` matrix of material properties with one property per row (see `sdtweb pl`)
- `il` matrix of element properties with one property per row (see `sdtweb il`)
- `Stack` cell array containing all other information (see `sdtweb Stack`)
- `unit` main model unit system (see `sdtweb fe_mat#Convert`)
- `name` model name

```
model=femesh('testubeam');disp(model)
    Node: [374x7 double]
     Elt: [161x10 double]
      pl: [1 4.1165e-001 2.1000e+011 2.8500e-001 7800 8.1712e+010]
      il: [1 6.2437e-004 0 -3 0 0]
    name: 'UBeam'
    unit: 'SI'
```

1.2.2 Viewing model with `feplot`

cf, cf.Stack, cf.CStack (pointers and values)

see `sdtweb dfeplot#FEPointers`

The `feplot` function deals with model viewing (see `sdtweb dfeplot`). `cf=feplot(model)` initializes the FE model displayed in the current figure. The model in the figure can then be access via `cf.mdl`, the stack via `cf.Stack` and the case stack via `cf.CStack`. The `fecom` function then gives commands to handle with the visualization options. Command options are more complete than options available through menus, icons and keys (use the `?` key to obtain a list of callbacks). Properties of model may be viewed using `fecom(cf,'pro')` which opens the `feplot` properties figure.

```
% sdtweb demosdt('DemoUbeam')

model=demosdt('demo ubeam mix NoPlot');
cf=feplot(5);   % empty model in figure 5

cf.model=model; % initialize model and display

fecom('pro');   % display property figure

% You should analyze the tabs in the propery figure
```

- materials and element properties, `sdtweb m_elastic` and `sdtweb p_solid` calls.
- boundary conditions `sdtweb fe_case`
- simul (standard solutions) `sdtweb fe_simul`, `sdtweb fe_eig`, `sdtweb fe_time`.

For other interfaces see `sdtweb femlink`.

```
%% Output of commands above

>> cf=feplot
cf =

   FEPLOT in figure 5
     Selections: cf.sel(1)='groupall';
   Deformations: [ { 1122x5} ]
   Sensor Sets: [ 0 (current 1)]
   Axis 1 (ScaleMode=max) objects:
    cf.o(1)='sel 1 def 1 ch 1 ty2 scc -0.15'; % mesh

    cf.o(2)='sel 1 def 1 ch 0 ty4 '; % undeformed

    cf.o(3) % title


>> cf.mdl
v_handle pointer in feplot(5)

       pl: [1 4.1165e-001 2.1000e+011 2.8500e-001 7800 8.1712e+010 2.0000e-002]
       il: [1 6.2437e-004 0 2 0 1 0]
      Elt: [161x10 double]
     Node: [374x7 double]
      DOF: []
    Stack: {'case'  'Case 1'  [1x1 struct]}
      bas: []

>> cf.Stack{'Case 1'}
ans =
    Stack: {4x3 cell}
        T: []
```

```
      DOF: []

>> cf.CStack
ans =
    'DOFLoad'     'Point load 1'     [1x1 struct]
    'DOFLoad'     'Point load 2'     [1x1 struct]
    'FVol'        'Volume load'      [1x1 struct]
    'FSurf'       'Surface load'     [1x1 struct]
>> cf.CStack{'Volume load'}
ans =
     sel: 'GroupAll'
     dir: [1 0 0]
    name: 'Volume load'
```

1.2.3 Controlling views, node and element selection

- orientation can be controled with toolbar. Or `fecom('view3')` calls see `sdtweb iimouse#view`.
- display material properties. `fecom('ColorDataMat')`. Use property figure to view specific parts.
- display node positions `fecom('ShowNodeMark', ... )`. See `sdtweb findnode`.
- display a part of the FEM model. `cf.sel= ...`. See `sdtweb findelt`.

Examples `cf.sel= ...` and `fecom('ShowNodeMark', ... )`.

```
cf=demosdt('DemoGartFE plot')
fecom('ColorDataPro-EdgeAlpha.05-alpha.5')
fecom('shownodemark','y>.5')
fecom('textnode 112','fontsize',20)
cf.sel={'inNode {y<0}','ColorData EvalZ'}
```

1.2.4 Viewing deformations

- `cf.def=def`, see `fecom.html#InitDef`, is the nominal procedure to view deformations is to define a structure with field values in `.def` defined at `.DOF`, see `def`.
- `fecom.html#ColorData` is used to control coloring. In particular this can be used to

1.3 Meshing and model manipulations

There are three main approaches to the definition of the nodal and elemental description matrices: explicit definition, functional definition and import from meshing software (see `sdtweb femlink`). When using the former all nodes and elements are declared individually and explicitly. Functional definition takes advantage of commands which allow the extrusion, repetition, translation of relatively simple models (such as those defined explicitly), enabling more complex models to be assembled.

1.3.1 Explicit definition

When defining nodal and elemental description matrices explicitly, only nodes and element matrices are required as all nodes and elements are defined collectively. Each row in node and element matrices represents a new node or element. The meshing of a simple truss structure will be used as an example - fig 1.3.

```
Node  X(m)  y(m)  z(m)
  1    0     0     0
  2    0     1     0
  3    1     1     0
  4    2     1     0
```

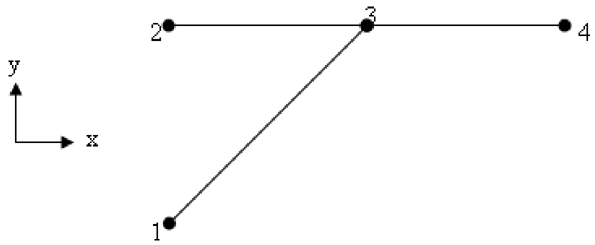


Figure 1.3: Truss structure mesh

The data required per node is `[NodeId PID DID GID x y z]` with

- `NodeId` node identification number
- `PID` coordinate system number for position (0 for global)
- `DID` coordinate system number for displacement (0 for global)
- `GID` group identification number (typically unused)
- `x,y,z` coordinates

At this stage only `NodeId` and the coordinates are required. `PID`, `DID` and `GID` are all set to default values of zero.

```
%           [NodeId PID DID GID x y z]}
model.Node=[1       0   0   0   0 0 0  % Warning: .Node not .node
            2       0   0   0   0 1 0
            3       0   0   0   1 1 0
            4       0   0   0   2 1 0];
```

`model.Elt` can be used to define any type of element. For the truss example only beam elements are required. The first line of the element declaration defines the type of element to be used. Each consecutive row holds the data for a single element.

```
model.Elt - [Element_type_declaration ;
             NodeNumbers MatId ProId EltId OtherInfo]
```

Element_type_declaration is a row of the matrix that defines which elements are described in following rows. The format of this row is [Inf abs('EltName')] where Inf marks a header line, EltName is the element type name (for example beam1, quad4, ...) and abs gives the ASCII value of the element name. See section 1.4 for more details.

NodeNumbers depending on the element type the number of nodes required to define it will vary. For a beam two nodes are required, the start NodeId and the end NodeId (entries must be separated by a space).

- MatId material ID number in model.pl (see sdtweb pl)
- ProId element property (e.g. section area) ID number in model.il (see sdtweb il)
- EltId element ID number which uniquely identifies element (this is rarely needed in SDT and can be fixed using (sdtweb feutil#feutil.EltId)
- OtherInfo additional options. Examples of additional options are beam node off-sets, orientation of bending planes etc.

In this case it can be assumed that all beam sections are identical. MatId and ProId can be set to 1 (only one material and one element property need to be defined), the properties of which will be discussed later. For beams, columns 5-7 specify the orientation and the EltId can be stored in column 8).

```
dir=[0 0 1];
model.Elt=[Inf abs('beam1') 0;
   2 3 1 1 dir ;
   3 4 1 1 dir ;
   1 3 1 1 dir ];
```

A graphical check can be performed using the feplot function - fig 1.4.

```
cf=feplot(model)
fecom('triaxOn'); % Show triax
fecom('textnode','GroupAll','FontSize',12); %node numbers
```

The desired orientation can be selected from the figure toolbar.

It is important to note at this stage that the true number of variables that can be entered for a beam element far exceeds those shown here. When using the explicit definition of elements it is essential that the dimensions of all rows and all columns are equal – this includes the first line. In the case of a beam the command [Inf abs('beam1')] accommodates six columns. All other rows must therefore have six columns. This is why the options column, although not being used, must be defined (as zero in this case) if following rows are larger. Use of the additional options is easier when using a functional definition of the model.

1.3.2 Functional definition of meshes

It is impractical to explicitly define all but the simplest of models. A number of functions are available in the SDT which allow manipulation of groups of elements in the assembly of a larger more complex model. This can be done in a piecemeal fashion, with sections of the complete model being added consecutively.

feutil function performs meshing operations (extrusion, repetitions, meshing some simple parts etc. ...).

The SDT uses command strings which define the specific action that the feutil (see sdtweb feutil) function performs. Typical call is following:

```
model=feutil('command string',model);
```

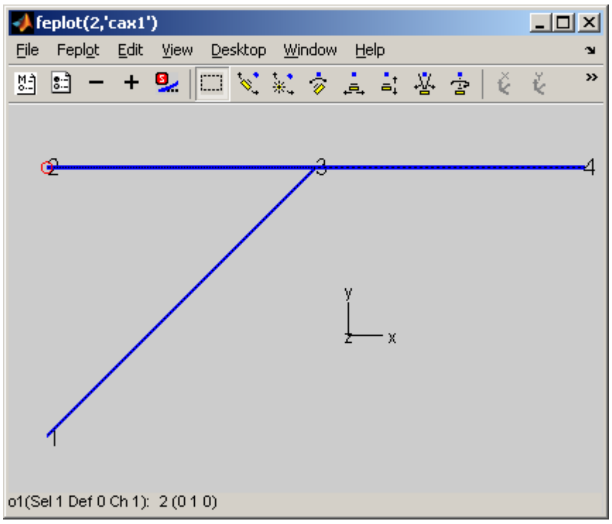


Figure 1.4: SDT plot of truss mesh

Some commands return only the element matrix instead of the whole model struture. Call is then of the form `model.Elt=feutil('command string',model)`.

One can build each part of the model to be meshed using successive call to `feutil` on different data structures `model1`, `model2`, ... and then combine each part using the `feutil AddTest` command (`model=feutil('AddTest',model1,model2);`).

- `AddTest` is a very important command in the meshing strategy as it performs merging or combination of 2 models. Option `Combine` (`AddTestCombine` command) states that some nodes are in common between the 2 models and their numbering is coherent (same node and same NodeId).

  Option `merge` forces detection of common nodes even with non coherent numbering (different NodeId and tolerance on position that can be defined as a command option `Epsl` *`value`*). This command should be preferred when merging 2 parts of the same model (but may have difficulties with distinct but coincident nodes)
  `model=feutil('AddTestMerge',model_part1,model_part2);`

- `RepeatSel`

  Allows repetition of the input model. Required inputs are the number of repetitions required (including the original) and the displacement vector in which the repetitions occur.

  `model=feutil('RepeatSel 10 1 0 0',model);`

  `model` will be repeated 10 times with each repetition separated by 1m in the x-direction. Irregular repetition is possible giving the direction and positions relative to origin `feutil('RepeatSel 0 1 0 0',model,[0 1 3 10]);`

- `TranSel` Allows translation of models. Thus with `model=feutil('TranSel 2 3 1',model);`, `model` is translated by the vector (2,3,1).

- `RotateNode`

  Allows rotation of model nodes about defined origin. Required inputs are origin `NodeId`, angle of rotation and the vector about which the rotation takes place. If no vector is given the default vector (z axis – [0 0 1]) is used. Thus `model=feutil('RotateNode 1 45 1 0 0',model)` rotates the model by 45 degrees about an axis passing by node 1 and the x direction.

`feutil` example 1:

A simple square-section made from beam elements will be manipulated into a 3D truss as an example of the `feutil` commands – fig 1.5.

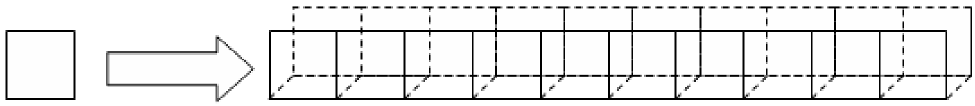


Figure 1.5: 3D truss mesh

An initial definition of the square geometry is required. This is done using an explicit definition.

```
mdl0=struct('Node',[],'Elt',[],'name','primer','unit','SI'); % init model
mdl0.Node=[1  0 0 0  0 0 0 ;
           2  0 0 0  0 1 0 ;
           3  0 0 0  1 1 0 ;
           4  0 0 0  1 0 0 ]; % define nodes
% define 4 beam1 elements
prop=[1 1  0 1 0]; % MatId ProId nx ny nz
mdl0.Elt=feutil('ObjectBeamLine 1 2 0 2 3 0 3 4 1',prop);
feutilb('write',mdl0) % display how to write this model
```

`mdl0` is used as the model shall be built up in pieces. It contains at this step a simple square composed of 4 beams.

The first step is to produce one side of the structure. Two nodes defined at the same coordinate by different elements counts as a single node, and those elements are joined together.

```
mdl0=feutil('RepeatSel 10 1 0 0',mdl0);
```

`mdl0` will now contain information for all ten square sections.

```
model=feutil('addtest',mdl0, ...
  feutil('TranSel 0 0 1',mdl0));
```

`model` now contains the information for both the initial model and translated elements. The final side of the truss is obtained by rotating initial side `mdl0` about a line through the origin parallel to the x axis.

```
mdl0=feutil('RotateSel 0 90 1 0 0',mdl0); % rotate the side
model=feutil('AddTestMerge',model,mdl0); % add rotated side
cf=feplot(model); % plot model
fecom('ColorDataGroup');
```

`model` now contains the full 3d truss model.

`feutil` other commands:

A number of more advanced commands are required as the complexity of the model increases. The following is a partial list of important ones

- `FindNode` (see `sdtweb findnode` for full documentation) selects a group of nodes determined by formal conditions. For example

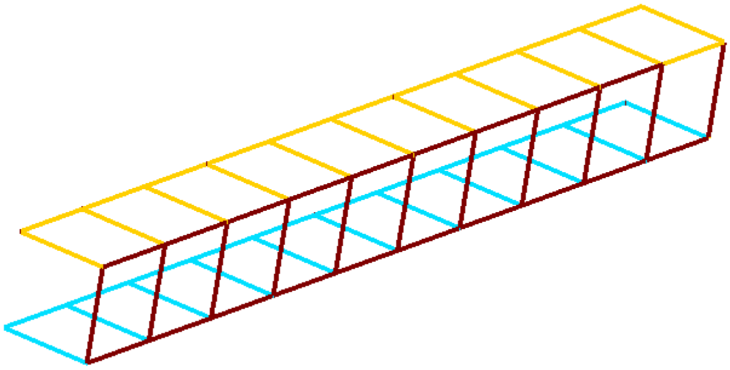


Figure 1.6: SDT plot of a 3D truss mesh

```
NodeId=feutil('FindNode x==0 & y>=1',model);
```

All nodes lying on the line `x=0` and with `y>=1` would be found. For display see `fecom TextNode` and `fecom ShowNodeMark`;

- `FindElt` (see `sdtweb findelt` for full documentation) selects elements trough formal conditions. For display see `fecom Sel`.

- `ObjectBeamLine` Creates multiple or single beams between defined nodes. Other object commands exist of plates, volumes, ...

  `model.Elt=feutil('ObjectBeamLine 1:5',prop)` generates four beam elements from nodes 1-2, 2-3 etc.

- `Extrude`

  Allows extrusion of the selected elements. Required inputs are number of repetitions of the extrusion and the displacement vector in which the extrusions occur. Extrusion changes the element type: mass → beam, beam → quad, quad → hexa, ...

```
model=feutil('Extrude 5 1 0 0',model);
```

If the input model was a mass then the resultant would be 5 beam elements each 1 m long, orientated in the x-direction.
The extrusion command reduces the number of explicit element definitions that the user has to make. For example, when meshing a plate the explicit definition of only two nodes is required.

`feutil` example 2:

The three commands listed above greatly reduce the volume of code required to generate larger meshes. The example of a right angled stiffener will be used. The mesh will consist of `quad4` elements. Each element is 2x1 cm.

Two nodes are defined at coordinates (0,0,0) and (0,1,0). The `ObjectBeamLine` command can now be used to define a beam element represented by the red line on figure 1.7 (nodes 1 and 2).

```
md0=struct('Node',[],'Elt',[],'name','primer','unit','SI'); % init model
mdl0.Node=[1 0 0 0 0 0 0 ;
           2 0 0 0 0 1 0 ];
mdl0.Elt=feutil('ObjectBeamLine 1 2'); % create beam between node 1 and 2
```

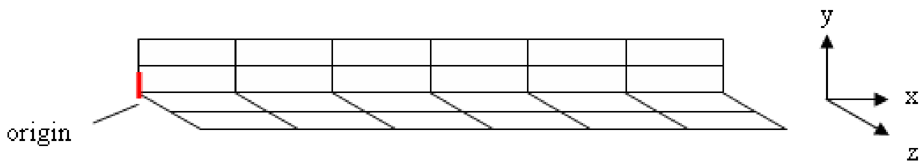


Figure 1.7: Right angled stiffener meshh

This single beam element can now be extruded into plate elements. *Note that the output argument is now a model since new nodes are added by the extrusion.*The six plate elements are then repeated in the y direction.

```
mdl0=feutil('Extrude 6 2 0 0',mdl0); % extrude the beam along x as 6 quads
mdl0=feutil('RepeatSel 2 0 1 0',mdl0); % repeat mdl0
model=mdl0; % first step of final model
```

The second surface must now be meshed. This could be done using rotation but a second example of the advanced commands will be used. The `ObjectBeamLine` command defines beams between all nodes along the line x=0 (obtained with a a `FindNode` command), this produces 6 beams). These are then extruded twice in the z-direction to form the right angled section. An `AddTest` is used to merge models.

```
mdl0.Elt=feutil('ObjectBeamLine', feutil('FindNode x>=0 & y==0',model)); % edge
mdl0=feutil('Extrude 2 0 0 1',mdl0); % extrude the edge
model=feutil('AddTestMerge',model,mdl0); % add mdl0 to model
cf=feplot(model);fecom('ShowPatch'); % plot model
```

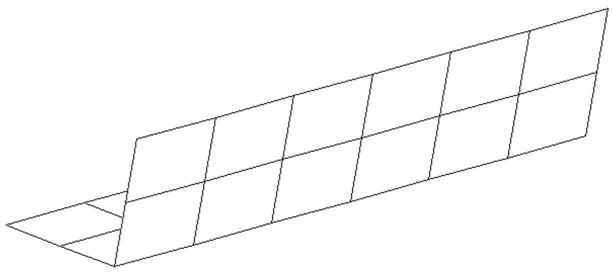


Figure 1.8: SDT plot of right angled stiffener mesh

The SDT plot of the stiffener can be seen in Figure 1.8. The desired orientation can be selected using the figure toolbar.

1.3.3 Automated (free) meshing from geometry (CAD)

While this is not the toolbox focus, SDT supports some free meshing capabilities.

`fe_gmsh` is an interface to the open source 3D mesher GMSH. Calls to this external program can be used to generate meshes by direct calls from MATLAB. Examples are given in the function reference.

`fe_fmesh` contains a 2D quad mesher which meshes a coarse mesh containing triangles or quads into quads of a target size. All nodes existing in the rough mesh are preserved.

```
% build rough mesh
model=feutil('Objectquad 1 1',[0 0 0;2 0 0; 2 3 0; 0 3 0],1,1);
model=feutil('Objectquad 1 1',model,[2 0 0;8 0 0; 8 1 0; 2 1 0],1,1);
feplot(model)
% start the mesher with a reference distance of .1
model=fe_fmesh('qmesh .1',model.Node,model.Elt);
model.name='Qmesh test'; model.unit='SI';
feplot(model);
```

1.4 Elements

1.4.1 Model description matrix

A model description matrix (typically stored in `model.Elt`) describes the model elements. The declaration is done through the use of element groups stacked as rows of a model description matrix and separated by header rows whose first element is `Inf` in Matlab and the following are the ASCII values for the name of the element. The lines following the header row each describe an element. A `beam` element is defined between two nodes

```
mdl0.Elt=[Inf abs('beam1') 0 0;
          n1 n2 MatId ProId vx vy vz EltId];
```

- `n1` – node 1 `NodeId`
- `n2` – node 2 `NodeId`
- `MatId` is the material identifier (see `sdtweb pl`)
- `ProId` is the beam section identifier (see `sdtweb il`)
- `vx,vy,vz` – are three components of the normal defining the bending plane
- `EltId` is a positive integer uniquely identifying each element. The `EltIdFix` command (`[Eltid,model.Elt]=feutil('EltIdFix',model);`) returns a model that verifies the unicity constraint.

A mass element is defined at a node

```
mdl0.Elt = [Inf abs('mass1') ;
           NodeId mxx myy mzz ixx iyy izz EltId];
```

- `NodeId` – node of the mass.
- `mxx`, `myy`, `mzz` – mass components in the directions given.
- `Ixx`, `Iyy`, `Izz` – inertia components about axis given.

Mass elements can also be defined using the `feutil` command `ObjectMass`.

```
mdl0.Elt=feutil('ObjectMass NodeId');
```

A mass is created at the node whose `NodeId` is given.
`mass2` elements allow definition of cross inertias (see `sdtweb mass1` for more details).

1.4.2 Element topologies and problem formulations

SDT distinguishes the topology (the shape of an element defined by its nodes) and the formulation (defined by its material and element properties). Supported topologies are

- point elements (mass1, mass2) to represent concentrated behavior. celas elements are spring between two nodes but one of them can be set to 0 thus leading to a point element.
- 2 node connections: celas (scalar spring or penalized rigid link, see sdtweb celas for details), cbush, beam1, bar1, ...

  rigid elements are a special case of linear constraints that are not really elements since no element matrix exists.
- shells tria3, tria6, quad4, quadb. The element names are typically used for shells but can be used to describe surfaces in 2 and 3D by defining the proper formulations.
- surface elements (or 2D volumes): t3p, q4p, see sdtweb q4p. These are equivalent to the shell topologies but are normally used to distinguish surfaces that are not shells.
- volume elements: tetra4, hexa8, penta6, ... (see sdtweb hexa8).

Formulations (see sdtweb feform) include elasticity 2 and 3D, acoustics, heat equation, piezoelectricity, laminated plate theory and more.

1.4.3 Identifier manipulations in element definitions

When defining elements using the feutil command a number of fields required may not have been entered. The ObjectMass command is a good example: only the NodeId is requested. The first few lines of the final model.Elt from feutil example 1 (see 1.3.2 above) are shown below.

```
[Inf abs('beam1')
  1  23  1  1  0  0
 23  25  1  1  0  0
 25   4  1  1  0  0
  4   1  1  1  0  0
  4  25  1  1  0  0
 25  28  1  1  0  0]
```

The first line holds a declaration of the element type and represents the Inf abs('beam1') command. The lines below this hold the data for each element. Each cell in the table can be selected and changed as the user wishes.

```
i1=feutil('FindElt EltName beam1'); % Indices of elements
model.Elt(i1,3)=2;                  % change MatId in column 3
```

This command would set the value of rows two to end of column three to 2. The material property has been changed from 1 to 2. The components a row describing and element have already been defined:

```
NodeNumbers MatId ProId EltId OtherInfo
```

The OtherInfo heading represents a group of columns which are only used by certain elements or formulations. These are therefore described in the associated help files. For example beam1 uses additional information to specify the section orientation.

1.5 Material and element properties

1.5.1 Material Properties

Materials are typically defined as rows of a `pl` matrix stored as the `model.pl` field (they can also be stored as `mat` stack entries in cases where a lot of data is needed : for temperature sensitivity for example).

Each material formulation should be associated with a `m_` or `p_` function. `m_elastic` for elasticity and acoustics, `m_piezo` for piezoelectric materials, `p_heat` for the heat equation, ...

High level calls use these functions which contain a databases. Typical high level calls are

```
m_elastic info % prints the database to the screen
model.pl=m_elastic('dbval 1 Steel')
model.pl=m_elastic(model.pl,'dbval 2 Aluminum')
```

These calls build `model.pl` where each row of the matrix represents a different material. The definition below is for standard isotropic materials.

Default units are SI, but you can use `m_elastic('dbval 1 -unit IN Steel')` for a conversion, see `sdtweb fe_mat#Convert`

```
pl=[MatId  E nu rho G eta alpha T0]
```

- `MatId` – material ID number as defined by user.
- `type` – type of material being used, typically `fe_mat('m_elastic','SI',subtype)`. In the present case should be `fe_mat('m_elastic','SI',1)`
- `E` – Young's modulus.
- `nu` – Poisson's ratio.
- `rho` – density.

These five are the only required inputs – the following four are optional and are set to their defaults if omitted.

- `G` – shear modulus (default set to $G = E/2(1 + nu)$).
- `eta` – hysteretic damping loss factor (default set to 0).
- `alpha` – thermal expansion coefficient.
- `T0` – reference temperature.

For example the material definition matrix for aluminum in SI units would be

```
model.pl=[1 fe_mat('m_elastic','SI',1) 7.2e9 0.3 2700]
```

1.5.2 Section Properties

The mathematical formulation of finite element approximations typically involves other parameters than just material properties. In the case of volumes, this is typically limited to the choice of the integration rule. For beams and shells, geometry parameters are also required. The property matrix stored in the `model.il` stores on property per row (unit conversion is supported by `fe_mat convert`.

- **Beam**
  High level calls use the `p_beam` function which contains a database. Typical high level calls are

  ```
  model.il=p_beam('dbval 1 rectangle .05 .01')
  model.il=p_beam(model.il,'dbval 2 circle 01')
  ```

  It is possible to convert to a desired unit system using command options `-unit` or `-punit`.

  - `-unit` command option converts the property *afterwards*, that is to say the property is generated in `SI`, and expects an input in `SI` before performing the conversion.
  - `-punit` command option only takes into account the desired system, and directly generates the property in the customized system. The input data must thus directly be in the desired unit system

  The following calls are equivalent, to define the previous model property but in the TM unit system:

  ```
  model.il=p_beam('dbval -unit TM 1 rectangle .05 .01') % Generate in TM, give data in SI
  model.il=p_beam('dbval -punit TM 1 rectangle 50 10') % Generate in TM, give data in TM
  ```

  These calls build `model.il` where each row of the matrix represents a different beam property.

  ```
  il=[ProId type J I1 I2 A k1 k2]
  ```

  - `ProId` – element property ID as defined by user.
  - `type` – type of property, typically, `fe_mat('p_beam','US',1)`.
  - `J` – torsional stiffness.
  - `I1` – second moment of area about bending plane 1 (for beam on x axis bp 1 in plane x-y).
  - `I2` – second moment of area about bending plane 2 (for beam on x axis bp 2 in plane x-z).
  - `A` – area.

  These six are the only required inputs. The shear correction factors can be omitted or set to zero if not desired (in this case the Euler-Bernoulli formulation is used)

  - `k1` – shear correction factor for bending plane 1.
  - `k2` – shear correction factor fort bending plane 2.

- **Plate**
  High level calls use the `p_shell` function which contains a database. Typical high level calls are

  ```
  model.il=p_shell('dbval 1 mindlin .01')
  model.il=p_shell(model.il,'dbval 2 kirchhoff 01')
  ```

  These calls build `model.il` where each row of the matrix represents a different shell property.

  ```
  il=[ProId type f  d  0  h  k]
  ```

  - `ProId` – element property ID as defined by user.

– `type` – type of property, typically, `fe_mat('p_shell',1,1)`.
– `h` – plate thickness.
– `f` – optional formulation selection.
– `d` – optional selection of stiffness coefficient for drilling DOF.
– `k` – shear correction factor for element formulations that support its use.

It is worth noting that a standard Matlab matrix must have the same number of columns in each row and vice versa. Care must be taken to ensure that this condition is not violated when storing properties of elements of different types in the same matrix. Additional rows or columns can be filled by entering a zero. Other elements will be described further in later sections. For example:

```
il=[1 fe_mat('p_beam','SI',1)  1e-6 1e-7 1e-7 1e-4   0.8 ;
    2 fe_mat('p_shell','SI',1) 1    1    0    5e-3   0 ];
```

The first row – element section property 1 (first column) – is a beam section definition. The second row is a plate definition. As the shear correction factor for plane 1 has been included for the beam property the final column of row two must be filled if all rows are to be of the same dimension.

- **Spring** and dashpot properties are supported with `p_spring`.
- `p_solid` supports integration rule selection for all elements that have all other properties defined in the material entry.

1.6 Loads and boundary conditions

All the loads and boundary conditions are handled with the `fe_case` function and currently only one global case is used at one time.

Before a model can be solved the boundary conditions must be considered.

The `fe_case` functions allows boundary conditions and loads to be declared. Each declaration is stored as a different entry of the case stack, allowing multiple conditions to be applied. Any declaration applies to the `model` data structure and uses the `fe_case` format

```
model=fe_case(model,command1,command2,  ...)
```

with commands following the format

```
'EntryType','Entry Name',Data
```

To understand the calls, a little information of degree of freedom coding is needed.

1.6.1 Degrees of freedom

Each node has up to 99 degrees of freedom. By convention, the first 6 DOFs are three translations and rotations (for other conventions see `sdtweb mdof`).

x-trans	y-trans	z-trans	x-rot	y-rot	z-rot	p	T	...
.01	.02	.03	.04	.05	.06	.19	.20	...

DOFs are coded with an integer part giving the node number and the first two dgits after the decimal giving the DOF number `NodeId.DofId`. Thus 220.06 is the rotation of node 220 about the z-axis. When manipulating matrices or results, DOFs are stored in a DOF definition vector (`def.DOF`, `model.DOF`, ...).

Before solving the model the active DOFs are resolved based on the element topologies and formulations by calling `model.DOF=feutil('GetDof',model)` (you really don't have to do it by yourself).

`fe_c(DOF)` returns a cell array giving the meaning of DOF in a vector. `ind=fe_c(DOF,adof,'ind')` is used to find the location of DOF âdof in vector `DOF`. `fe_c` supports a number of other DOF manipulations.

1.6.2 Fixed boundary conditions

The entry type is FixDof (declared DOFs fixed). The name is arbitrarily defined by the user and is used as a reference to the boundary condition being applied. The data holds the list of DOFs that the command will apply to. There are a number of options on the type of data that can be entered.

- A global declaration of DOFs can be given as a column vector. For example `[.03 .04 .05]'` would activate a 2-D simulation with only DOFs in x-trans, y-trans and z-rot being active because all z-trans, x-rot and y-rot DOF are fixed.
- A specific declaration of DOFs can be given as a column vector . For example `[1.04 7.01 12]'` will fix x-rot at node 1, x-trans at node 7 and all DOFs at node 12).
- A `FindNode` command can be given as a string (see sdtweb FindNode for details). For example

  ```
  model=fe_case(model,'FixDof', 'LeftEdge', 'x==0 -DOF 2 3');
  ```

  fixes DOFs 2 and 3 (translation y and z) along the line of nodes that verify `x=0`. In the structures defined in the examples above this represents the left edge, hence the name given.

1.6.3 Loads

DofLoad entries are used to apply loads on specified DOFs. When giving a list of DOFs, a unit load is applied to each DOF (there are as many loads as DOFs in the list). To apply combined loads (a single load on multiple DOF), you need to define a set of values on these loads through a structure

```
data=struct('DOF', Dof_declaration, 'def', load_array);
```

The DOF declaration is a column vector of the DOFs to which the loads apply. The load array contains a number of column vectors, each of which represents a different loading condition. Thus for

$$DOF = \left\{ \begin{array}{c} 1.03 \\ 2.01 \\ 3.01 \end{array} \right\} \quad load = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \tag{1.1}$$

DOFs 1z, 2x and 3x are being loaded. The first column of the def array returns a unit load on DOF 1z. The second column returns a positive unit load at DOF 2x and a negative at DOF 3x in the x, this is typical of a relative load.

For distributed loads, see sdtweb fe_load FVol and FSurf.

1.7 Solving

There are a number of solver functions. fe_simul provides high level integrated solutions that combine model assembly and resolution for static, modal and time and frequency responses. More specific solvers fe_eig for eigenvalues, fe_time for time domain or non-linear statics, fe2ss for state-space model building or fe_reduc for model reduction provide more specialized calls with more options.

1.7.1 fe_simul generic integrated solver

The fe_simul function is the generic function to compute various types of response. Once you have defined a FEM model, material and elements properties, loads and boundary conditions calling fe_simul assembles the model (if necessary) and computes the response using the dedicated algorithm. The generic call is

```
[def,mo1] = fe_simul('command',model)
```

Where model is your FE model data structure and def is the deformation result (data structure detailed in the post-processing section).

Accepted commands are 'Static', 'Mode', 'Time', or 'Dfrf'.

To control the assembly steps use the optimized assembly strategies (see sdtweb simul#s*feass) or low level fe_mknl calls.

1.7.2 fe_eig real eigenvalue solution

The fe_eig function returns the eigendata – including both mode shape and natural frequencies – in a structured matrix with fields .def for shapes, .DOF to code the DOFs of each row in .def and .data giving the modal frequencies in Hz. The model matrix is the required input.

```
eigopt=[SolutionMethod nm Shift Print Thres];
def=fe_eig(model,eigopt);
```

Of the five possible options only the first two are required. The final three should be set to zero (the default values will be assigned).

- SolutionMethod – integer defines solution method. Default method is a full solution which can only be used for simple models. For larger models use Lanczos solver 5.

- nm – number of modes required. The default value is a full analysis i.e. one mode for every degree of freedom.

1.7.3 fe_time full model transient and NL analysis

The fe_time function allows analysis of the transient or non-linear response. It returns the deformation data structure def and the model data structure (optional). Each column of the def.def array represents a time step or non-linear increment and the rows represent the DOF.

```
[def,model] = fe_time('command',model)
```

command – type of solver used. Newmark or dg (Discontinuous Galerkin) for example. You can also use a call of the form

```
model=stack_set(model,'info','TimeOpt',data)
[def,model] = fe_time(model)
```

with time options in data having at least fields .Method and .Opt .
TimeOpt.Method – string defining time integration algorithm ('Newmark', 'dg'...).
TimeOpt.Opt – line vector containing numeric options. For Newmark algorithms [beta gamma t0 dt Ns] with beta=0.25, gamma=0.5 as defaults, t0 – Beginning of time simulation, dt – Time step, Ns – Number of steps to be computed.

For example:

```
model=stack_set(model,'curve','q0',[]); % no initial deformation
TimeOpt=struct('Method','newmark', ...
              'Opt',[.25 .5 0 .1 20]);
model=stack_set(model,'info','TimeOpt',TimeOpt);
def=fe_time(model);
```

Newmark solver used for model case 1. Zero initial conditions. Default values of beta and alpha used. Simulation begins at t=0, the time step is 0.1 s, there are 20 cycles computed.

1.7.4 nor2xf, modal frequency response

The nor2xf function allows FRFs to be evaluated using a modal basis. A required input is a set of modes (as calculated by fe_eig). The location of both sensor and loading point by which it is excited are also required. A case must therefore be defined, but it is done in a slightly different way.

```
load=struct('DOF',[14.03],'def',[1]);
Case=fe_case('DofLoad','loading',load,'SensDof','sensor',[14 15 16]'+.03);
```

As the deformations have already been calculated the model matrix is not needed. Loading and sensor placement are given as before. Before the response can be calculated the frequency points at which it is required must be given. This is commonly done using the Matlab linspace function. The range of values (start and end) and the total number of data points are inputted.

```
freq=linspace(start,end,no_points);
xf=nor2xf(def,damping,Case,freq,'command');
```

- def – deformation matrix. This is obtained using the fe_eig function.
- damping – damping ratio (0-1).
- Case – load case and sensor placement definition.
- freq – points at which frequency response calculated.
- command – optional command used to define units of frequency response.

For example:

```
freq=linspace(5,70,500);
nor2xf(def,.01,Case,IIw,'Hz iiplot -po "CurveName"');
```

Response measured at 500 equally spaced frequency points between 5 to 70 Hz (the units are defined by the Hz in the command). A damping ratio of 0.01 is used. The iiplot -po "CurveName" appends a dataset CurveName to the current iiplot figure (see section 1.8.2 or sdtweb diiplot).

1.8 Post-processing

1.8.1 feplot

The feplot function allows geometry display and deformation animation for both analysis and test. In order to manipulate the feplot you must open the figure and the associated GUI, display a model and possibly deformations. A typical call is thus

```
cf=feplot(2); % open feplot in figure 2, store handle in cf variable
cf.model=model; % store model in the figure that cf points to
cf.def=def; % display deformation in this figure
```

cf=feplot(model,def) is a shortcut declaring model and deformations in a single call. Declaring a figure number is useful to force multiple feplot figures. The figure stores data using data structures detailed below.

- model=cf.mdl is a pointer to the model (see sdtweb model) stored in the figure
- def=cf.def deformation data structure (see sdtweb def). Fields are .DOF field (column vector), a .def field (matrix) whose number of rows is the same as .DOF vector and each column is a set of deformation (one modal deformation or one deformation at one time step, ...) and .data with as many rows as .def columns defining time steps, frequencies, ...

In this example an animated plot of natural frequencies and mode shapes follows from fe_eig

```
model=femesh('Testubeam'); % simple ubeam example model
def=fe_eig(model,[5 10 1e3]);
cf=feplot(model,def)
```

The def structure holds mode shape data for the first 10 natural frequencies (including rigid body modes here). Having a figure handle cf, one can use cf.def=def to show def on the model displayed, and cf.def=[] to reset them.

In this other example a time response is considered

```
model=fe_time('demo 2d');
model=fe_time('TimeOpt Newmark .25 .5 0 1e-4 50 10',model);
def=fe_time(model); % compute the response
cf=feplot(model,def);
fecom('AnimTime')
```

The AnimTime command switches animation mode (time scans trough steps, while frequency uses a complex scaling factor). Scaling of the amplitude is often necessary with the time response, this can be done using the feplot GUI button , the l,L key (see sdtweb iimouse#key). Alternatively the fecom commands can be used in the code to make changes programmatically (see sdtweb fecom).

1.8.2 iiplot

iiplot supports frequency/time data viewing. The data is stored into the figure and can be accessed through a pointer ci=iiplot;ci.Stack, see more details with sdtweb diiplot#CurveStack. The function support data superposition, scanning through multiple channels, signal processing (illustrated in the complete example below), ... Use the properties button to open the GUI tabs giving you graphical access to a wide range of capabilities and see sdtweb iicom for a more complete list.

1.9 A complete example

The following example illustrates all functions and commands that have been introduced to this point. The structure being modeled is the right angled stiffener already described.

```
% Mesh ------------------------------------------------------------
mdl0=struct('Node',[],'Elt',[],'name','primer','unit','SI'); % Init
% initial node declaration
mdl0.Node=[1 0 0 0  0  0 0;
           2 0 0 0  0 .1 0];

mdl0.Elt=feutil('ObjectBeamLine',feutil('FindNode x==0 & y<=1',mdl0));
mdl0=feutil('Extrude 6 .2 0 0',mdl0);
% beam element created and extruded to make first row of plate elements
mdl0=feutil('RepeatSel 2 0 .1 0',mdl0);
% plates repeated to make one side of stiffener
model=mdl0;
% elements in temp mdl0 are put in model as group 1

mdl0.Elt=feutil('ObjectBeamLine',feutil('FindNode x>=0 & y==0',mdl0));
% beam elements created between nodes along line y=0
mdl0=feutil('Extrude 2 0 0 .1',mdl0);
% beam elements extruded to plate elements
model=feutil('AddTestMerge',model,mdl0);
% merge model and mdl0, mdl0 is in group 2 of new model

% Material and element properties ------------------------------------

model.pl=[1 fe_mat('m_elastic','SI',1) 7.2e9 .3 2700 2.68e10]; % material
model.il=[1 fe_mat('p_shell','SI',1) 0 0 0 5e-3]; % plate properties
% force MatId and ProId of both groups to be 1
model.Elt=feutil('SetGroup 1 mat1 pro1',model);
model.Elt=feutil('SetGroup 2 mat1 pro1',model);

% Loads and boundary conditions ---------------------------------------

load=struct('DOF',[14.03],'def',[10]);
% load condition defined at node 14 orientated along z axis

model=fe_case(model, ...
    'FixDof','left edge','x==0',... % fix edge x==0
    'DofLoad','endload',load,...    % apply load on 14z
    'SensDof','tipsensor',[14.03]); % place sensor on 14z

% Normal modes ---------------------------------------------------------
cf=feplot(model); model=cf.mdl;

cf.Stack{'info','EigOpt'}=[5 10 1e3];

% deformation calculated for model
% options: 5 - Lanczos solver
```

```
%           10 - number of modes
%           1e3 - shift (needed with rigid body modes)

d1=fe_simul('Mode',model); cf.def=d1;

% Time simulation and signal processing --------------------------------
cf.Stack{'curve','q0'}=[];
% initial conditions for time response set to zero
% define options
cf.Stack{'info','TimeOpt'}=...
struct('Method','newmark', ...
'Opt',[.25 .5 0 1e-3 10000], ... % beta,gamma,t0,dt,N
'NeedUVA',[1 1 0]); % compute disp, vel, not acc
% Generate a step over the first ten time steps with the load
cf.CStack{'endload'}.curve={fe_curve('TestStep t1=1e-2')};
% Launch the simulation
def=fe_simul('Time',model);
cf.def=def;
fecom(';colordataEvalZ;view3;viewh+180');
fecom('AnimTime2'); % animate displacement, using 1 every 2 time steps

def.name='Time';ci=iiplot(def); % Do some signal processing
ii_mmif('fft',ci,'Time');
iicom(ci,'iix:fft(Time)Only');
```

2

Experimental modal analysis

Contents

Experimental modal analysis has the objective of characterizing modal properties (poles and mode-shapes) trough experiment, this tutorial will focus on the case where inputs are measured, but output only methods are also useful in many cases.

2.1 Testing

This is a very crude summary of the many issues associated with testing that are analyzed in more detail in classical books [1, 2] or the course notes [3] available at `www.sdtools.com/pdf/PolyId.pdf`.

2.1.1 Measuring transfer functions

Individual transfer functions are measured using a setup similar to that depicted in figure 2.1.

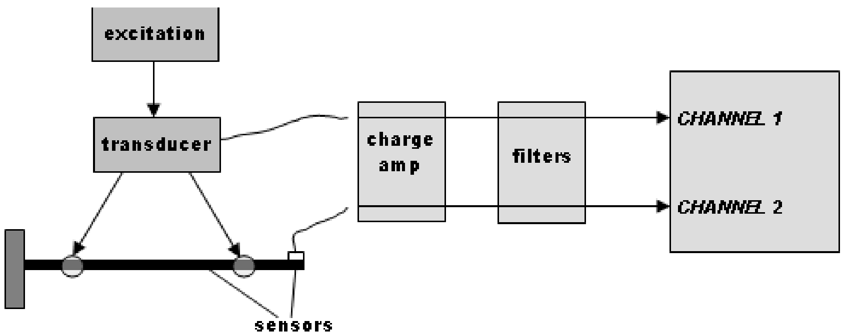


Figure 2.1: Test setup illustration

The output of the transducer (load cell) and sensors (accelerometer, vibrometer, ...) will be a voltage or a charge. The most common produce a charge and require a charge amplifier to convert the signal to a voltage. The analyzer will be a separate piece of hardware or a card in the PC.

Each test will involve one excitation location and one or more sensor location. The time response of both force transducer and sensor are measured and filtered in a similar way. From these power spectral densities are derived through averaging (strongly recommended in most cases) and transfer functions are computed.

The easiest excitation to implement is usually based on a hammer with force transducer used to measure excitation and an accelerometer is used to measure the response. The energy delivered by a hammer during and impact excitation has a frequency spectrum as shown in fig 2.2.

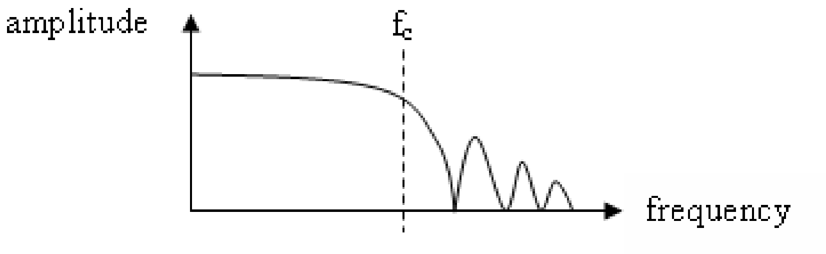


Figure 2.2: Frequency spectrum of hammer excitation

f_c is the cut-off frequency – the hammer cannot be relied upon to excite natural frequencies beyond this point. The type of tip controls the cut-off frequency. A softer tip has a lower f_c. It is recommended that the softest possible tip is used to ensure that the majority of the energy excites the structure in the frequency range of interest.

In shaker excitation, the input is driven by a signal generator (nowadays this is generally a digital to analog converter). It is important to note that the signal generator and shaker is a dynamic system in itself, so the output of the shaker will not match the signal exactly. The output of the shaker must therefore be monitored using a force transducer (except in the case of current driven shakers).

Typical signals considered with a shaker are sine, chirp or pseudo-random noise. In `SDT`, these signals can be generated using `fe_curve` commands.

2.1.2 Multiple locations to get shapes

A single shaker/sensor pair does not give access to shapes which correspond to the relative amplitude of motion at multiple points. To obtain shapes one considers multiple-sensors or input locations (typically by impacting a range of points while keeping a sensor fixed).

To scale modeshapes properly (obtain a modal mass) a driving point FRFs (also called collocated measurement) is needed. This corresponds to cases where the displacement corresponding to the applied load is measured (same location and direction).

Test validation typically requires the visualization of modes and correlation with finite element results a sufficient number of sensors to properly distinguish modes. Sensor placement is thus crucial and addressed in the `fe_sens` commands.

2.2 Identification (mode extraction)

2.2.1 Importing measurements into iiplot, idcom

Identification is performed using the `iiplot, idcom` interface in much the same way as FE model plots use the `feplot` function. See `sdtweb diiplot` to learn about features of the interface.

Starting from scratch, you can use a script of the form

```
ci=idcom % Open idcom interface
XF1=struct('w', f, ...   % frequencies in Hz
           'xf',[col_1 col_2 col_3] ... % responses (size Nfreq x n_io_pair)
            );
ci.Stack{'Test'}=XF1;          % sets data and verifies
```

DOF data

Sensor and excitation information will also be needed for shape display (not for identification itself). If not imported from universal file, this data can be manually defined by the user in `ci.Stack{'Test'}.dof`.

```
ci.Stack{'Test'}.dof=[response DOF, excite DOF, channel options]
```

The format of the response and excitation DOF follows that in section 1.6.1. Each row of `ci.Stack{'Test'}.dof` holds data for a different FRF. The channel is an integer label which links the DOF data to the corresponding frequency response (held in `ci.Stack{'Test'}.xf`). There are additional options that are not required at this stage.

Standard datasets

idcom uses standard datasets (for idcom the names are mandatory, but you can have other sets in the same figure)

- Test measured transfer functions (main fields are .w frequencies, .xf responses, .dof sensor actuator definitions), see sdtweb('curve#Response data') for more details).
- IdFrf last identification result obtained using idcom commands
- IdMain principal set of identified modes, main fields are .po poles (first column with frequencies, second with damping ratio) and .res residues (one row per pole), see sdtweb('curve#Shapes at DOFs') for more details).
- IdAlt alternate set of identified modes

These datasets are stored in the figure and more easily accessed by name

```
ci=idcom % obtain pointer to the figure
ci.Stack{'Test'} reference a dataset by its name
ci.Stack{'Test'}.dof(:,1) % get input DOFs
```

Identification options

Options relevant for the identification can be set in the idcom IDopt tab. They should be modified graphically or using the ci.IDopt pointer. Typical values are shown below

```
>> ci=idcom; ci.IDopt
  (ID options in figure(2)) =
   ResidualTerms : [ 0 | 1 (1) | 2 (s^-2) | {3 (1 s^-2)} | 10 (1 s)]
   DataType : [ disp./force | vel./force | {acc./force} ]
   AbscissaUnits : [ {Hz} | rd/s | s ]
   PoleUnits : [ {Hz} | rd/s ]
   SelectedRange : [ 1-3124 (4.0039-64.9998) ]
   FittingModel : [ Posit. cpx | {Complex modes} | Normal Modes]
   NSNA : [ 1 sensor(s) 24 actuator(s) ]
   Reciprocity : [ {Not used} | 1 FRF | MIMO ]
   Collocated : [ none declared ]
```

Typical script modifications for data imported manually would be (see sdtweb idopt for more details)

```
ci.IDopt.Residual=3; % Force low and high frequency residual
ci.IDopt.DataType='Acc'; % if acceleration was not set
ci.IDopt.Fit='Complex';  % Use normal modes
ci.IDopt.NSNA=[24 1]; % declare number of sensors/actuators
```

Learn more

Further details on import are discussed in sdtweb diiplot#xfread for both cases with raw vectors of frequencies and responses and cases where universal files are available (when this is the case, more info is already available and should be imported).

Results can be saved with idcom('CurveSave') or File:Curve save ... menu. They can be reloaded with ci=iicom('CurveLoad','FileName.mat'), or File:Load curves from ... menu.

2.2.2 Identified modal model

Identification is the process by which a mathematical representation of FRF in the form of a series of modal contributions is obtained. The nominal spectral decomposition is associated to complex modes and leads to a representation of the transfer function of the form

$$[\alpha(s)] = \sum_{j=1}^{2N} \left(\frac{[R_j]}{s - \lambda_j} \right) \tag{2.1}$$

Each row of `ci.Stack{'IdMain'}.po` represents a pole λ_j. Except when forced otherwise, the first column contains the frequency (magnitude of pole) and the second column the damping ratio.

Each row of `ci.Stack{'IdMain'}.res` represents mode shape R_j. Each column represents a different response channel i.e. a different actuator/sensor pair.

2.2.3 Single mode peak picking method

The modal model is constructed using FRFs. There are a number of methods used to extract this data, the method preferred in SDT is a gradual building of the model using sequential peak pickings, followed by refinement.

-3dB method, a reminder

An FRF for a multi-degree of freedom (MDOF) system will contain a number of peaks. Each peak represents a resonant frequency of a particular mode. It can be assumed that the influence of other modes in this region is minimal. The SDOF method utilizes this assumption by treating each mode independently.

For an isolated mode and a velocity measurement, the natural frequency corresponds to the frequency of the peak amplitude $|\hat{H}|$. The values ω_a and ω_b of points at -3dB amplitude can be used to calculate the damping ratio:

$$2\xi_r = \eta_r = \frac{\omega_a^2 - \omega_b^2}{2\omega_r^2} \approx \frac{\Delta\omega}{\omega_r}$$

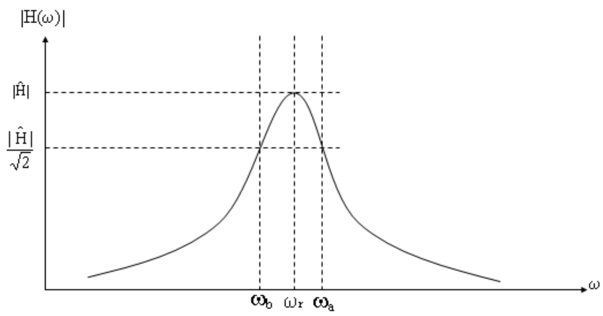


Figure 2.3: SDOF peak-amplitude method

The residue can then be derived.

Each mode on the FRF is considered in turn. The minimum data requirement is a single point FRF, and multiple transfer FRFs for all locations required in the test mode shape. It is advisable to use more FRFs than the minimum (both point and transfer) to improve the mode shape accuracy.

The SDT has built in functions to aid identification of the modal model.

`idcom e`

The `idcom('eband frequency')'` command is a single pole estimator that is similar to the -3dB method but uses all transfers in `Test` simultaneously and allows for the presence of nearby modes. You are expected to provide a frequency bandwidth trough *band* (number of points if ¿1, or fraction of center frequency if ¡1) and a center frequency for the search through *frequency*.

With no argument, `idcom('e')` uses the current bandwidth set in the `idcom` figure (default to 1%) and waits for a graphical input of the frequency in the `iiplot` figure.

Once a pole has been estimated it will appear on the `iiplot` as a `IdFrf` green curve over the test data. If the estimate is accurate (this should determined by visual inspection) it should be added to the main set of modeshapes `ci.Stack{'IdMain'}` with the `idcom('ea')` command or right arrow in the GUI.

A typical example would be

```
ci=idcom('curveload gartid');
idcom('e .01 6.5');  % Estimate a first mode
idcom('ea')          % Move it to the main set of modes
idcom('e .01 34');   % Estimate a second mode
idcom('ea')          % Move it to the main set of modes
idcom('TableIdMain') % Display poles in IdMain
```

2.2.4 Multi-mode estimation and refinement method

Once a sufficient number of modes are identified with peak picking, the next step is to obtain a broadband model with multiple modes.

`idcom('est')` or the associated button will estimate the modes using the whole data currently selected (shown in `iiplot`). When the estimate is not accurate enough you should learn how to tune the poles using the `eopt` and `eup` commands/buttons (see `sdtweb idrc` for details).

When dealing with data that is not very clean, you may find useful to estimate modeshapes using only a fraction of the measured data around each resonance. This obtained with `idcom('estLocalPole')`.

With both commands shapes and poles are stored in `ci.Stack{'IdMain'}` while synthesized responses are in `ci.Stack{'IdFrf'}`.

2.3 Test geometry and visualization

2.3.1 Wire frame model

A wire frame model of the test structure is generated to visualize (view and animate) test shapes with no need of an underlying FE model. The wire frame model maps out the geometry of the test structure and should encompass all test node locations. As an example consider a helicopter airframe with sensor locations as shown in fig 2.4. The wire frame model must capture the motion of all sensors to enable later comparison with the finite element mesh.

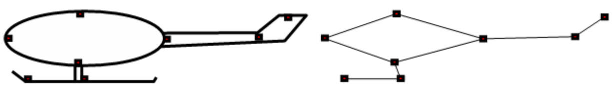


Figure 2.4: Wire frame model

A wire-frame geometry is composed of nodes and elements. It is important to remember that the elements used for the representation have no mechanical meaning. Steps for the definition of a wire-frame model are (see `sdtweb pre` for more details).

- Declaration of nodes these should correspond to all the locations where a sensor is placed along with a number of additional nodes which aid visualization. Nodes can be declared in a script (see the `gartte` demo for example) or added graphically in a `feplot` figure.

  Input argument of `fecom AddNode` is a standard model node matrix (with 7 columns, `[NodeId 0 0 0 x y z]`), for example `fecom('AddNode',[1001 0 0 0 1 0 0; 1002 0 0 0 2 2 2]);`. One can also only give a 3 column matrix with `x y z` positions but `NodeIds` are then assigned automatically. With no input, a dialog box opens that allows cut and paste (from Excel for example).

- Declaration of connectivity. Classically lines are used, with a call of the form `fecom('AddLine',L)` with the `L` array holding the `NodeId` numbers for those nodes being connected, the line will be continuous between all nodes given unless separated by a 0. For example

  ```
  L=[1020 1023 1034 0 1012 1034 1039];
  fecom('AddLine',L);
  ```

  represents two lines between nodes: 1020-1023-1034 and 1012-1034-1039.

  In the example below additional nodes (1003, 1007, 1009 ...) are introduced to obtain a better modeshape visualization. But response at these nodes is not measured, extrapolation (called *expansion*) for proper animation will be discussed later.

- Declaration of sensors. In the simple case of sensors in global directions, just provide the `NodeId.DofId` format. If you have more general configurations, look the translation sensor documentation (`sdtweb sensor#trans`)

The right angled stiffener clamped at one end will be used in this example (see fig 1.8). `sdof=fe_sens('mseq 10 type',def)` is used to determine optimum locations for 10 sensors, leading to

```
 sdof=[35.02 ; 21.03 ; 18.03 ; 32.02 ; 19.03 ; 33.02 ;
       16.03 ; 30.02 ; 35.03 ; 21.02]+1000;
```

The following lines give a typical test setup script.

```
if ishandle(2);delete(2);end;cf=feplot(2);
% 1. Define nodes
node=[1001 0 0 0   0 0 0   ;  1003 0 0 0 0.2 0 0
      1007 0 0 0 0.6 0 0   ;  1009 0 0 0 0.8 0 0
      1013 0 0 0 1.2 0 0   ;  1015 0 0 0 0 0.2 0
      1016 0 0 0 0.2 0.2 0;  1018 0 0 0 0.6 0.2 0
      1019 0 0 0 0.8 0.2 0;  1021 0 0 0 1.2 0.2 0
      1029 0 0 0 0 0 0.2   ;  1030 0 0 0 0.2 0 0.2
      1032 0 0 0 0.6 0 0.2;  1033 0 0 0 0.8 0 0.2
      1035 0 0 0 1.2 0 0.2];
fecom('AddNode',node)

% 2. Define connectivity
% define straight edges
L=[1001 1003 1007 1009 1013]; fecom('AddLine',L);
L=[1015 1016 1018 1019 1021]; fecom('AddLine',L);
L=[1029 1030 1032 1033 1035]; fecom('AddLine',L);
```

```
% Define 5 L shaped edges as single 4th group
L=[1015 1001 1029 0 1016 1003 1030 0 1018 1007 1032 0 ...
   1019 1009 1033 0 1021 1013 1035 0]; fecom('AddLine',L);

% 3. Define and show sensors
 sdof=[35.02 ; 21.03 ; 18.03 ; 32.02 ; 19.03 ; 33.02 ;
       16.03 ; 30.02 ; 35.03 ; 21.02]+1000;
cf.mdl=fe_case(cf.mdl,'SensDof','Test',sdof);
fecom('CurtabCases','Test');fecom('ProviewOn');
fecom('TriaxOn');fecom('TextNode','GroupAll','FontSize',12)
```

The lines have been added to `cf.mdl` with an element group ID of `-1`. This distinguishes them from the real elements. The wire frame is refreshed after each `fecom` call. The additional commands generate the plot shown in fig 2.5.

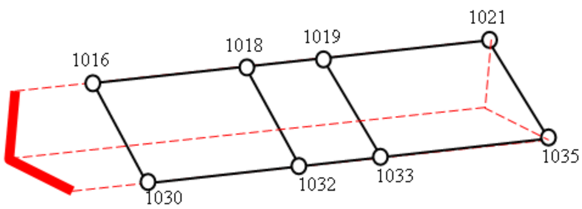


Figure 2.5: Test node and sensor locations

2.3.2 Sensor placement

The following section refers to sensor location, but the directions given apply equally to impact location. A good test setup will enable all mode shapes in the desired frequency range to be defined, with the minimum of response measurements. A FE model can be analyzed before testing to ascertain which DOFs will give the most complete picture of the mode shape. For example, the first two modes of a cantilevered beam are in the bending plane.

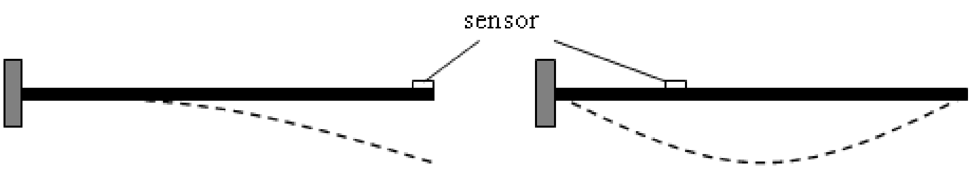


Figure 2.6: Sensor locations for cantilevered beam

The two sensor locations highlighted in fig 2.6 would give a good indication of the presence of both modes.

`fe_sens` is the SDT function handling sensor placement. The maximum sequence placement algorithm [4] is preferred and uses a call of the form

`sdof=fe_sens('mseq n',def,sdof0);`

where `sdof` is a vector containing the optimal sensor DOFs (see `sdtweb mdof` for the standard DOF format). The *n* in the command is a user input and specifies the number of sensor locations desired. A set of deformations (see `sdtweb def`) typically corresponding to modes must be given as well as an optional initial set of sensors `sdof0` that must be retained. Rotational DOFs are ignored du to the difficulty in measuring them.

A cantilevered beam will be used as an example. The model dimensions are shown in fig 2.7. Aluminum material properties used for material and beam section properties are $J = I2 = 10^{-9}m^4$, $I1 = 310^{-9}m^4$ and $A = 10^{-4}m^2$.

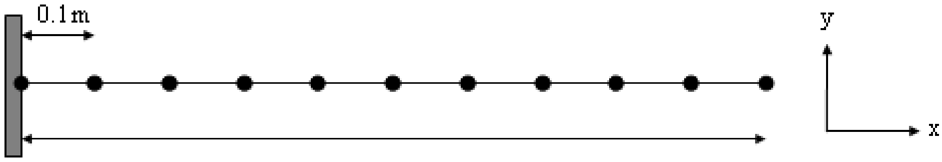


Figure 2.7: Cantilevered beam mesh

The model is generated and modes computed as follows

```
% Generate model
Node=[1 0 0 0 0 0 0
      2 0 0 0 .1 0 0];
model=feutil('ObjectBeam 1 1',Node,linspace(0,1,10));
model.pl=[1 fe_mat('m_elastic','SI',1) 7.2e9 .3 2700 0];
model.il=[1 fe_mat('p_beam','SI',1) 1e-9 1e-9 3e-9 1e-4];
cf0=feplot(model); model=cf0.mdl;
model=fe_case(model,'FixDof','2D',[.01 .02 .04 .06]',...
                    'FixDof','leftedge','x==0');

% Compute and display 10 modes
cf0.def=fe_eig(model,[5 10 0]);
```

The `fe_sens` function is used to place 3 sensors. Then `fecom` commands allow viewing

```
% place sensors
sdof=fe_sens('mseq 3 type',cf0.def);

%Declare optimal list as sensors and view
cf0.mdl=fe_case(cf0.mdl,'SensDof','Test',sdof)
fecom('CurtabCases','Test');fecom('ProViewOn')
```

`sdof` is the sensor set. It is a column vector containing the sensor DOFs ranked in order of importance (the most influential being listed at the top).

For shaker placement one typically wants to guarantee good commandability (thus find the location that has the maximum minimum commandability over the modes). The result here is the trivial excitation at the tip

```
i_trans=fe_c(cf0.def.DOF,[.01;.02;.03],'ind');
i_mode=1:10; % selected modes
[r1,i1]=sort(-min(abs(cf0.def.def(i_trans,i_mode)),[],2));
idof=cf0.def.DOF(i_trans(i1)); % potential shaker DOF
cf0.mdl=fe_case(cf0.mdl,'SensDof','IN',idof(1))
fecom('CurtabCases',{'Test';'IN'});fecom('ProViewOn')
fecom('Textnode',idof(1),'FontSize',12)
```

It should be noted that more than one shaker may be needed to excite all modes (for example for modes orthogonal to the shaker axis).

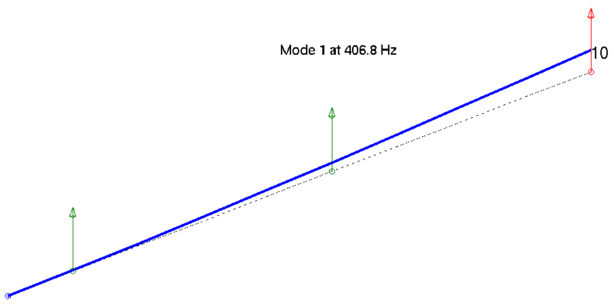


Figure 2.8: SDT plot of cantilever beam sensor placement

`fecom` commands can also be initialized using the `feplot` properties GUI (open with a click on the button).

2.3.3 Visualizing test shapes (ODS, modes, ...)

Having extracted the modal model a means to visualize the mode shapes is required. The deformations are linked in with the wire frame model to produce an animated plot. The method used is a standard SDT `feplot` . It is assumed here that a wire frame has already been defined in the variable `model`.

```
cf=feplot(model);
fecom(cf,'ShowLine');
```

To plot the mode shapes simply use `cf.def=ci.Stack{'IdMain'}`. You can also display FRFs with `cf.def=ci.Stack{'Test'}` (but in that case use the `Cursor ...:ODS start` context menu in `iiplot`).

It may be useful to understand the relation between test and FEM storage. Typicall def (as `cf.def`) has following field:

- `.DOF` – list of degrees of freedom to which the deformation applies. For test first column of `ci.Stack{'IdMain'}.dof(:,1)` gives the same information and `id_rm` is used to deal with MIMO cases where sensors are repeated for multiple inputs).
- `.data` – natural frequencies corresponds to `ci.Stack{'IdMain'}.po` for poles or `ci.Stack{'Test'}.w` for frequencies.
- `.def` – columns give mode shapes while `ci.Stack{'IdMain'}.res` rows give residues and `ci.Stack{'Test'}.xf` rows responses at sensors.

2.4 A complete modal test example

The example structure being used here is a tubular structure shown below.

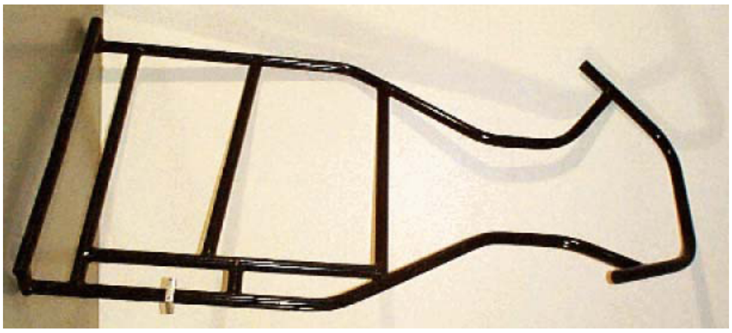


Figure 2.9: Structure used in the modal test

The test data used was generated using SDT – a full testing example will be given separately.

```
% The data can be downloaded with
demosdt('download http://www.sdtools.com/contrib/kart_example.mat')
% www.sdtools.com/contrib/kart_example.mat
%----------

% WIRE FRAME DEFINITION (present in WIREFRAME) built with
%----------------------

WIREFRAME.Node=[ ...
  1 0 0 0 0.613 0 0      ;   2 0 0 0 1.133 -0.198 0
  3 0 0 0 1.548 -0.208 0 ;   4 0 0 0 1.538 -0.487 0
  5 0 0 0 1.133 -0.51 0  ;   6 0 0 0 0.613 -0.742 0
  7 0 0 0 0.32 0 0       ;   8 0 0 0 0.772 -0.287 0
  9 0 0 0 0.772 -0.521 0 ;  10 0 0 0 0.33 -0.7483 0
  11 0 0 0 0.41 -0.218 0 ;  12 0 0 0 0.38 -0.545 0
  13 0 0 0 0.1 -0.22066 0;  14 0 0 0 0.1 -0.55167 0
  15 0 0 0 0 -0.2177 0  ; 16 0 0 0 0 -0.544285 0

];

WIREFRAME.Elt=[];feplot(WIREFRAME)
fecom('TextNode');

% You can use the contextmenu (right click) cursor ... -> 3dline pick

feplot('addline',[4 3 2 1 7 15 16 10 6 5 4]) % add a first line
feplot('addline',[8 11 13 0 9 12 14 13 0 9 8 0 12 11]) % other line

% POLE ESTIMATION
%-----------------

% download and load test data :
demosdt('download http://www.sdtools.com/contrib/kart_example.mat')

ci=idcom; % open interface and get pointers ci
ci.Stack{'Test'}.w=TESTDATA.w; % Frequencies
```

```
ci.Stack{'Test'}.xf=TESTDATA.xf; % Responses
ci.Stack{'Test'}.dof=TESTDATA.dof; % Dofs

iicom('SubMagPha');  % frequency response data plotted

% poles must now be identified one by one

idcom('e .01 44.6');
iicom('CurTabIdent')

% pole estimated at freq of 44.6Hz with possible range of .01 percent
% around that frequency.
% estimate checked on freq plot, if correct it is added

idcom('ea');
idcom('e .01 48.8'); idcom('ea');
idcom('e .01 95.3'); idcom('ea');
idcom('e .01 125.3');idcom('ea');
idcom('e .01 141');  idcom('ea');

% five poles estimated in total and are stored in ci.Stack{'IdMain'}.po.
% modeshapes are based on narrowband estimate.

idcom('est'); % build a broad-band identification

% VISUALISATION
%---------------

cf=feplot;  cf.model=WIREFRAME; % plot WIREFRAME test model
cf.def=ci.Stack{'IdMain'}; fecom('view3') % display identified shapes
```

3

Correlation

Contents

This section is very incomplete. You should really look up chapter 3 of the documentation, `sdtweb cor`.

3.1 Topology correlation

The first step in correlating test and analysis is to observe the motion on the same sensors. There is no particular reason to force the model to use test sensors and SDT supports arbitrary placement of sensors with respect to test, see `sdtweb topo`.

3.2 Correlation criteria

3.2.1 Modal Assurance Criteria (MAC)

A quantifiable correlation between experimental and analytical mode shapes can be determined based on the Modal Assurance Criteria (MAC).

Each mode shape is a vector of nodal displacements. The correlation between two vectors Φ_a and Φ_b is given by

$$MAC(\Phi_a, \Phi_b) = \frac{|\{\Phi_b\}^T \{\Phi_b\}|^2}{(\{\Phi_b\}^T \{\Phi_b\})(\{\Phi_a\}^T \{\Phi_a\})} \tag{3.1}$$

The product of this is a scalar quantity. If Φ_a and Φ_b are identical the numerator and denominator are equal, giving a MAC value of 1. If the two vectors are orthogonal to one another the numerator is 0 and hence the MAC value is zero. Each vector from data set A is compared with each vector from data set B. The data is displayed on a MAC plot, see fig 3.1.

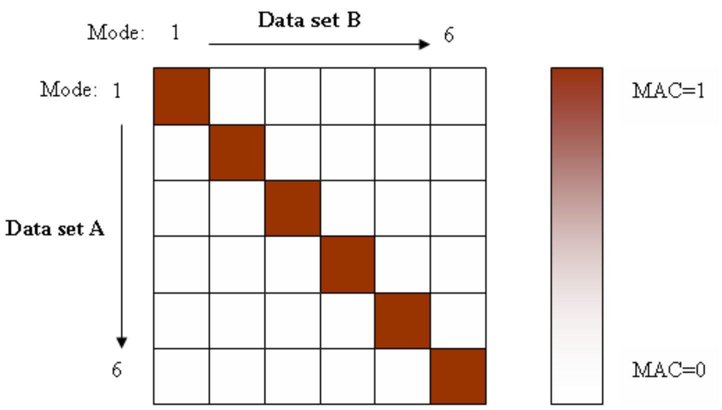


Figure 3.1: MAC plot example

The MAC plot is an n x n grid (n is the number of modes being compared) enabling comparison of all modes from data set A with all modes from data set B. The leading diagonal represents the correlation between modes of the same number from data sets A and B. If the ordering of the modes in each set is correct (modes have been paired correctly) then the correlation here should be high. The MAC values can be ascertained using the color bar. Alternatively some MAC plots contain the actual MAC values in each cell of the grid.

Correlation is achieved in the SDT using the `ii_mac` function.

```
ii_mac(data sets, 'command string');
```

The data sets are eigenvector matrices required in the correlation. The command string determines the type of correlation performed. A `plot` option can be included in the command string and generates an `ii_plot`.

3.2.2 Auto MAC

The MAC can be unreliable when a vector is under-populated. This is often the case with experimental data where the number of DOF's included is far lower than the total. It is good practice to compare such data with itself to gain an indication of how well the vectors are defined. The values on the leading diagonal will equal unity. High off-diagonal terms (> 0.2) indicate poor definition between modes. A high correlation between modes which are close in frequency indicates the possibility of coupling between modes.

The auto MAC is a self-correlation test and so only one data set is required. `ii_mac(dataA, 'MAC auto plot')`

3.2.3 Standard MAC

This is the most common correlation test applied in modal analysis. MAC values are calculated for all grid values as indicated in fig 3.1. Automatic pairing of the modes is optional; based on the highest MAC value. The initial pairing of modes is based on their frequency, but discrepancies between model and test are common. It is also possible that a test mode will have a high correlation with more than one analytical mode, and as a result will appear more than once on the MAC plot.

The standard MAC pairs the uncorrelated vectors.

```
ii_mac(dataA, dataB, 'MAC pair plot');
```

If the `pair` option in the command string is omitted a direct correlation without pairing is performed.

3.2.4 COMAC

The coordinate modal assurance criteria (COMAC) are an extension of the MAC and are used to identify those DOF's responsible for lowering the MAC. For each DOF the correlation is assessed between analytical and experimental mode shapes using every mode. It requires the modes to be paired using the standard MAC.

$$COMACf = \frac{\sum_{m=1}^{M}\left\{|\Phi_{amf}\Phi_{bmf}|^2\right\}}{\sum_{m=1}^{M}\left\{\Phi_{amf}^T\Phi_{amf}\right\}\sum_{m=1}^{M}\left\{\Phi_{bmf}^T\Phi_{bmf}\right\}}$$

Where Φ_{amf} is the nodal displacement at DOF f, mode m from data set a. As with the standard MAC a value approaching unity implies a good correlation.

The `COMAC` command automatically pairs the modes using the MAC pair command. The equivalent eCOMAC command is also given.

```
ii_mac(dataA, dataB, 'comac plot');
ii_mac(dataA, dataB, 'comac e plot');
```

In all `ii_mac` commands given, omission of the `plot` command option will prevent the MAC plot being displayed.

There are a number of additional plot features that can be used with the `ii_mac` function. A list of these can be found in the SDT help files. The plot features can also be accessed using the `iiplot` GUI.

3.2.5 eCOMAC

The enhanced COMAC (eCOMAC) is an adaptation of the COMAC. All modes are normalized to unity, thus making the assessed error insensitive to scaling.

$$eCOMAC = \frac{\sum_m^M |\hat{\Phi}_{amf} - \hat{\Phi}_{bmf}|}{2M} \text{ with } \hat{\Phi}_{amf} = \frac{\Phi_{amf}}{|\Phi_{am}|} \text{ and } \hat{\Phi}_{bmf} = \frac{\Phi_{bmf}}{|\Phi_{bm}|}$$

The eCOMAC is often presented as a bar chart. The x axis contains each DOF and the y axis contains (1-eCOMAC). The peaks in the chart thus represent areas of poor correlation.

3.3 Modeshape expansion

This section needs documentation.

Bibliography

[1] D. Ewins, *Modal Testing: Theory and Practice*. John Wiley and Sons, Inc., New York, NY, 1984.

[2] W. Heylen, S. Lammens, and P. Sas, *Modal Analysis Theory and Testing*. KUL Press, Leuven, Belgium, 1997.

[3] E. Balmes, *Methods for vibration design and validation*. Course notes ENSAM/Ecole Centrale Paris, 1997-2012.

[4] E. Balmes, "Orthogonal maximum sequence sensor placements algorithms for modal tests, expansion and visibility.," *IMAC*, January 2005.