Uncertainty propagation in modal analysis

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ABSTRACT

Errors in predicted modal behavior can result from modeling errors or actual variations of the properties due to flight conditions, change in payload or manufacturing tolerances. Launcher models used by EADS-ST for modal analysis studies have a nominal definition and adaptations for each flight. Experimental modal analysis and model updating are used to validate the nominal model giving very satisfactory results for the considered Ariane 5 applications. Given a test validated model, one then seeks to exploit an uncertainty characterization on physical parameters to predict uncertainties on modal properties. These predictions are then used for dynamic response validation and control design. Predictions are obtained using reanalysis techniques thus allowing very fast estimates within a design hypercube while having an error evaluation strategy that is detailed. Model reduction is obtained using the multi-model approach and possible choices in implementation are discussed. The results are illustrated on a model of upper cryogenic stage of Ariane 5.

1 INTRODUCTION

Model updating methods use test/analysis correlation to validate the properties of a model and possibly correct some of its parameters. Variability studies seek to exploit a characterization of uncertainties on model parameters to predict uncertainties on modal properties. Such predictions can then be used for dynamic design and control.

Both updating and uncertainty propagation use parametric studies that are very demanding in terms of computational time. Reanalysis techniques [1, 2, 3] are typically considered to obtain approximations in an acceptable time. Section 2 summarizes, the principles of the Residue Iteration method [4], which introduces an error control mechanism in reanalysis techniques.

Section 3 then considers an application to the model updating of the Upper Stage of the Ariane 5 launcher. Section 4 finally discusses, for the same model, uncertainty propagation and ideas on how to use the results of such computations.

2 RESIDUE ITERATION METHOD

Reanalysis techniques [1, 2, 3] build approximation bases that span a subspace where one seeks solutions to a parametric family of problems. Many methods to build bases have been developed for eigenvalue computations, substructuring, sensitivity analyses, ... But the problem specific nature of these methods limits their usefulness.

The Residue Iteration (RI) method, outlined in figure 1, gives a general framework to extend reanalysis techniques while giving an error control and enrichment mechanism.

Starting with an initial basis $T^{(o)}$ obtained with a classical reanalysis technique, one computes an approximate solution and defines a dynamic residual characterizing the error level for the approximate solution. This residual is very classical in iterative methods. For example, the residual associated with the computation of a normal mode $\{\phi_j(p)\}, \omega_j(p)$, is given by

$$R_L(\{\phi_j(p)\}, \omega_j(p), p) = \left[K(p) - \omega_j^2 M(p)\right]\{\phi_j(p)\} \tag{1}$$

To the load residual R_L, one associates a displacement residual $R_D = \left[\hat{K}\right]^{-1} R_L$ where $\hat{K}$ is usually taken to be the nominal stiffness matrix and can be seen as a preconditioner [4].

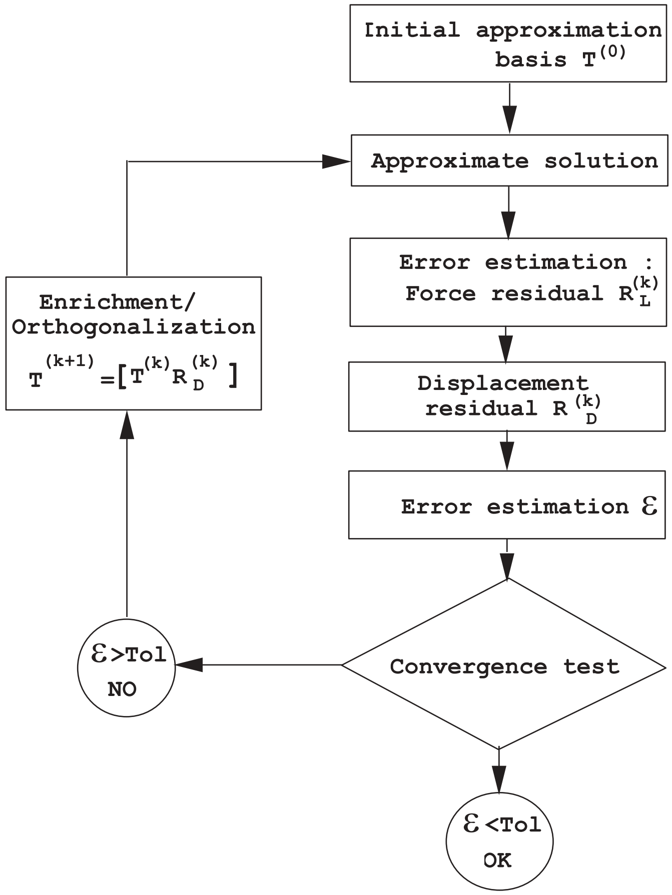

Figure 1: Principle of enrichment strategy of the Residue Iteration Method [4].

The displacement residual leads to a relative error in strain energy

$$\epsilon(\{\phi_j(p)\}, \omega_j(p), p) = \frac{\|R_D\|_{K(p_0)}}{\|\phi_j(p)\|_{K(p_0)}} \tag{2}$$

and also gives a new direction to enrich the subspace by using $T^{(k+1)} = \begin{bmatrix} T^{(k)} & R_D^{(k)} \end{bmatrix}$. In practice, an orthonormalization must be performed before appending $R_D^{(k)}$ to $T^{(k)}$. Error computations shown in section 4.5 are based on this technique.

This approach has been shown to be efficient for the computation of normal and complex modes [4], multiple field problems found in fluid structure interaction and updating [5, 6], as well as direct frequency response and sensitivities [7, 4].

3 MODEL UPDATING

Model updating is a parametric optimization of model parameters in order to minimize test/analysis distance. Many questions remain opened in this area. In particular one can cite

- the effect of equivalent representations of joints or parts with internal structure which are very often used to simplify the representations of some parts [8].
- the need to aggregate the parameters affecting various elements to obtain global parameters that are sufficiently influent to be estimated during the updating phase. The problem of default detection will be illustrated in the oral presentation.
- the effect of residual errors, which for practical reasons, one does not wish to correct in the model.

To answer all those questions, one needs to have easy access to a whole panel of tools : identification methods, topology correlation, test/analysis correlation criteria, parameter selection methods, reanalysis for large parameterized models. The last item in particular is still not widely available for large industrial models and motivated the RI method developments outlined in section 2.

The updating of the Ariane 5 upper cryogenic stage (ESC-A) was performed in [4]. As shown in figure 2, the initial correlation is quite good as soon as one accounts for the dissymmetry which induces a rotation of the main bending planes.

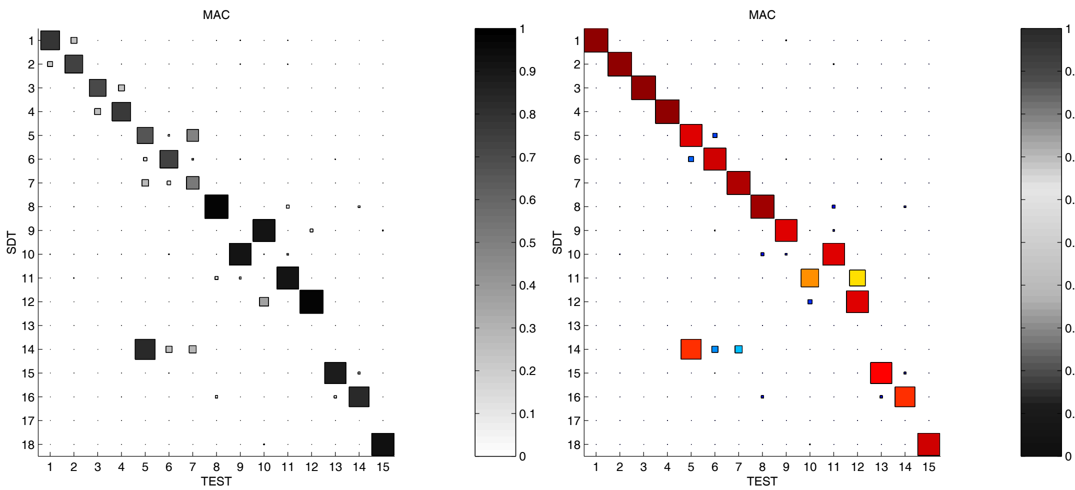

Figure 2: TEST vs. FEM MAC. Initial without and with accounting for bending plane rotation.

The updating improved correlation of the 15 retained modes with no deterioration of correlation for the following modes (see details in table 1. It appeared however that most of the remaining difference could be explained by errors on the localization of payload center of gravity below 5 cm, which is a very realistic value.

4 UNCERTAINTY PROPAGATION IN MODAL ANALYSIS

Launch vehicle models used by EADS-ST for modal analysis have a *nominal* definition, which is then adapted for each flight. Here one seeks to exploit a characterization of uncertainties on model parameters in order to predict uncertainties on modal properties. These uncertainties can then be used in dynamic design and control synthesis.

The main results of [9] are summarized here. Section 4.1 lists classical representations used to account for uncertainties in definition, modeling and manufacturing processes. Section 4.2 details an original implementation of reanalysis techniques adapted for the prediction of modal properties inside a parametric hypercube. The use of reanalysis is mandated for statistical studies and one pays particular attention to error control on the predictions. The following sections apply the proposed tools to the case of the dynamic model of the Ariane 5 Upper Cryogenic Stage.

4.1 Modelling of uncertainties

Mechanical structures are, in practice, described by a set of geometric and material properties. An uncertainty model in the space of physical parameters is thus a characterization of possible changes in the model parameters (geometry, constitutive material properties, choice of element formulation, etc.) One can formally write a nominal model and possible variations

$$p = p_0 + \Delta p \tag{3}$$

If the possible parameters p are described in terms of their probability distribution, one talks about a stochastic model. The propagation of probability distributions on physical parameters to probability distributions on modal properties is the subject of this study.

On a given mesh, the properties of each element can theoretically vary independently. In practice however, there is a strong correlation between values at various points. A realistic handling of parameter uncertainties thus uses a small number of uncertainty functions $P(x)$ describing how the properties vary as a function of the position or element number x. This results in a description

$\Delta f/f$ initial	MAC initial	$\Delta f/f$ final	MAC final
1.82 (%)	99	1.62(%)	99
0.18	99	0.03	99
-0.58	100	-0.59	100
-7.63	99	-7.62	99
-0.59	**92**	-0.21	**95**
-0.65	93	0.03	93
-0.08	96	0.25	97
-0.76	97	0.62	94
2.64	**91**	2.57	**98**
-2.34	**92**	**-0.80**	**97**
-3.73	**92**	-3.54	**98**
-0.00	97	0.09	97
0.73	**89**	0.73	**96**
-0.99	**83**	-0.70	**92**
-0.76	93	-0.74	93
0.17	96	0.24	96
1.76	75	1.77	76
-4.54	72	-4.54	72
-1.88	67	-1.88	66
-3.56	69	-3.56	69
1.38	77	1.44	79
0.61	76	0.81	71
38.50	59	38.47	51
-13.94	25	-13.84	24
-2.66	41	-3.31	33

TABLE 1: Correlation before and after updating

of p as a linear combination of a small number of aggregated parameters $\tilde{p}$

$$p(x) = \sum_{k=1}^{NPI} P(x)\tilde{p} \tag{4}$$

which are usually assumed to be uncorrelated. The aggregation of element mass and stiffness parameters in (6) corresponds to the choice of fields $P(x)$ equal to one on elements within a group and zero elsewhere.

The building of $P(x)$ fields based on correlation distances for random property fields has been well studied in the literature on stochastic finite elements [10]. A representation of spatial variations of an auto-regressive field, leads to a decomposition of spatial variations of the associated operator, still of the form (4).

In practice, the computation of element matrices for many different values of parameters has a high numerical cost. One is thus usually limited to cases where the global matrices can be expressed as linear combinations of constant matrices with variable coefficients. The constant matrices are often taken to be element matrices, thus leading to

$$[M(p)] = \sum_{e,k} \alpha_k(p)[M_k^e] \quad [K(p)] = \sum_{e,k} \beta_k(p)[K_k^e] \tag{5}$$

where element matrices are grouped in selections (e_i, e_j) with associated mass p_i or stiffness parameters p_j. Thus for a mass parameter p_i associated with element selection e_i, one has

$$\begin{array}{l} \alpha_k, \beta_k = 1 \quad \text{for} \quad k \notin e_i \text{ or } e_j \\ \alpha_k = p_i \quad \text{for} \quad k \in e_i \\ \beta_k = p_j \quad \text{for} \quad k \in e_j \end{array} \tag{6}$$

This representation easily accounts for variations in constitutive relations (modulus, density, shell thickness, etc.). Some alternatives exist but are not widely considered.

The choice of uncertain parameters and their aggregation is a key aspect of any uncertainty study. The basic assumption is that the initial FEM mesh can really be used to represent all the possible responses of the family of models being considered.

This assumption, although unavoidable, is of doubtful validity when uncertainties are associated with simplifying assumptions in the model. The use of detailed local models seems the only valid procedure to establish an uncertainty model in this case. Such validations are often considered, but they require significant engineering time. There will thus always be a compromise.

The last difficulty is to build an uncertainty model for each parameter. In the present study, uncertainty bounds were generated based on static tests on the launch vehicle. But there is no consistent justification for why this is a valid procedure. Model updating results [4] do not currently give a procedure to build an uncertainty model, in particular because of remaining bias in the model. The subject of building an uncertainty model thus remains a very open subject that has not been addressed in this study.

The last assumption used here is that probability distributions are uniform over an interval. Using a bounded support for the probability distribution is needed for the error control procedure shown in the next section, but the form of the probability distribution is an arbitrary design choice.

4.2 Propagation through reanalysis techniques

As will be shown in section 4.4, variations of modal properties are quite non linear. The use of perturbation methods, even of high order, thus does not yield sufficient accuracy. Reanalysis techniques [1, 2, 3] will thus be preferred here.

These methods seek approximate solutions in a subspace T independent of p by solving for each value of p

$$\left[\left[T^T K(p) T\right] - \omega_{jR}^2(p)\left[T^T M(p) T\right]\right]\{\phi_{jR}(p)\} = \{0\} \tag{7}$$

As long as K and M are a matrix polynomials in p, one can project the matrix coefficients once and for all. The resolution of (7) can thus be very fast. Restitution of responses on all DOFs is then simply given by $\{\phi_j\} = [T]\{\phi_{jR}\}$.

The fundamental question is the procedure to build a basis T giving good predictions for all desired values of p. A few classical solutions should be considered

- the basis of nominal modes $T = [\phi(p_0)_{1:NM}]$ [1],
- the enrichment of this basis by inclusion of mode shape sensitivities [2] (for their computation one can see [11]), thus $T = \left[\phi(p_o)\ \frac{\partial \phi}{\partial p_i}\right]$
- the multi-model approach [2] exact modes at a number of design points p_i are retained, thus $T = [\phi(p_1)\ \phi(p_{NE})]$

For a basis of nominal modes, predictions are valid for a fairly narrow parametric zone because the reduction basis does not account for actual model variations [2]. By including sensitivities, one significantly extends the range of validity but this range is not easily controlled. For a multi-model basis, mode shapes predictions are exact for the retained values $p_i, i = 1 : NE$ and fairly good between those points. In the rest of the paper, one considers a basis where one keeps the nominal modes and a number of other design points.

Building a multi-model reduction basis requires exact computations at target design points. The choice of these points is an important problem treated in the design of experiments [12].

As shown in figure 3 the uncertainty intervals are assumed to form an hypercube. A classical experiment would evaluate objectives at all corners, thus leading to 2^{NP} evaluations.

Building a multi-model basis using this full experiment is not realistic for the applications of interest. For the ESC-A model, keeping 15 modes uses 13 MB. Keeping modes for 2^{10} corners of a 10 parameter hypercube would require 13 GB which is not realistic both in terms of disk space and computational time.

The retained experiment uses the exact $NP + 1$ responses at the face centers. Which is much more practical. This however introduces an arbitrary selection of the *upper* face of the hypercube. In practice, one thus verifies the accuracy of reanalysis results on the opposite face using the error evaluation technique detailed section 2.

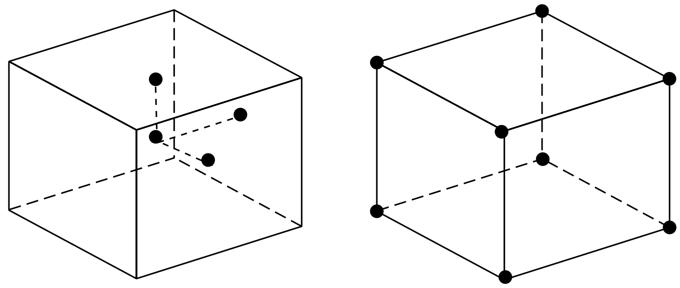

Figure 3: Positions of exact mode evaluations in parameter space. a) Hypercube face center. b) Classical 2^{NP} factorial plan.

The basis combining modes computed at each design point is generally poorly conditioned (the vectors are very collinear). One thus uses a complete reorthonormalization of this basis with respect to the nominal mass and stiffness matrices (the need to this orthonormalization is discussed in more details in [4]).

Finally, the uncertainty model to be propagated being defined by the parametric hypercube $p_i \in \left[p_i^{\min} p_i^{\max}\right]$, it is important to validate the precision of reanalysis predictions on the full domain. Possible strategies on the selection of points where this accuracy is estimated will be discussed in section 4.5.

4.3 Objectives and parameters

Quantities of interest in this study are modal frequencies and excitabilities. Excitability is defined by the value of j^{th} mode contribution at its resonance frequency, that is

$$e_j = \frac{[c]\{\phi_j\}\{\phi_j\}^T[b]}{2\zeta_j\omega_j^2\omega} \tag{8}$$

For applications, one considered the first 15 modes and 6 excitabilities associated with the transfer between engine gimbal joint and inertial measurements (SRI) in roll (θ_x to θ_x), pitch (u_z to θ_y), and yaw (u_y to θ_z).

One will see later that transmissibilities undergo strong variations during modal crossing. It thus appeared important to use concepts derived from MIMO control design methodologies.

Rather than considering individual transmissibilities, one can consider the 3 inputs (engine θ_x, u_z, u_y) and 6 outputs (rotations at both SRI). e_j then is a 6×3 matrix whose singular values can be used as objectives since they are much more stable. Similarly for close modes, one can consider the sum of transmissibilities for nearby modes [13].

The number of computations increasing exponentially with the number of parameters, it is important to only retain parameters that are really necessary. In the initial decomposition into substructures shown in figure 4 a single multiplicative stiffness parameter is retained.

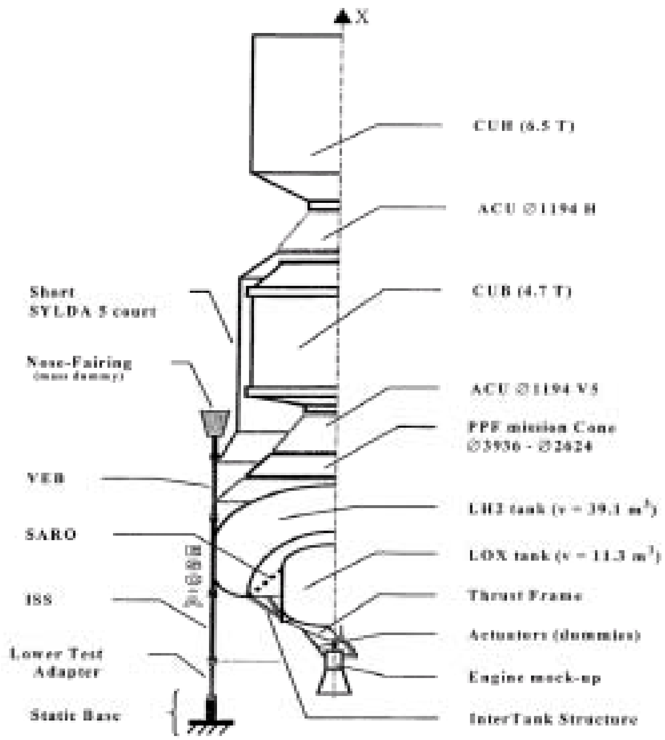

Figure 4: Model decomposition into sub-structures.

This is clearly a rough approximation but finding a logical basis for any aggregation of parameters is difficult. Engineering judgment and analysis of static test results led to retain 10 parameters shown in table 2.

TABLE 2: Paramètres retenus et variations maximales sur les 15 premières fréquences

#	Name	typ	cur.	min	max	$\max \Delta f$
1	461 ACU Basse	k	1	0.8	1.2	11 % (3)
2	430 Sylda5 court	k	1	0.9	1.1	6 % (12)
3	530 IS Skirt	k	1	0.9	1.1	2 % (15)
4	RLH2 upper	k	1	0.82	1.07	2 % (9)
5	RLH2 lower	k	1	0.82	1.07	1 % (8)
6	410 Case C	k	1	0.9	1.1	1 % (3)
7	481 Cone PPF	k	1	0.95	1.15	0 % (3)
8	520 BMA	k	1	0.95	1.05	1 % (7)
9	ITS	k	1	0.95	2.2	11 % (7)
10	570 Saro	k	1.12	1.12	1.27	1 % (6)

Analysis of target mode sensitivities was then used to allow further parameter elimination [9]. The end result of these analyzes is a table giving, for each target modes, the important parameters by order of importance.

4.4 Propagation analyses

Figure 5a shows evolutions of the first mode frequency. The figure shows the edges of the dimension 4 hypercube associated with parameters 2,3,4,6 as a function of the two parameters inducing the largest variations.

This display is generated using 160 evaluations, which is much below a coverage of the hypercube using random or structured experiments necessary for the constitution of histograms shown in figure 5b, while still giving an excellent indication of the range of variations for the objectives (frequency or excitability).

The first few modes are favorable cases, because they do not lead to modal crossing phenomena. Similar results are obtained for modes that do not show strong sensitivity to parameters. In other cases, one has difficulties illustrated for modes 5-7 in figure 6. There, one clearly sees that the range of variation of ITS properties can lead to two mode crossings. Such behavior could not be reproduced by perturbations or polynomial response surfaces. One will note that reanalysis can be seen as the creation of a response surface characterized by a rational fraction with the inverse of a polynomial dynamic stiffness.

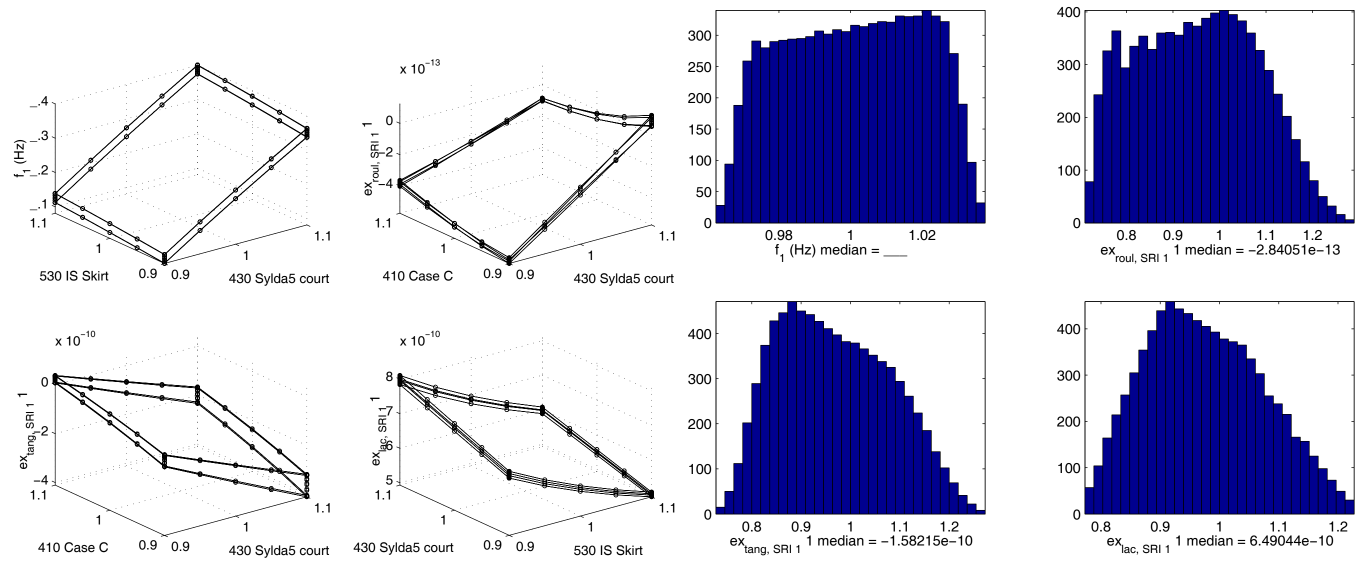

Figure 5: a) Evolution of mode 1 frequency on the edges of a 4 parameter hypercube. b) Corresponding histograms

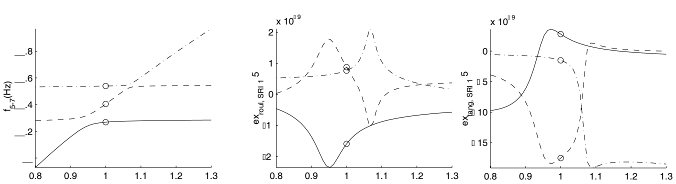

Figure 6: Evolution of frequencies and excitabilities during crossing of modes 5:7 for changes in properties of ITS (Inter Tank Struts). o **exact values.**

4.5 Computational strategies and error control

For the 105 000 DOF model of ESC-A, the evaluation of 15 modes at a parametric point is performed in 80s (PIII 1 GHz Linux, SDT [14] with `spfmex` static solver). A propagation study requiring several thousand points, find a compromise between accuracy and computational time is an obligation.

For the considered problem, building the reanalysis basis requires 500 s (for 6 full order solution) and reanalysis itself is approximately 1000 faster than a full order solve. The main factor influencing that acceleration is the size of the reanalysis basis, which depends on the number of parameters and the selection of target modes. One thus shows in [9] that is possible to speed computations up by focusing on a small number of target modes that need not be the first few.

The error evaluation strategy shown in section 4.2 is here only four times faster than an exact result. This is in great part due to the heavy cost of matrix / vector product for the mass matrix that has very large blocks linked to added mass for the fluids. Error evaluation could still be significantly optimized but gaining orders of magnitude is unlikely.

To control error, the initial idea was to select a richer experiment design than for the reanalysis building. A first control is actually performed by checking accuracy levels on negative face centers of the hypercube of figure 3. This check is coupled with the initial sensitivity analysis on parameters.

This first verification does not however validate the effects of correlated variations of multiple parameters. For a small number of parameters, the 2^{NP} factorial experiment (see [12]) associated with the corners of the hypercube gives a good confidence in results. But beyond ten parameters the cost associated with this check becomes prohibitive.

The retained strategy is to check error on points of hypercube edges shown in figure 5a where at least one of the objective scalars reaches an extremum. For the application extrema of frequency and transmissibility to SRI1 of modes 5-6-7, a number of extrema are reached at the same parametric points which leads to only 16 error evaluations.

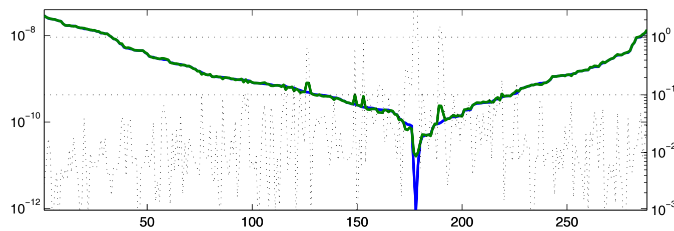

Figure 7: Errors on the prediction of transmissibilities on extreme points of objectives (modes 5:7, 6 parameters). Left scale and continuous lines: predicted values and results with one step enrichment sorted by value. Right scale and dotted line : relative error on the prediction.

Figure 7 shows that errors are small when transmissibilities are important. The few points where transmissibilities are not very small and the error visible could be used to enrich the reanalysis basis. It is interesting to note that the relative error on strain energy is always small ($< 10^{-6}$), which shows that this error is not a good indicator of errors on transmissibilities (local error on shape).

5 CONCLUSIONS AND PERSPECTIVES

A number of new results obtained during a collaboration between *EADS-ST* and *Ecole Centrale Paris* on the theme of model updating and uncertainty modeling were summarized here.

The main theoretical development is the residue iteration method, which generalizes reanalysis methods and provides a class of iterative solvers that are well suited for advanced problems in modal analysis. For applications in uncertainty propagation

- building of bases using the multi-model approach seems the most suited for a control of prediction error levels;
- the fundamental limit of the proposed approach is linked to the size of the retained bases. One can however control that size by eliminating parameters or focusing on a restricted number of target modes.

For model updating, most of the tools developed are now available in the SDT [14] environment. The initial correlation of the ESC-A model being very good, updating only allowed the correction of a few parameters. The main limitation of the model is a poor accounting for payload dissymmetry. One thus has a situation where the model cannot accurately represent reality without introducing significant modifications that could not easily be represented as updating parameters.

In the subjects that remain open, one will note the need to refine the strategy used to build an uncertainty model based on static tests of the structures; to introduce new parameters such as payload dissymmetry; to improve sensitive parameter selection tools and controlled basis enrichment; to define more precisely objectives that are pertinent for dynamic design; to improve the handling of mode crossing problems.

REFERENCES

[1] **Tourneau, P., P., B., Mercier, F.** and **Klein, M.**, *Applications of Scatter Analyses in Satellite Development*, CNES/ESA International Conference *Spacecraft structures and mechanical testing*, Cepadues Editions, 1994.

[2] **Balmès, E.**, *Parametric families of reduced finite element models. Theory and applications*, Mechanical Systems and Signal Processing, Vol. 10, No. 4, pp. 381–394, 1996.

[3] **Bouazzouni, A., Lallement, G.** and **Cogan, S.**, *Selecting a Ritz Basis for the Reanalysis of the Frequency Response Functions of Modified Structures*, Journal of Sound and Vibration, Vol. 199, No. 2, pp. 309–322, 1997.

[4] **Bobillot, A.**, Méthodes de réduction pour le recalage. Application au cas d'Ariane 5, Ph.D. thesis, Ecole Centrale de Paris, 2002.

[5] **Bobillot, A.** and **Balmès, E.**, *Iterative techniques for eigenvalue solutions of damped structures coupled with fluids*, SDM Conference, 2002.

[6] **Bobillot, A.** and **Balmès, E.**, *Expansion par minimisation du résidu dynamique. Résolution et utilisation pour la localisation d'erreur.*, Revue Européenne des Éléments Finis, numéro spécial, 2002.

[7] **Balmès, E.** and **Germès, S.**, *Tools for Viscoelastic Damping Treatment Design. Application to an Automotive Floor Panel.*, ISMA, September 2002.

[8] **Deraemaeker, A.**, Sur la maîtrise des modèles en dynamique des structures à partir de résultats d'essais, Doctoral dissertation LMT/ENS Cachan, 2001.

[9] **Balmès**, *Propagation de méconnaissances en analyse modale.*, Rapport EADS-LV, December 2002.

[10] **Ghanem, R.**, *Uncertainty characterization, propagation and management in model-based predictions.*, Cinquième Colloque National en Calcul des Structures, pp. 59–73, 2001.

[11] **Bobillot, A.** and **Balmès, E.**, *Iterative Computation of Modal Sensitivities*, Accepted by AIAA Journal, 2003.

[12] **Myers, R. H.** and **Montgomery, D. C.**, Response Surface Methodology, Wiley Inter Science, New York, 1995.

[13] **Balmès, E.**, Modèles analytiques réduits et modèles expérimentaux complets en dynamique des structures, Mémoire d'habilitation à diriger des recherches soutenue à l'Université Pierre et Marie Curie le 10 juillet 1997, 1997.

[14] **Balmès, E.** and **Leclère, J.**, Structural Dynamics Toolbox 5.1 (for use with MATLAB), SDTools, Paris, France, http://www.sdtools.com, October 2003.