Prediction of the influence of structural modifications including viscoelastic materials

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Abstract
EDF is in charge of a large industrial fleet of electric power stations. Because of ageing and strong environmental conditions, unexpected vibration problems can appear on machines like pumps or electric motors. These problems have to be fixed rapidly (by structural modifications) in order to avoid electricity shortages. Stiffness or mass additions are often used to solve the problem when only a few modes of the defective structure are involved in the faulty behaviour. However, the use of such modifications is sometimes irrelevant. In such cases, damping modifications could be an efficient alternative to reduce the amplitude of in operation vibrations. This work deals with the estimation of the influence of modifications including viscoelastic materials on the dynamic behaviour of a structure. Current structural modification methods basics and reduction methods of complex problems are first presented. Then some numerical results of prediction of the effect of damping modification are shown through a mock up example.

1 Introduction
EDF is in charge of a large industrial fleet of electric power stations. Because of ageing and strong environmental conditions, unexpected vibration problems can appear on machines. Harmonic or wideband excitations can in fact cause troubles. In the first case, mass or stiffness additions can fix problems rapidly [1]. In the second one, such a method is in general irrelevant and damping modifications could be a good alternative. Thus this work deals with the estimation of the influence of modifications including viscoelastic material on the dynamic behaviour of a structure.

In many cases, no tuned F.E. model of the target structure exists, and time allowed to the study is to short to build one. One must then deal with an experimental model of the structure to be cured.

Structural modification has been a major topic in modal analysis for several years until the beginning of the 1990s. In the way to overlay problems where only limited action on the structure is allowed, several authors have dealt with hybrid coupling of a numerical model on one side and experimental model on the other side but restricted to mass and stiffness additions. In this paper is raised the question of hybrid coupling problem between an experimental model of a structure, only known by its first modes and a given numerical model of a modification including viscoelastic material (high damped modification).

The method used to derive the dynamical behaviour of the modified structure proceed as follow:

- an experimental modal analysis is performed on the initial structure
- these experimental data are then extended on to a crude un-updated model (local model)
- a numerical model of viscoelastic dampers is designed
- a hybrid coupling between the extended experimental model of the structure and the numerical model of dampers is performed in such a way that normal modes, damping and frequency response of the modified structure are calculated.
Throughout a mock up example, some well-known numerical results on sub-structuring, on the effect of modal truncation and on the importance of the contribution of high-frequency modes or residual flexibility are exhibited. Methods of estimation of residual flexibility from experimental data as well as innovative reflections on the use of not complex but normal modes to characterize the mechanical behaviour of the treated structure are presented.

2 State-of-the-art

2.1 Dynamic sub-structuring

Considering two structures without any load, one can write equation 1

\[
\begin{bmatrix}
Z_{CC}^B & Z_{CI}^B & 0 \\
Z_{IC}^B & Z_{II}^B + Z_{II}^M & Z_{IC}^M \\
0 & Z_{IC}^M & Z_{CC}^M
\end{bmatrix}
\begin{bmatrix}
q_C^B \\
q_I^B \\
q_C^M
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
f_I^B \\
0
\end{bmatrix}
\]

Reduce the problem on a particular subspace is in many cases interesting in such way we obtain equation 2

\[
\{q\}_{N \times 1} = [T]_{N \times Ng} \{q\}_{Ng \times 1}
\]

Each sub-structure is then described by a few generalised degrees of freedom (DOF) \(\{q\}_{Ng \times 1}\) with \(Ng << N\). The choice of the subspace is a very important point. Free and fixed interface methods are the two major methods to reduce a problem on a subspace. MacNeal [2] and Craig & Bampton [3] methods are respectively ambassadors of those methods. The first one uses a base \([T_{MN}]\) (equation 3) composed of:

- a truncated base of \(N_{free}\) eigenvectors with interface DOF free
- a base with \(N_A\) attachment modes (displacements of DOF for a static unitary load input on interface DOF)

\[
[T_{MN}] = \begin{bmatrix}
[\phi_{free}]_C & [\phi_{free}]_I & [K_{CC}]^{-1} & [0]_N \\
[\phi_{free}]_C & [K_{IC}] & [K_{II}] & [Id]_N
\end{bmatrix}_{N_{free} \times N_{free} + N_{free} \times N_{free} + N_{free} \times N_{free} + N_{free} \times N_{free}}
\]

Craig & Bampton base is made of:

- a truncated base of \(N_{fixed}\) eigenvectors with interface DOF fixed
- a base with \(N_C\) constraint modes (Guyan condensation)

\[
[T_{C&B}] = \begin{bmatrix}
[\phi_{fixed}]_C & [0]_N \\
[0]_N & [K_{CC}]^{-1} [K_{CI}] & [Id]_N
\end{bmatrix}_{N_{fixed} \times N_{fixed} + N_{fixed} \times N_{fixed} + N_{fixed} \times N_{fixed} + N_{fixed} \times N_{fixed}}
\]
2.2 Structural dynamic modification (SDM)

In the particular case of structural modification, experimental data from a structure are considered. Only partial data are then available to describe the mechanical behaviour of the structure. In order to couple the basic structure and the modification one need properly expand experimental data (from measured DOF) to interfacing DOF. The different steps of the method are then presented.

1. expansion bases building

- with static modes

Using static modes related to measured DOF to expand displacements from instrumented DOF may appear natural at first sight. It is however irrelevant. Static modes actually introduce very local deformation. The reduction bases takes the form of equation 5

\[ [T_{It}] = [K]^{-1}[C_{tL}]^T \] (5)

- with normal modes

Using normal modes is often insufficient except in the case of using a tuned FE model. But in many cases no tuned FE model is available.

- with interface modes

Interface modes are in fact the best way to build the expansion bases [1]. In order to give a smoothly characteristic to the notion of static modes one would rather solve the reduced problem of the equation 6

\[ (-\omega_g^2[T_{It}]^T[M][T_{It}] + [T_{It}]^T[K][T_{It}]\{\phi_g\} = \{0\} \] (6)

and obtain the expansion bases of equation 7

\[ [T] = [T_{It}][\{\phi_1^g\}...\{\phi_{N_g}^g\}] \] (7)

The static modes bases \([T_{It}]\) can include different DOF sets. Static reduction on measured and interface DOF is the best choice according to [1].

2. experimental data expansion

Once the expansion bases is built one will solve the least square problem related to equation 8

\[ \{\eta_{red}\} = ArgMin_{\eta} \left( \| \{q_{test}\} - [C][T]\{\eta\}\right)^2_{[X]} \] (8)

Experimental data are now extended to the whole local model \([\phi_{Lg}] = [T]\{\eta_{red}\}\) and so to the interface DOF \([\phi_{Ig}] = [T_{I}]\{\eta_{red}\}\).
3. **coupling**

Thus coupling can be written like the equation 9

\[
[Z^{B+M}] = \begin{bmatrix}
-\omega^2 [I_d] + j\omega [\Gamma^B] + [\Omega^B]^2 \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
[Z^M_C1]^T [\phi^B] \\
[Z^M_CC]^T [\phi^B]
\end{bmatrix}
\] (9)

In our case hysteretic damping hypothesis set us to use a complex rigidity matrix for the modification sub-structure. For the initial structure, identified damping characteristics are critical damping ratios \(\xi\), thus damping is modeled by a viscous damping matrix \([C^B_{ii}] = 2\omega_0 \xi_i\).

4. **optimum expansion bases a posteriori choice**

The choice of the optimal size of \([\phi_{Lg}]\) is a critical point. In [1], two indicators based on strain and kinetic energy of the modification are defined. They are based on the comparaison of displacements of the interface for the coupled problem obtained by a static manner in one hand and a dynamic one in the other.

2.3 **Solve complex eigenvalues problem**

Solving eigenvalues problem directly is often unreasonable because of the size of the considered problem. One actually uses approximations using subspace methods (model reduction). Dynamic rigidity of equation 10 is then considered.

\[ [Z] = -\omega^2 [M] + [K] + i[B] \] (10)

2.3.1 **Model reduction**

Classical bases used in modal analysis and dynamic sub-structuring are made of free eigenvectors and terms related to imposed loading \([b]\) (attachment modes \([K]^{-1}[b]\)). One can complete the bases with correction modes which take into account the damping effect of the free eigenvectors on the structure [4] [5] (equation 11).

\[ [K]^{-1}[B][\phi_{1:N}] \] (11)

The reduction base takes thus the form of the equation 12

\[ [T] = [[\phi_{1:N}] \ [K]^{-1}[b] \ [K]^{-1}[B][\phi_{1:N}] \] (12)

2.3.2 **Residue based iteration method**

In [4] is described a residue based iteration method which allows very good estimates and great time savings. Using a subspace \([T^n]\) Ritz approximations takes the form of equation 13
\[
\{q^n(s)\} = [T^n] \left[ [\phi_{1:N}] [K]^{-1} \right] [B] [\phi_{1:N}]^T [T^n] b \{u(s)\} 
\]

At a given frequency \(s\), an energy error estimator is thus given by equation 14:

\[
\{R_d^n(s)\} = [K_0]^{-1} \left( Z \{q^n(s)\} - \{F(s)\} \right) 
\]

From this displacement residue, one can then calculate its stain energy \(\| \{R_d^n(s)\} \|_{K_0}\) and check the convergence thanks to the criterion of equation 15:

\[
\epsilon^n = \frac{\| \{R_d^n(s)\} \|_{K_0}}{\| \{q^n(s)\} \|_{K_0}} 
\]

While the criterion is over a given tolerance the corresponding residue is added to the base (equation 16):

\[
[T^{n+1}] = [T^n \{R_d^n(s)\}] 
\]

3 Application

3.1 The mock up example

3.1.1 Geometric and Modal Characteristics

The mock up example has been chosen because its mechanical behaviour correspond to a type of vibration problems interesting for EDF. The oval-shaped modes at low frequencies motivated this choice. It consists in a cylinder welded on to a circular plate which is bolt screwed on to a square plate. Figure 1 shows the corresponding Finite Elements Model (FEM), instrumented and driving points configuration.

![Figure 1: FEM of the mock up example](image)

Table 1 presents the geometrical characteristics of the structure.

In appendix A, identified modes are presented in the table 4 and one can see on the figure 7 the good accuracy of the FEM.
Table 1: Geometric characteristics

<table>
<thead>
<tr>
<th>Cylinder</th>
<th>Diameter</th>
<th>60 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Height</td>
<td>80 cm</td>
</tr>
<tr>
<td></td>
<td>Thickness</td>
<td>0.5 cm</td>
</tr>
<tr>
<td>Ears</td>
<td>Height</td>
<td>15 cm</td>
</tr>
<tr>
<td></td>
<td>Width</td>
<td>10 cm</td>
</tr>
<tr>
<td></td>
<td>Thickness</td>
<td>0.4 cm</td>
</tr>
<tr>
<td>Circular Plate</td>
<td>Diameter</td>
<td>82 cm</td>
</tr>
<tr>
<td></td>
<td>Thickness</td>
<td>0.5 cm</td>
</tr>
<tr>
<td>Square Plate</td>
<td>Width</td>
<td>120 cm</td>
</tr>
<tr>
<td></td>
<td>Thickness</td>
<td>1 cm</td>
</tr>
<tr>
<td>Mass</td>
<td></td>
<td>255 kg</td>
</tr>
</tbody>
</table>

3.1.2 Modification

The modification is made up by a stiff ring and viscoelastic dampers. Figure 2 is an example of what could be the modification. It shows the hybrid coupling between the experimental mesh and the FEM of a modification.

The damping devices are made of viscoelastic material and steel plates. Some authors ([8] for aeronautical domain and [9] for building) depict different designs of such damping devices. Viscoelastic materials considered there are homogenous isotropic polymeric materials. [10] is a good reference for representing constitutive laws of viscoelastic materials.

In this article, the behaviour of viscoelastic materials is averaged on a frequency band. Let us consider an average Young modulus between $4.5 \times 10^5$ and $2 \times 10^7 Pa$ and an average loss factor $\eta = 1$. The sizing parameter for viscoelastic dampers is the equivalent axial stiffness which actually correspond to the shear stiffness of the viscoelastic panel given by $k = \frac{G S}{\tau}$ where $G$ is the shear modulus, $S$ and $\tau$ the surface and the thickness of the viscoelastic material.

The optimization of the viscoelastic dampers is not trivial. Damping of oval-shaped modes depends on position and equivalent axial stiffness of the dampers. One has thus to compare characteristics of available materials and possible configurations.

Using 9 dampers allows the modification to be supported by itself. Figure 3 shows how damping and frequency of different modes can change with the equivalent axial stiffness of dampers.
Figure 3: Evolution of damping and frequency of different modes for $4.5 \times 10^5 < k < 2 \times 10^7 N.m^{-1}$.

3.1.3 Reference

Reduction model methods often allow to find out a good estimate to eigenvalues problems. Many methods have been devised for the construction of real projection bases. One will refer to the literature on condensation [6] and Component Mode Synthesis [7]. Reduction bases usually considered combine normal modes and static corrections defined in section 2.3.1.

Figure 4 shows Frequency Response Functions (frf) for different reduction bases. On the left-side plot one can see effects of using truncated bases in a light damped case (structure before being modified). On the right-side one can see that using correction modes in addition of normal modes is efficient. Correction modes take into account the effect of damping on normal modes. Even with a real base one is able to compute high damped frf. Moreover computing direct frf is 60-time longer than with projection on a real base in our case.

Figure 4: frf for different reduction bases before and after the modification

Frf computed with normal and correction modes are thus used in following sections as a reference to judge the ability of Structural Dynamic Modification method (SDM) to predict eigenvalues, corresponding damping ratios and frf from experimental data.

3.2 Damping prediction

In next sections a local model is introduced. It is a un-updated model which is used to extend experimental data from measured DOF to interface DOF. In the paper the local model has been computed from the initial
model and some transformations have been computed to change its mechanical behaviour. Thus the link between the circular and squared plates makes the local model different from the initial model. In spite of a 4-screw link, local model considers a perfect constraint link. That makes the local model stiffer and easier to model than the initial model.

The different steps (described in section 2.2) of the SDM are recalled in figure 5.

![Figure 5: SDM step by step](image)

### 3.2.1 Numerical

From numerical simulated data the method is run. First results of prediction show how the method is able to give a good estimate of damping. Table 2 gives frequency and damping prediction for first modes with an equivalent axial stiffness $k = 9 \times 10^5 \text{N.m}^{-1}$. One can see the efficiency of the modification to damp oval-shaped modes. A minimum damping ratio of 6.69% is achieved within the affected modes. Moreover it shows that SDM methods are able to predict not only natural frequencies (max. 5%-error) but damping ratios. Damping ratios are predicted with a maximum 30%-error which is a good prediction given that the initial damping ratio is light (0.5%) and modified structure’s very high.

In appendice B, figure 8 shows how frequency and damping prediction depend on the size of $[\phi_L g]$. Such a plot reveals some stable bands which give a good estimate of prediction. Error estimators built in [1] to select the best band will be developed.

Figure 6 shows frf computed from predicted complex modes for different equivalent axial stiffness of the dampers. Effects of viscoelastic dampers are very positive.
Table 2: Eigenvalues and corresponding damping of the initial structure and of the modified structure (reference and prediction)

<table>
<thead>
<tr>
<th>mode shape</th>
<th>initial structure</th>
<th>modified structure</th>
<th>SDM prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f (Hz)$</td>
<td>$\xi$ (%)</td>
<td>$f (Hz)$</td>
</tr>
<tr>
<td>pumping</td>
<td>15.16</td>
<td>0.5</td>
<td>13.81</td>
</tr>
<tr>
<td>tilting 1</td>
<td>22.73</td>
<td>0.5</td>
<td>18.18</td>
</tr>
<tr>
<td>tilting 2</td>
<td>23.58</td>
<td>0.5</td>
<td>19.57</td>
</tr>
<tr>
<td>2-lobe-oval 1</td>
<td>53.88</td>
<td>0.5</td>
<td>63.25</td>
</tr>
<tr>
<td>2-lobe-oval 2</td>
<td>59.07</td>
<td>0.5</td>
<td>73.79</td>
</tr>
<tr>
<td>3-lobe-oval 1</td>
<td>114.41</td>
<td>0.5</td>
<td>120.1</td>
</tr>
<tr>
<td>3-lobe-oval 2</td>
<td>118.3</td>
<td>0.5</td>
<td>133.0</td>
</tr>
</tbody>
</table>

3.2.2 Experimental

From experimental modal analysis data, prediction using the SDM was computed. Table 3 gives frequencies and damping ratios of the first modes for the initial structure on one hand (experimental modal analysis data) and for the modified structure (SDM prediction) on the other. A $9.5 \times 10^5 N.m^{-1}$-equivalent-axial stiffness for the viscoelastic dampers have been used to compute those results.

![Figure 6: Frf before and after modification for different stiffness](image-url)
<table>
<thead>
<tr>
<th>Mode shape</th>
<th>Initial structure</th>
<th>Modified structure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental modal analysis</td>
<td>SDM prediction</td>
</tr>
<tr>
<td></td>
<td>$f (Hz)$</td>
<td>$\xi (%)$</td>
</tr>
<tr>
<td>pumping</td>
<td>17.1</td>
<td>0.7</td>
</tr>
<tr>
<td>tilting 1</td>
<td>23.06</td>
<td>1.60</td>
</tr>
<tr>
<td>tilting 2</td>
<td>23.06</td>
<td>1.60</td>
</tr>
<tr>
<td>2-lobe-oval 1</td>
<td>45.43</td>
<td>0.36</td>
</tr>
<tr>
<td>2-lobe-oval 2</td>
<td>49.38</td>
<td>0.30</td>
</tr>
<tr>
<td>3-lobe-oval 1</td>
<td>104.97</td>
<td>0.23</td>
</tr>
<tr>
<td>3-lobe-oval 2</td>
<td>107.40</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Table 3: Frequencies and damping ratios of initial (from experimental modal analysis) and modified (from SDM prediction) structures

### 4 Conclusion

Major techniques used in Structural Dynamic Modification methods have been presented in the paper. They have been extended to complex eigenvalues problem. It is thus now possible to predict effects of very high damped modifications on a only-experimental-known structure. A mock up example with a mechanical behaviour similar to some EDF problems has been built. First numerical results of prediction using SDM have been presented.

Dynamic behaviour of the modified structure is well estimated. Errors on frequency and damping prediction are low. Predict FRF from complex modes is a good way to save computing time.

The paper shows the ability of SDM methods to predict effects of a high damping modification on a structure. First numerical results presented in the paper are encouraging. Nevertheless some parameters have to be studied. For example the choice of the size of the expansion base is sometimes difficult. And indicators built in [1] to estimate the quality of the expansion over the interface are quite unstable in very high damped cases.

An experimental modal analysis have been conducted and the modification will be realized very soon to provide confirmation of our experimental prediction.

### References


A Identified modes

<table>
<thead>
<tr>
<th></th>
<th>Pumping</th>
<th>Bascule 1 &amp; 2</th>
<th>2-lobe oval.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>17.1 Hz at 0.7%</td>
<td>23.06 Hz at 1.6%</td>
<td>45.43 Hz at 0.36%</td>
</tr>
<tr>
<td></td>
<td>2-lobe oval.</td>
<td>3-lobe oval.</td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>49.38 Hz at 0.3%</td>
<td>104.97 Hz at 0.23%</td>
<td>107.40 Hz at 0.21%</td>
</tr>
</tbody>
</table>

Table 4: Modal characteristics

Figure 7: MAC between experimental ($\phi_{TEST}$) and FEM ($\phi_{FEM}$) modes
B Prediction of natural frequency and damping ratio

Figure 8: Frequency and damping ratio of the first 2-lobe-oval-shaped mode vs. size of $[\phi_{Lg}]$