Error characterization in modal analysis and model updating.

An overview of tools and procedures using the Structural Dynamics Toolbox.

Etienne BALMES (ENSAM PIMM & SDTools)
Guillaume MARTIN

SURVISHNO
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Model validation and verification

CAD Model

Experimental model

Continuous model + interfaces

Verification

Design

Validation (Updating)

Dispersion

FE Model
Identification

known @ sensors

\{ y(t) \} = [c] \{ q(t) \}

Correlation

distance between test & FEM

Topology correlation

= observe FEM @ sensors

Updating

= estimate parameters optimizing correlation

Expansion

= estimate all DOF knowing test and model
Tutorial objective: where is the error?

TEST CASE: drum brake plate

G. Martin, « Méthodes de corrélation calcul/essai pour l’analyse du crissement »
Ph.D. CIFRE SDTools, Arts et Metiers ParisTech, Paris, 2017

Identification error
• Noisy measurements
• Identification bias
• NL, time varying, ...

Topology errors
• sensor/act position
• matching

FEM error
• Geometry
• Material parameters
• Contact properties
Direct and inverse problems in vibration

### Direct

- **Family of models**
  - FEM (necessary for geometry)
  - Linear / time invariant / band limited / fixed inputs
  - Non-linear: constitutive 2-3D / equivalent 0D

- **Parameters**
  - Model parameters (previous direct/inverse problems)
  - Loading (space/time)
  - Operating conditions

- **Solver**
  - Frequency / transient
  - Reduction / component mode synthesis

- **Objective functions**
  - Resonant frequency placement
  - Displacement, stresses, ...
  - Max, mean, statistics, ...

### Inverse

- **Family of models**
  - Signal coming from a system with low noise
  - LTI system, non-linear, time varying, ...
  - Hybrid test/FEM (extended parametric LTI)

- **Parameters**
  - Transfer function, non-linear kernels
  - Modal parameters
  - Noise sources

- **Objective functions**
  - Time / frequency domain output error
  - Subspace error

- **Solver (identification / optimization)**
  - H1 estimator, Hilbert transform, ...
  - LSCF, NL-Optim
Experimental modal analysis : data

- Laser vibrometer (measure)
- Measurement point
- Plate
- Shaker (excitation)

Diagram:
- Computer to drive acq.
- Signal generation
- Power amplifier
- Shaker
- Force measurement
- Response sensors
- Signal conditioning (amplification)
- Analyzer
Assumption LTI = transfers exist

\[ \{Y(\omega)\} = [H(\omega)]\{U(\omega)\} \]

ONE input

ONE output

MANY resonances

Transfers estimated from time response

Test 1001z, 12-z

Some standard texts for the vibration community

**LTI. Space frequency separation**

![Graph showing frequency response with peaks and valleys.](https://youtu.be/LCQljBOtuYQ)

<table>
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<th>Type</th>
<th>Name</th>
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$s_2 \text{ freq.}=446.875\text{Hz (6.64\%)}$
Learning shapes in squeal event

Time measurement during squeal

Variability
- influence of wheel angle

Reproductibility
- Multiple events

Sample time/frequency responses

Braking event 1
Braking event 2
Braking event 3
What is expected from theory?

Shape 1: real mode with normal contribution

\[ \sum_{i} \left\{ \begin{array}{c} U_i \\ \downarrow \text{sensors} \end{array} \right\} \{a_i(t) \rightarrow \text{time}\} \]

2 DOF: amplitudes of each shape

Parameter: friction = non symmetric coupling

normal displacement \(\Rightarrow\) tangential load

\[
[K_u] = [b_{TAN}] \begin{bmatrix} \mu_w J \frac{\partial p}{\partial q} \end{bmatrix} [c_{NOR}] \\
\mu=0.1 \text{ at } 5052 \text{ Hz } -0.13 \% 
\]

[\{q\}_N = \begin{pmatrix} T \\ N_x \end{pmatrix}_{N_R}]

Shapes constant / DOF (function of parameter)

- Start $\mu = 0$ small damping
- increase $\mu$ : coupling and transition towards instability
Participation of real shapes to complex modes

Normal

Tangent

Amplitude of real modes

Real mode phase

Complex

Damping [%]

Frequency [kHz]

Real mode participation

Angle [rad]
Subspace learning & basis selection

\[ y(c, f_{\text{max}}(t)) \rightarrow \text{time} \]

- only 2 significant real shapes & 2 associated DOF
- shapes independent of parameter
- Parameter: difficult to control (wheel position, brake event, ...) but exists

SVD

\[ \sum_i \left\{ \begin{array}{c} U_i \\ \downarrow \text{sensors} \end{array} \right\} \{a_i(t) \rightarrow \text{time} \} \]

Details: G. Martin & al. ISMA 2018
SVDCur button in SDT

\[ f(\text{max } v_i), \sigma_i \]

\[ a_i(f) = \sigma_i v_i(f) \]

\[ U_i \]
Experimental modal analysis: an inverse problem

Data
- Transfer
- Hankel mat

Objective $H_{\text{test}} - H_{\text{id}}$

Optimization

Result: parameters

Family of models

SVD $\sum_i \{ a_i(t) \to \text{time} \}$

State space $\{ \dot{x} \} = [A]\{x\} + [B]\{u\}$

Pole residue $\sum \frac{[R_j]_{NS \times NA}}{s - \lambda_j}$

Rational fraction, modal model, second order ...
Classical linear system Id
Id Phases

• **Initialization**:
  - Pick (stab or single pole estimate [1])?
  - Stabilization diagram [2]

• **Estimate by band (why ? [1,3])**

• **How can problems be detected ? [3-4]**

• **Re-optimize poles (why ?)**

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Noise induced bias

Inconsistence between data and family of models induces bias/variability

Quality : an error criterion

Is the model well identified?

- Superpose measured and identified FRF for each sensor, around each mode
- Compute error on Nyquist

\[ e_{j,c} = \frac{\int_{\omega_j}^{\infty} (1 + \zeta_j) \left| H_{\text{Test,c}}(s) - H_{\text{id,c}}(s) \right|^2}{\int_{\omega_j}^{\infty} (1 - \zeta_j) \left| H_{\text{id,c}}(s) \right|^2} \]

“Around each mode”:
- Half power : +/- \( \zeta_j \omega_j \)
- Visible peak : +/- 5\( \zeta_j \omega_j \)
For each sensor, each mode may have strong error/noise & low contribution.

Test error per mode/sensor

Evaluate quality of identification
Other errors: NL ≠ one system

- Non linear system: resonance dependent on input point / accurate positioning
- MMIF or identification per impact location shows significant dispersion
- Multiple identification results are not perfectly coherent
Other errors: local modes

- "Same mode" multiple times

- Non-structural masses generate global mode duplicates
- "Small mass" ≠ "Tiny peak" (if this were true, proof mass dampers would not work)
Summary: experimental modal analysis

• Space/time decomposition applies very often

• Base assumption LTI (linear time invariant) should be challenged

• Inconsistent data & model family $\Rightarrow$ bias rather than variance

• Error classification is per mode/sensor

• General handling of NL & local modes is beyond state of the art
Identification error
- Noisy measurements
- Identification bias
- NL, time varying, ...

Topology errors
- sensor/act position
- matching

FEM error
- Geometry
- Material parameters
- Multi-scale problems & equivalent parameters
- Junction representation
- Design change & modal property tracking
Geometry updating

- Up to 2 mm on surface
- 11% error on volume

Nominal geometry

Measured geometry

Geometry correlation

Inverse CAD (Geomagic, ...)
Morphing (SDT)
Geometry updating

- Frequency band: 0 - 6kHz
- Notable shape difference >3 kHz
- Frequency error
  - 2 % mean
  - 5 % max

Checking geometry should always be the first step
Geometry parametrization / morphing

- **Shape optimization / morphing**

  \[ P(x) = \sum_{i} \{p_i(x)\}q_{\text{imaster}} \]

  `fe_shapeoptim BuildFromSel`
  1) fix bottom face, 2) prescribe edge motion
  3) deform edges (straight)
  4) deform faces (flat)
  5) deform interior (good elements ?)

- **Process simulation + field projection**

  `fe_shapeoptim Interp`

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Direct problems: material parameters

- Uniform
- Field
- Equivalent (at certain scales)

- Geometry
- Material parameters
- Junction representation
- Equivalent parameters

Basic parametrization tool: dependence on constitutive parameters $C_{ij}$

$$K = \int_{\Omega} \{\epsilon\}^T [C_{ij}] \{\epsilon\} = \sum_g B^T [C_{ij}] B w_g = \sum_{ij} C_{ij} \left[ \sum_g B^T [C_{ij}^u] B w_g \right] = \sum_{ij} C_{ij} [K_{ij}]$$
Element/model weights

\[ K(p) = \sum_e \alpha_e(p)[K^e] \]

**Weighted element matrices = standard**

- Element-wise (topology optimization)
- Field/groupwise
  - parameter groups ...
  - solution of eigenvalue problem (polynomial chaos, Ghanem, Soize, ...)
  - clustering

Clustering of Parameter Sensitivities: Examples from a Helicopter Airframe Model Updating Exercise. Shahverdi, Mottershead & All
SDT: upcom / zCoef

- **Element wise** \( K(p) = \sum_e \alpha_k^e [K^e] \)

\[
\text{mind} = \begin{bmatrix}
M_s & M_e & K_s & K_e & \alpha_m & \alpha_k \\
\vdots & & & & & \\
elt & & & & & \\
\end{bmatrix}
\]

- **Group wise** \( K(p) = \sum_g \alpha_g [K^g] \)

www.sdtools.com/help/upcom.html

www.sdtools.com/help/zCoef.html

```matlab
zCoef={'Klab','mCoef','zCoef0','zCoefFcn';
'M' 1 0 '-w.^2';
'Ke' 0 1 1+i*fe_def('DefEta',[]);
'Kv' 0 1 'par(1)';
```
Updating of material properties

Modal frequencies

- Weight / volume ⇒ density
- Frequencies ⇒ modulus
- Test/FEM distance below physical dispersion (df/f ≈ 2%)

Weight

Frequency correlation

\[ J = \min_{E} \left( \sum_{i} \left| \frac{\Delta f_i}{f_i} \right| \right) \]

Wrong volume = bias
Equivalent materials / homogeneization

Family

• **Micro**: cell walls, glue, face-sheet, viscoelastic material
• **Macro**: shell/ orthotropic volume/shell

Objective

• Equivalence: waves/modes

Parameters

• orthotropic law
• Scale separation

Direct problems: visco junction parameters

- Geometry, material
- Junction representation
- Equivalent parameters

**Test of visco mount**

\[ F_x = f(x, \dot{x}) \]
- Measure disp/load
- Macro: results mix material and geometry

**0D / macro / meta-model**

Behavior
- material & structure
- \[ F_x = f(x, \dot{x}, x_i, p) \]

**Identification (p)**


**3D physical / micro Knowledge**

- FEM structure
- Implicit/explicit time
- \( \{\sigma\} = f(\{\varepsilon\}, \{\dot{\varepsilon}\}, \{\varepsilon_i\}, p) \)
- Hyper-visco-hysteresis, mat

**Multi-body / system simulation**

Use cases
- Engine suspension
- Drive line
- Comfort, endurance


Visco junctions a few keywords

Material
• 3D finite strain (visco/hysteretic)
• Effects: Mullins, Payne, Madelung, Masing, ...

Structure
• Micro/macro transition
• Reduction / hyper-reduction (macro-Gauss points)
Direct: contact/friction junction parameters

Test

Identification (p)

2D physical /

Identification (p)

Reduction & Hyper-réduction

0D / macro / meta-model

System simulation

Transient simulation

[1] FUI CLIMA (Conception de Lliaisons Mécaniques Amortissantes)
Junctions: contact / 0D strategies

Typical simplification: mass & stiffness

Kinematic junctions (pivots, ...) often difficult with 2D

Equivalent spring build may require complex numeric/test identification
Sample junction problems

- **Master nodes for multi-body**

  Type B links : 82 Hz  
  Type A links : 49 Hz

- **Weld spots**

- **Contact around WS**
Junction parameters: contact surface/stiffness

- Variable contact surface and stiffness
- Comparable impact frequencies and shapes

Equivalent stiffness: frequency/stiffness map

- Sensitivity to prestress/amplitude $\rightarrow$ tangent stiffness map
Numerical methodologies useful/necessary to treat 2D

- **StaticStatus** iterate on nodes either bilateral or no contact

- **StaticModeTraj** [1] series of statics with \([M]\{\phi _j\}\) load

- **Multi-model reduction** [2] Ritz analysis on a few learning points

On the need for equivalent laws

Physical contact laws are sensitive to accuracy on

• Time events

• Spatial events

Equivalent 2D law needed to approximate both
Design change & pole tracking

- Choose a reference $Z(p) = (K - \omega^2 M) = Z_0 + (Z(p) - Z_0)$
- Compute reference modes $\{\phi_j(p_0)\}, \omega_j(p_0)$
- Use generalized coordinates/energies [1] rather than MAC
  
  $\alpha_j(t) = \{\phi_j^0\}^T [M(p_0)] \{q(t)\}$
  
  $\{e_j(t)\} = \dot{\alpha}_j^2 + \omega_j^2 \alpha_j^2$

- Possibly use component modes

  - CMT [2]: free +global
  
  $\begin{bmatrix}
  \phi_A^0 & \phi_{global}^0 \\
  \phi_B^0 & \phi_{global}^0
  \end{bmatrix}_A$

  $\begin{bmatrix}
  I_A \\
  \phi_{rig}^0
  \end{bmatrix}_B$ ⊥

  - Flexible with rigid global [3] in subspace

KnKt study (SDT contact)

https://youtu.be/A9V2MtKRcS8
Summary: parametric direct problems

- **Geometry updating key**: inverse CAD or morphing
- **Material parameter classics**
  - grouping/clustering/field generation
  - homogeneization & scale separation
- **Junction parameters**
  - 0D or 2/3D laws
  - Equivalent 2D for contact (time/space constraints)
- **Parametric studies**
  - Reduction is key for **CPU time AND memory**
  - Tracking tools needed to make sense of data

Design exploration 1000 points
- Full 80 days CPU, 22 TB
- CMT a few hours off-line learning, <1h exploration, 10 GB
Hybrid test & FEM

Identification error
- Noisy measurements
- Identification bias
- NL, time varying, ...

FEM error
- Geometry
- Material parameters
...

**Topology correlation** = observe FEM @ sensors

**Correlation** = distance between test & FEM

**Updating** = estimate parameters using correlation

**Expansion** = estimate all DOF knowing test and model
Basic topology correlation process
Topology correlation process

- Initial orient (coarse)
- *Give reference points & reposition*

- Optimize orientation to minimize distance

- Analyze errors
Test topology objectives: view & correlate

- Connect to allow viewing
- Use regularity to avoid distortion
- Use exact topology
- Triax measurement is not the universal answer (⇒ expand)
Basic shape correlation

https://youtu.be/tVV-R2r-3M8
Correlation: mode crossing & MAC combine

- High sensitivity for close modes associated with mode crossing
- MAC can drop to 0.5
- MAC combine linear combination of close modes = stays at 1.
- Orthonormal combination of modes in a group
  \[ A = \arg \min ||[\phi_{test}]_{gA} - [c\phi_{FEM}]_{gB}[A]|| \]
MAC & sensors : historical methods

- **MAC on sensor sets**
  - 😊 Localize area of poor correlation
  - 😞 No automated set definition

- **COMAC : is a specific sensor bad ?**
  - correlation of multiple modes at one sensor
  - eCOMAC : allow incorrect modeshape scaling
  - Not per mode / sensor

- **MACCo : does correlation improve with less sensors ?**
  - 😊 Easy to implement
  - 😊 Sensor sets per mode or global
  - 😞 No understanding of why each sensor is removed.

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MAC & sensors : MACCo variants

MACErr : Ignore sensors with bad identification

MACCo : sort sensors by impact on MAC
Removed sensors may indicate
• Bad measurements
• Model errors
Updating : freq-MAC objective

Family : FEM with updated geom.
Parameters : mat/junction
Data : test modes

Objective function \( J_{\text{freq-MAC}} \)

\[
R_j(p) = \left\{ \begin{array}{c}
\frac{f_{EF,i} - f_{Test,j}}{f_{Test,j}} \\
\beta(1 - MAC_{i,j})
\end{array} \right\}
\]

\[
J_{\text{freq-MAC}}(p) = \sqrt{\sum_{j=1}^{NM} \| R_j(p) \|^2}
\]

Associated problems

- Mixed quantity (scaling)
- Jumps on mode pairing
- Parameter choice
- Sensitive range
Parameter equivalence / conditionning

Vertical bending

Tests

Observable modes

Identification

Torsions

Solution is not unique
Mode-shape very similar

Assumption must be made to improve conditioning

- $G_{31}/E_1$ constant ?
- $E_1$ traction ?

$c^{-1} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$
Sample clustering method

- Compute cosine distance

\[
\cos^{-1} \left( \max \frac{\langle u|u \rangle}{\|u\|\|v\|} \right)
\]

\[
A, B \rightarrow U_A, U_B \rightarrow \sigma_{\text{max}} \left( U_B - U_A (U_A^H U_B) \right)
\]

- Recursively group elements with smallest distance

Shahverdi, Mottershead & All, Clustering of Parameter Sensitivities: Examples from a Helicopter Airframe Model Updating Exercise.
Expansion / hybrid models

Test error:
Reverse sensor direction

\[ [c] \{\dot{\phi}\} - \{y_{Test}\} = \epsilon_{Test} \]

Model error:
coupling stiffness

\[ \{R_L\} = [K - \omega_j^2M] \{\dot{\phi}\} \neq 0 \]
\[ \{R_D\} = [K]^{-1}\{R_L\} \]

Residuals for test and model error

Load residual
Displacement residual
MDRE : multi-objective cost function

\[ J(\phi_{exp}) = \| \epsilon_{Test}(\phi_{exp}) \| + \| \epsilon_{Mod}(\phi_{exp}) \| \]

multi objective cost function

Test error

Model error

\[ \epsilon_{Test} = \| [c] \{ \phi_{exp} \} - \{ y_{Test} \} \| \]

\[ \{ R_L \} = [K - \omega_j^2 M] \{ \phi_{exp} \} \neq 0 \]

\[ \epsilon_{Mod} = \| [K]^{-1} \{ R_L \} \|_K \]
Expansion: robust to model error

Softer coupling

Stiffer coupling

Robustness band

Coupling stiffness

True
Model error = clues for parameterization

Model error concentration at sensors
Decoupling model error on
• Model modes
• Enrichment

Concentration of model error close to the coupling

Model error: guides model parameterization
Test error: can be used to locate poor sensors
Expansion: an old method with renewed interest

Early work: Ladeveze 72 -> [1] present
Independent start: Balmes [2] 00 -> present
Not much used for 40 years due to computation challenge (see [3])

Renewed interest at SDTools (last 3 years) due to reduction
- Modes / multi-model + sens \([K]^{-1} [b]\)
- A few thousand DOF, a few seconds per point

Potential uses
- Updating
- Frequency domain
  structural dynamics modification [4]
- Time domain (extended Kalman Filtering)

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Summary: hybrid test/FEM

• With enough sensors (extreme case field measurement)
• Reasonable match between model and reality (LTI, no local modes, ...)

... updating (=parametric identification) works

If it does not, don’t throw the model away
• Lots of things will never be measured: hidden faces, internal loads, ...
• Many can be estimated even with somewhat wrong model
• Hybrid models have used efficiently for decades
• Reduction extends usability to vibration problems

“Digital twin” is the current hype keyword for this old idea
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