Error characterization in modal analysis and model updating.

An overview of tools and procedures using the Structural Dynamics Toolbox.

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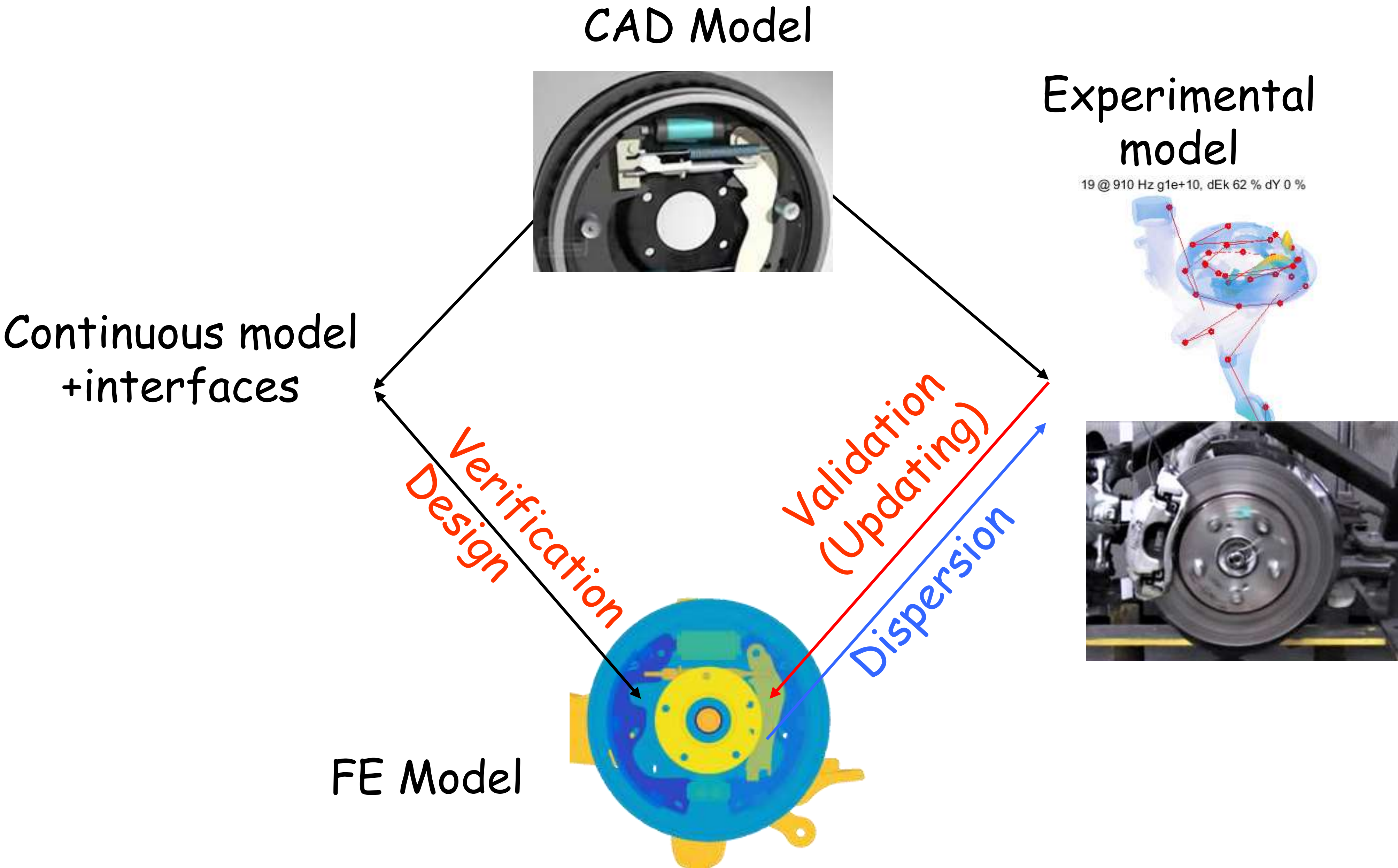
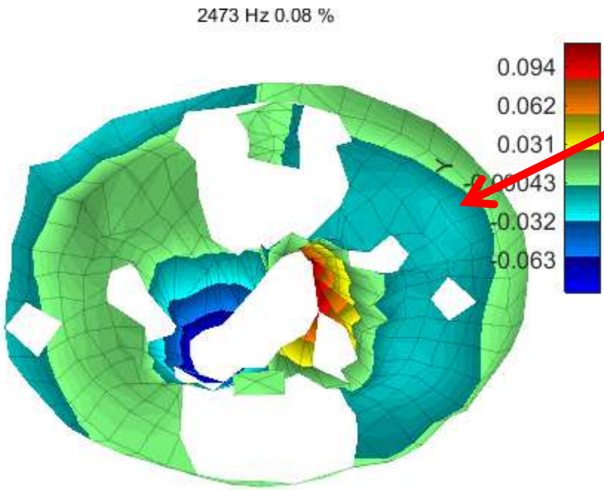
Lyon, July 8, 2019

Model validation and verification



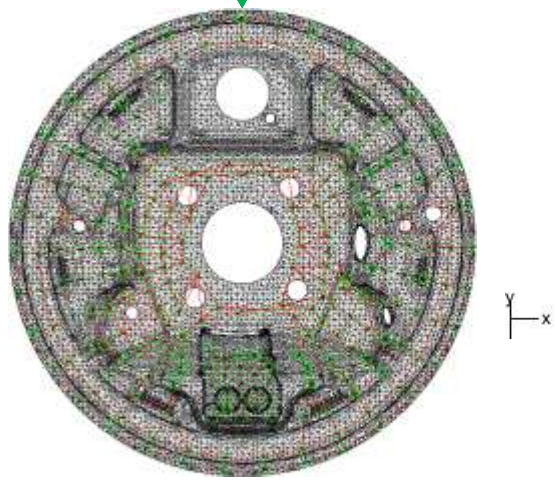
Test, FEM, hybrid models

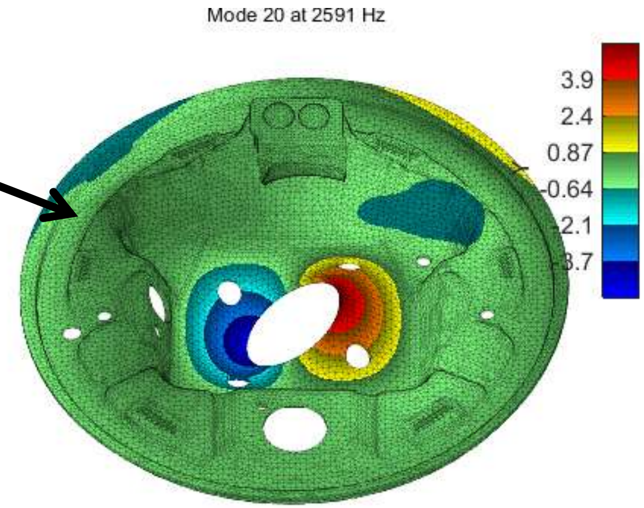
$$\{y(t)\} = [c]\,\{q(t)\}$$



Identification

known @ sensors





FEM known @ DOF

Topology correlation = observe FEM @ sensors

Correlation = distance between test & FEM

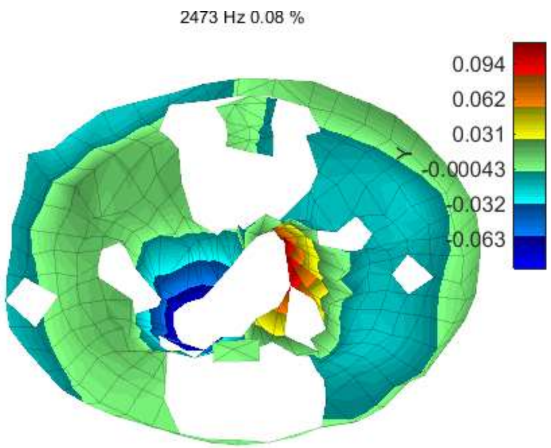
Updating = estimate parameters optimizing correlation

Expansion = estimate all DOF knowing test and model

Tutorial objective : where is the error?

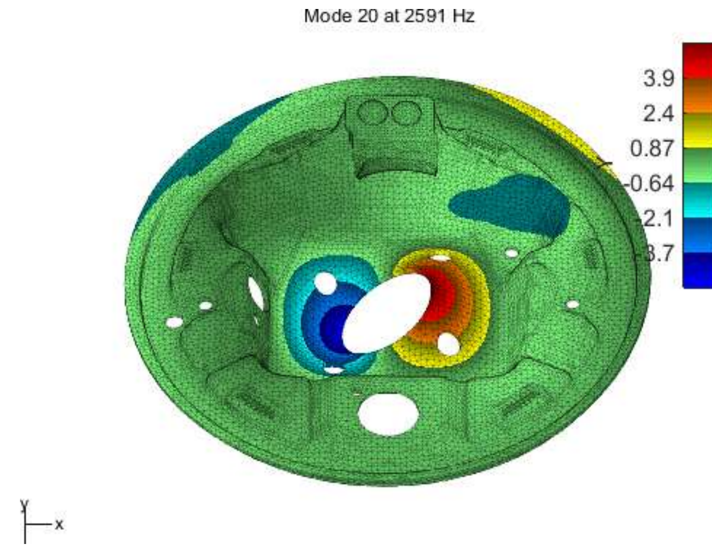
TEST CASE : drum brake plate

G. Martin, « Méthodes de corrélation calcul/essai pour l'analyse du crissement »
Ph.D. CIFRE SDTools, Arts et Metiers ParisTech, Paris, 2017



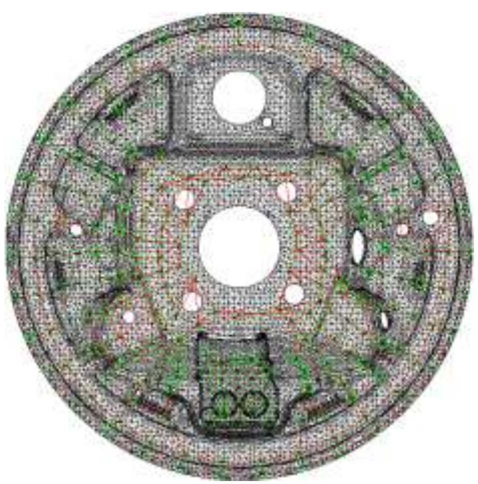
Topology errors

- sensor/act position
- matching



Identification error

- Noisy measurements
- Identification bias
- NL, time varying, …

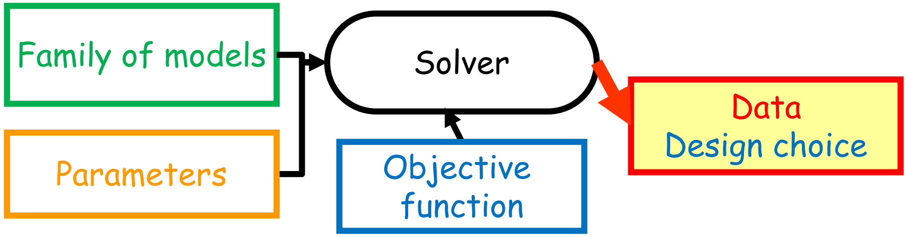


FEM error

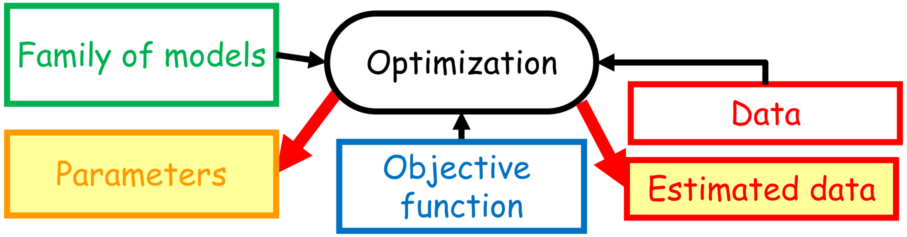
- Geometry
- Material parameters
- Contact properties

Direct and inverse problems in vibration

Direct



Inverse



Family of models
- FEM (necessary for geometry)
- Linear / time invariant / band limited / fixed inputs
- Non-linear : constitutive 2-3D / equivalent 0D

Parameters
- Model parameters (previous direct/inverse problems)
- Loading (space/time)
- Operating conditions

Solver
- Frequency / transient
- Reduction / component mode synthesis

Objective functions
- Resonant frequency placement
- Displacement, stresses, …
- Max, mean, statistics, …

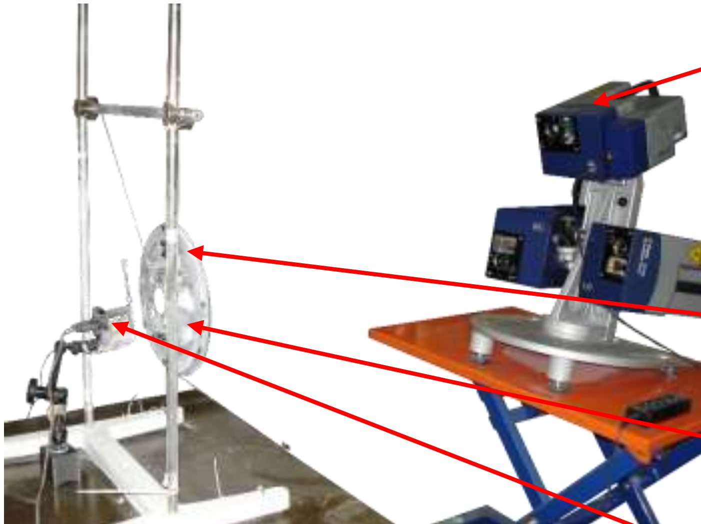
Family of models
- Signal coming from a system with low noise
- LTI system, non-linear, time varying, …
- Hybrid test/FEM (extended parametric LTI)

Parameters
- Transfer function, non-linear kernels
- Modal parameters
- Noise sources

Objective functions
- Time / frequency domain output error
- Subspace error

Solver (identification / optimization)
- H1 estimator, Hilbert transform, …
- LSCF, NL-Optim

Experimental modal analysis : data

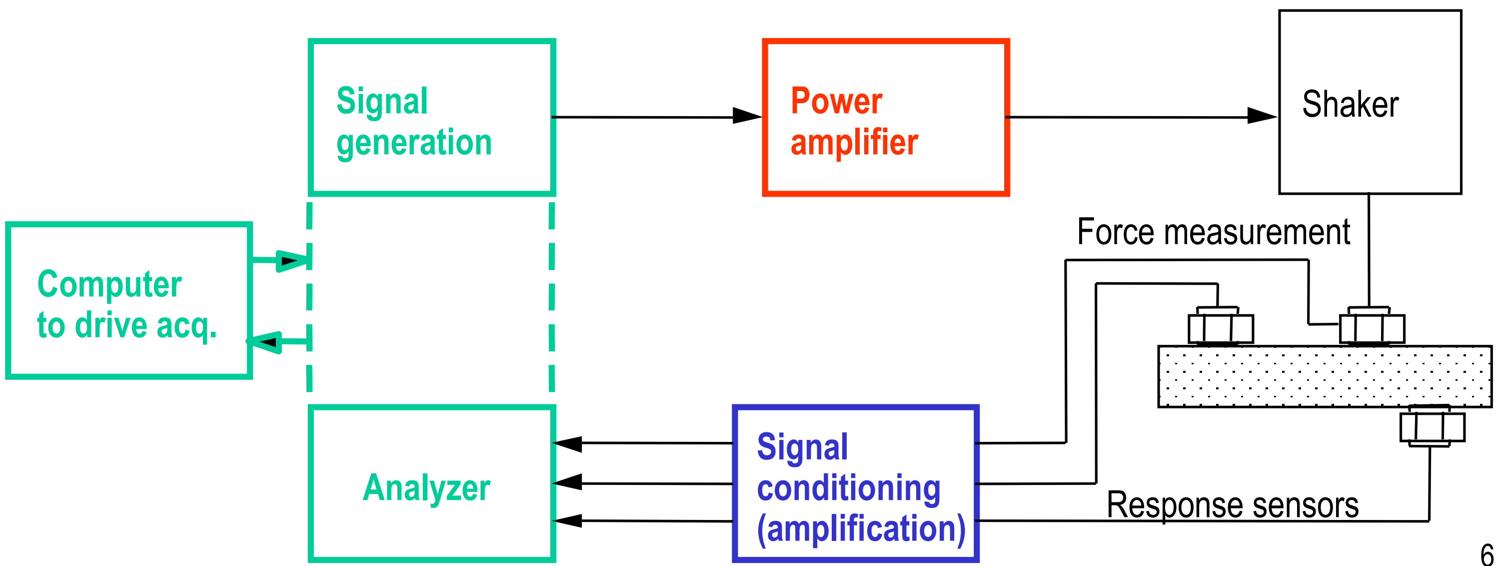


Laser vibrometer (measure)

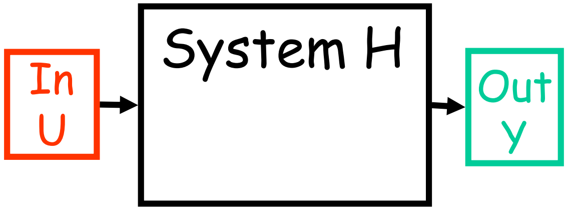
Measurement point

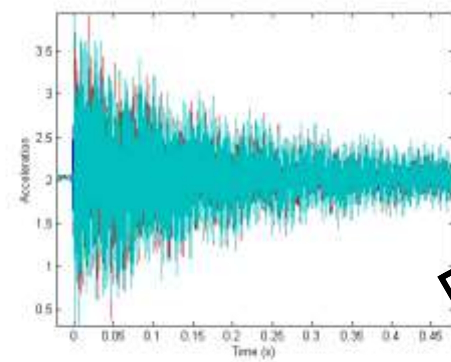
Plate

Shaker (excitation)



Assumption LTI = transfers exist





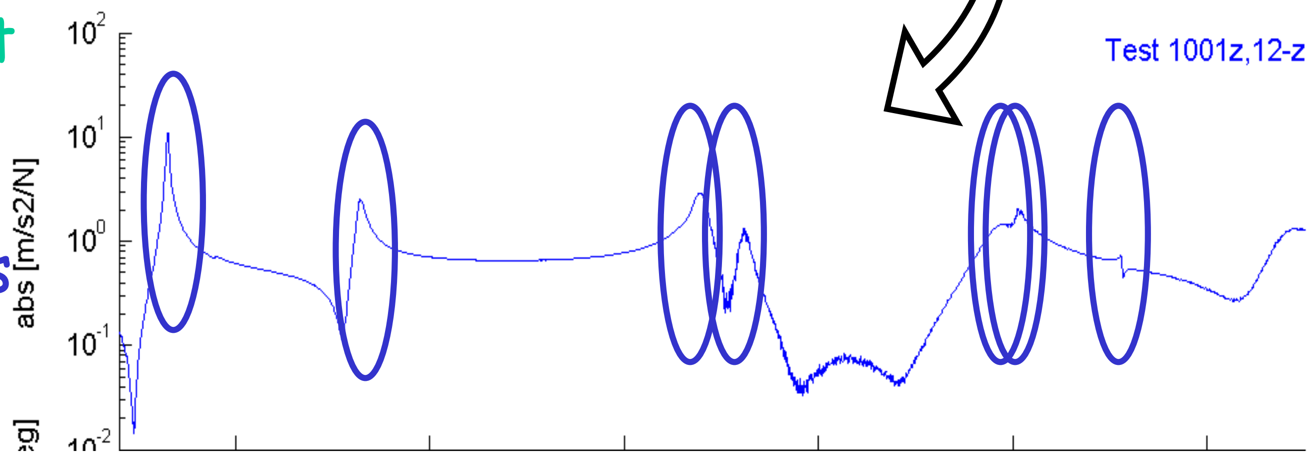
Transfers estimated from time response

ONE input

ONE output

MANY resonances

$$\{Y(\omega)\}=[H(\omega)]\{U(\omega)\}$$



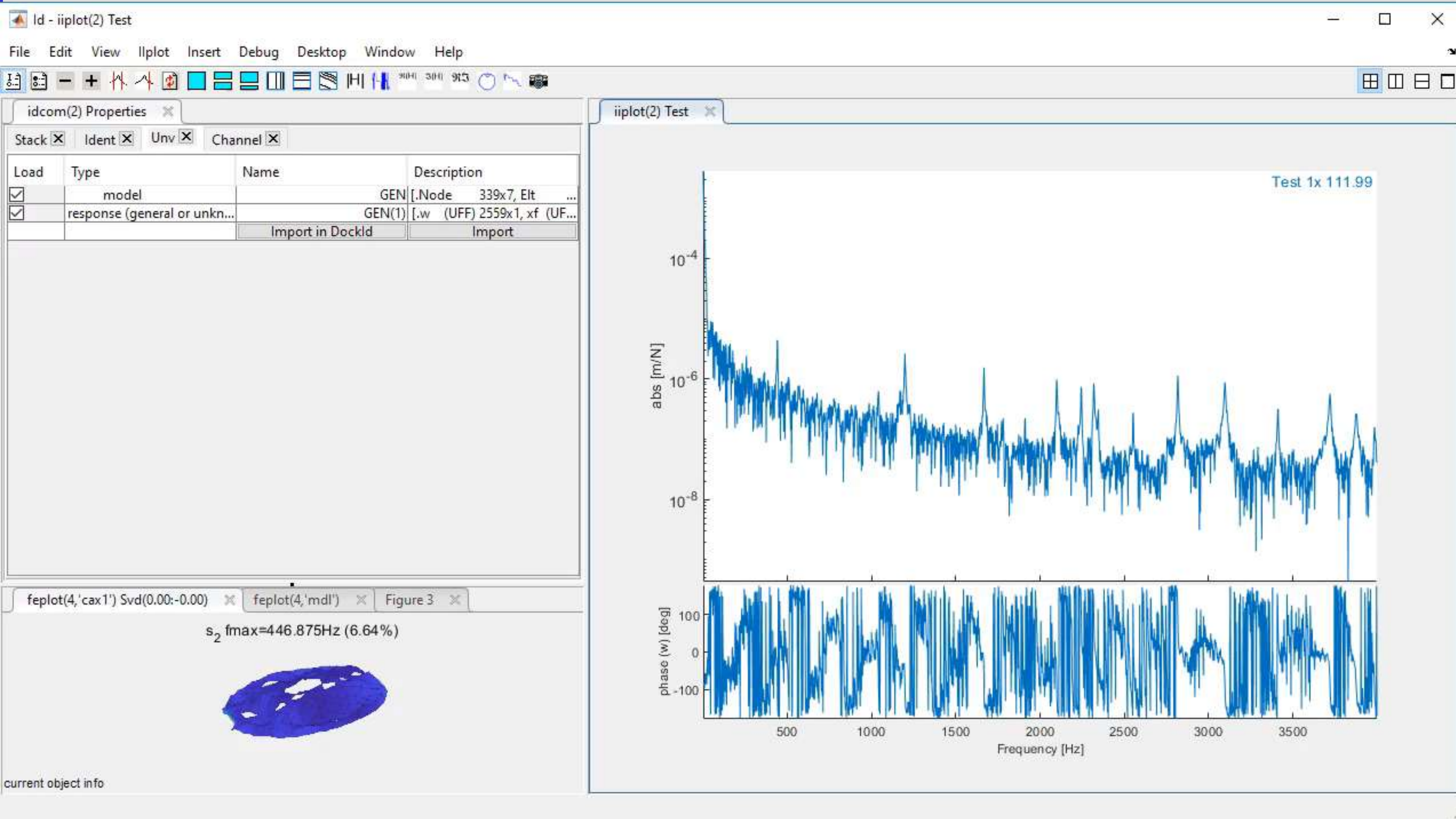
Some standard texts for the vibration community

[1] D. J. Ewins, Modal Testing: Theory and Practice. John Wiley and Sons, Inc., New York, NY, 1984.

[2] W. Heylen et P. Sas, Modal analysis theory and testing. Katholieke Universteit Leuven, Departement Werktuigkunde, 2006

[3] K. G. McConnell, Vibration Testing. Theory and Practice. Wiley Interscience, New-York, 1995

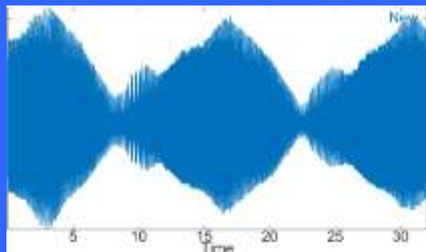
LTI. Space frequency separation



https://youtu.be/LCQljBOtuYQ

Learning shapes in squeal event

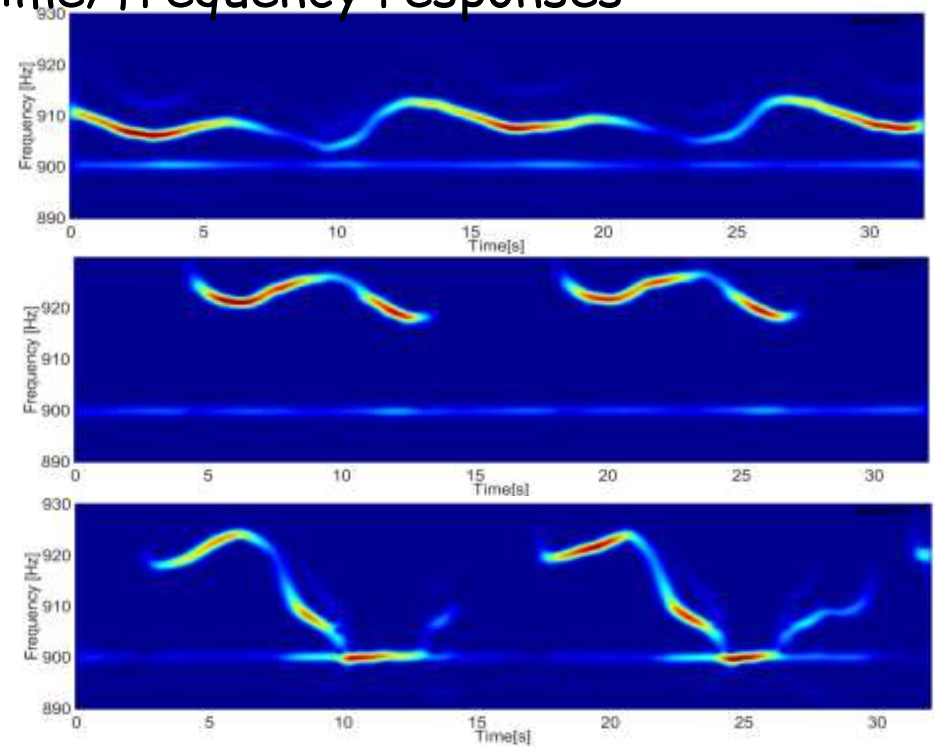
Time measurement during squeal



Variability
- influence of wheel angle

Reproductibility
- Multiple events

Sample time/frequency responses



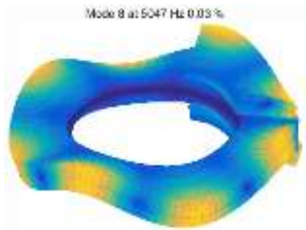
Braking event 1

Braking event 2

Braking event 3

What is expected from theory?

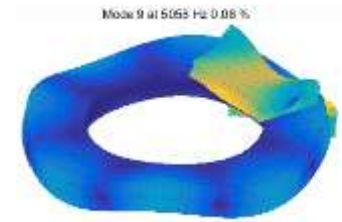
Shape 1 : real mode with normal contribution



$$\sum_i \begin{Bmatrix} U_i \\ \downarrow \\ sensors \end{Bmatrix} \{a_i(t) \quad \rightarrow \quad time\}$$

2 DOF : amplitudes of each shape

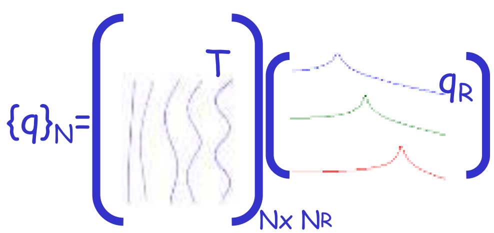
Shape 2 : real mode with tangential contribution

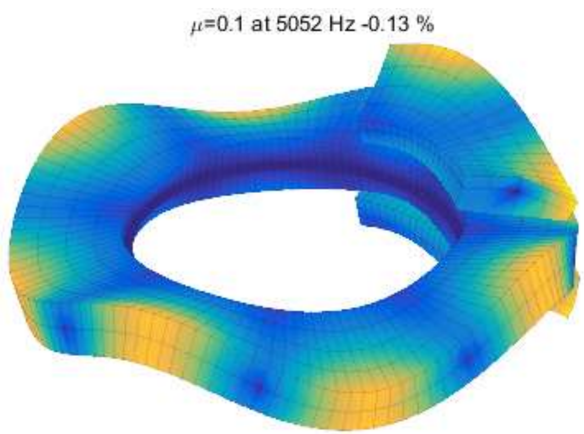


Parameter : friction = non symmetric coupling

normal displacement $\Rightarrow$ tangential load

$$[K_u] = [\boldsymbol{b}_{TAN}] \left[\diagdown \mu w_j J \frac{\partial p}{\partial q} \diagdown \right] [\boldsymbol{c}_{NOR}]$$

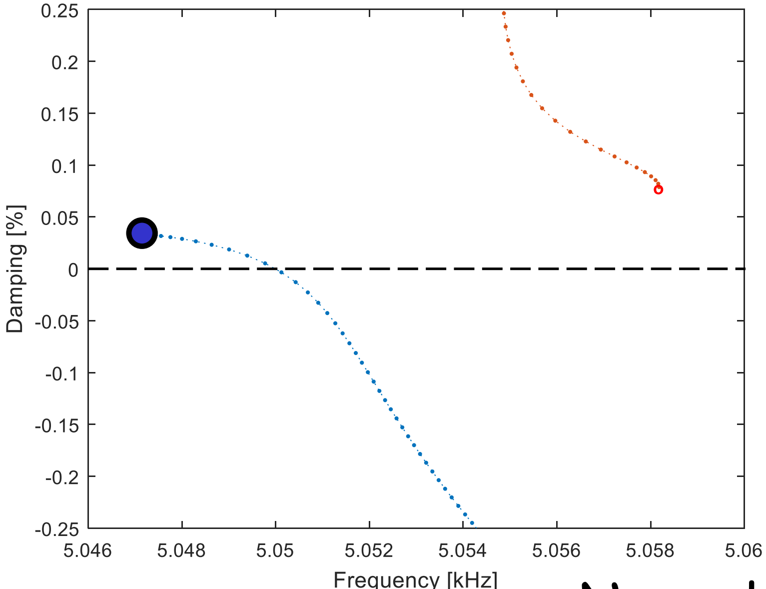




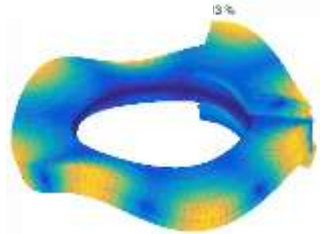
G. Vermot des Roches, « Frequency and time simulation of squeal instabilities. Application to the design of industrial automotive brakes », Ph.D. ECP, CIFRE SDTools, 2011

Shapes constant / DOF (function of parameter)

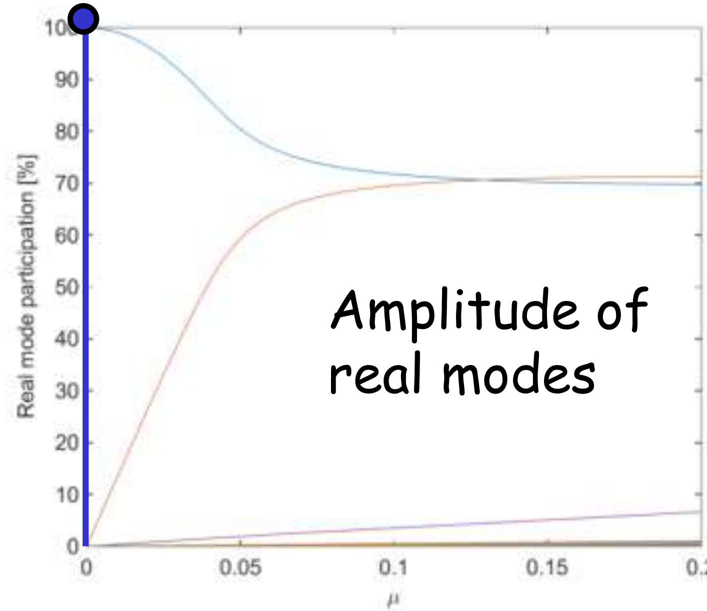
- Start $\mu = 0$ small damping
- increase μ : coupling and transition towards instability



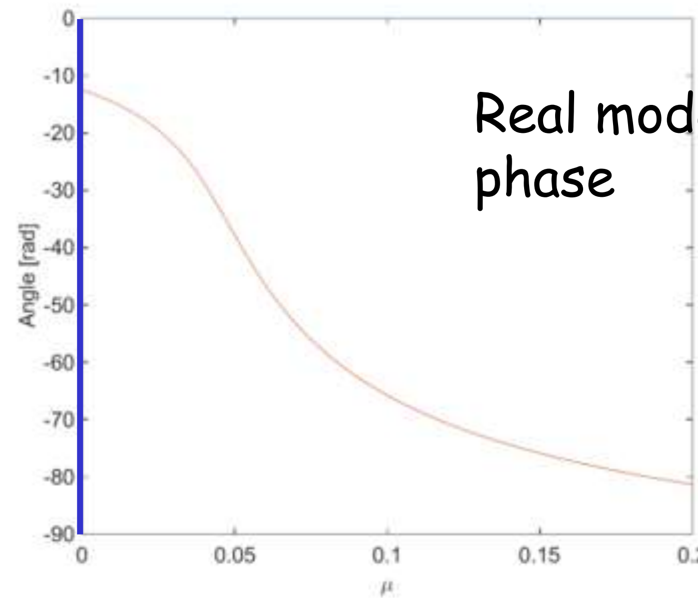
Normal



Tangent

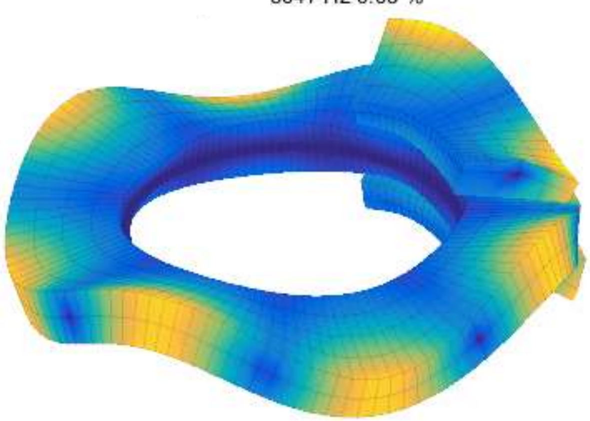


Amplitude of real modes

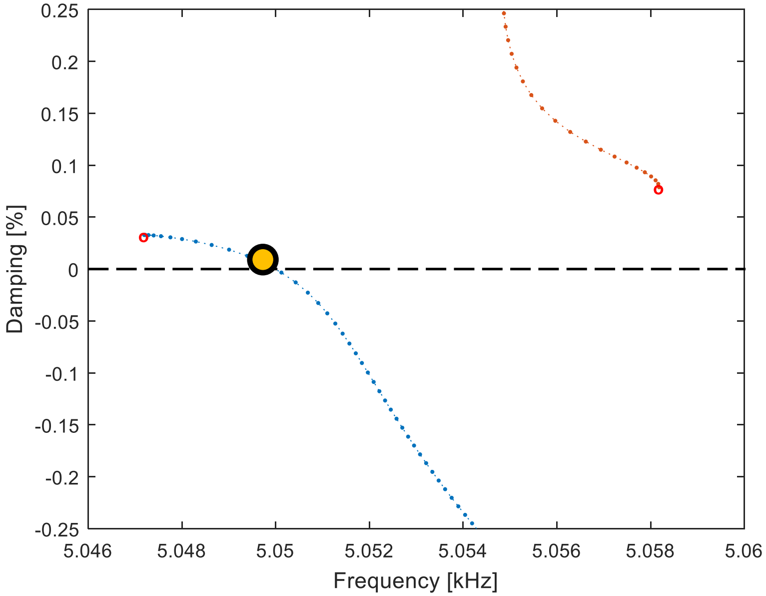


Real mode phase

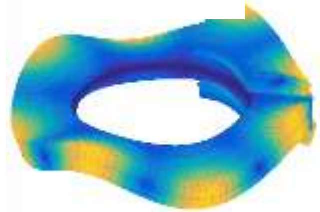
Normal



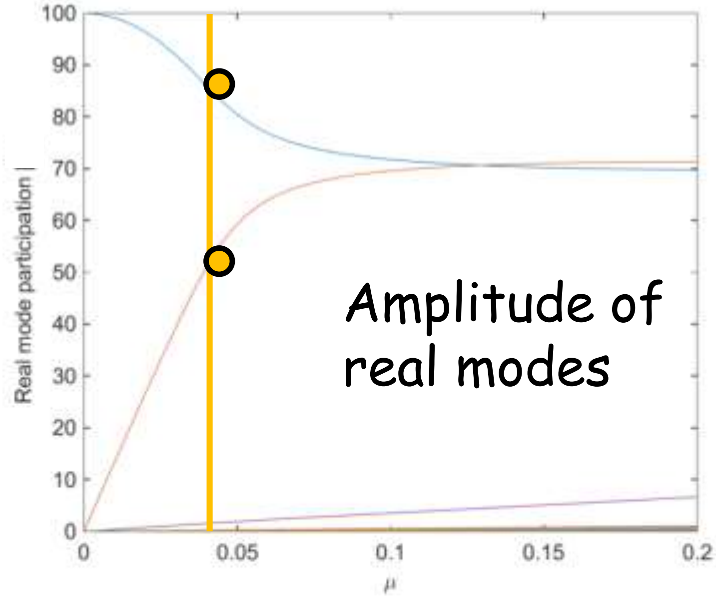
Participation of real shapes to complex modes



Normal

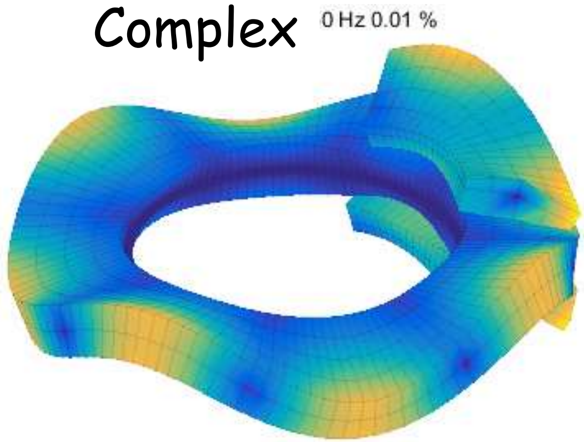


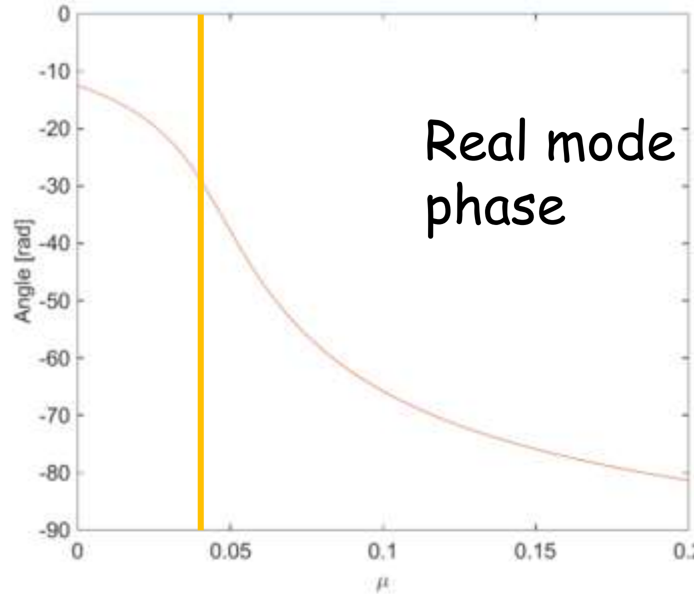
Tangent



Amplitude of real modes

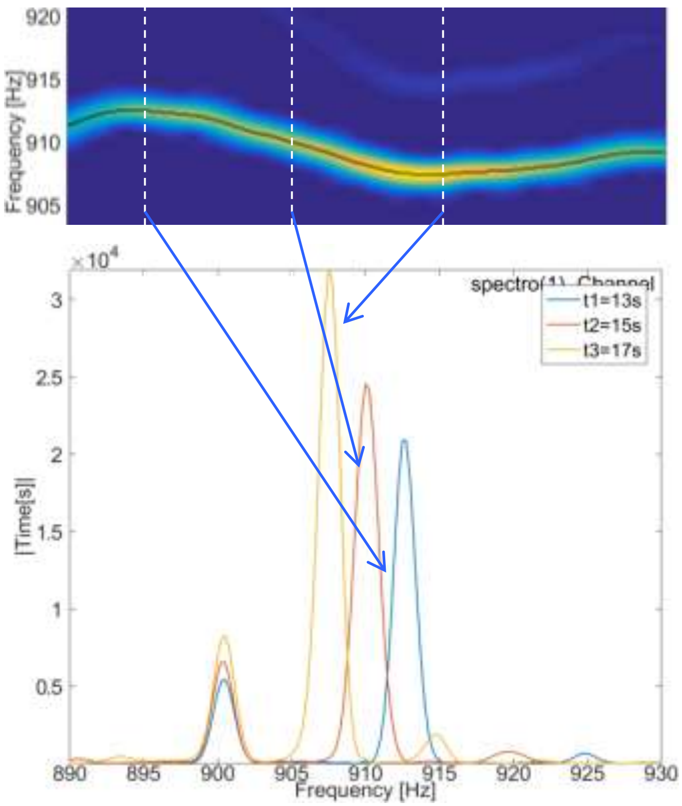
Complex





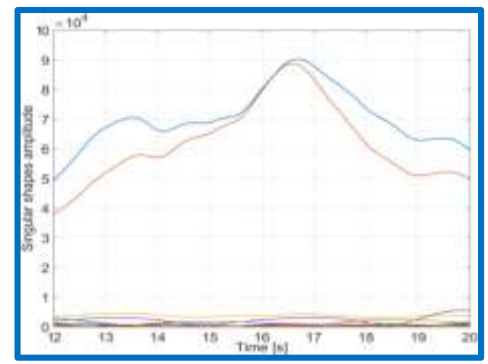
Real mode phase

Subspace learning & basis selection



$y(f_{max}(t),t)$ Time varying Complex shape	=	**Space :** Principal real shapes	**Time :** Complex amplitudes

SVD $\qquad \sum_i \left\{\begin{matrix} U_i \\ \downarrow \\ sensors \end{matrix}\right\} \{a_i(t) \quad \rightarrow \quad time\}$

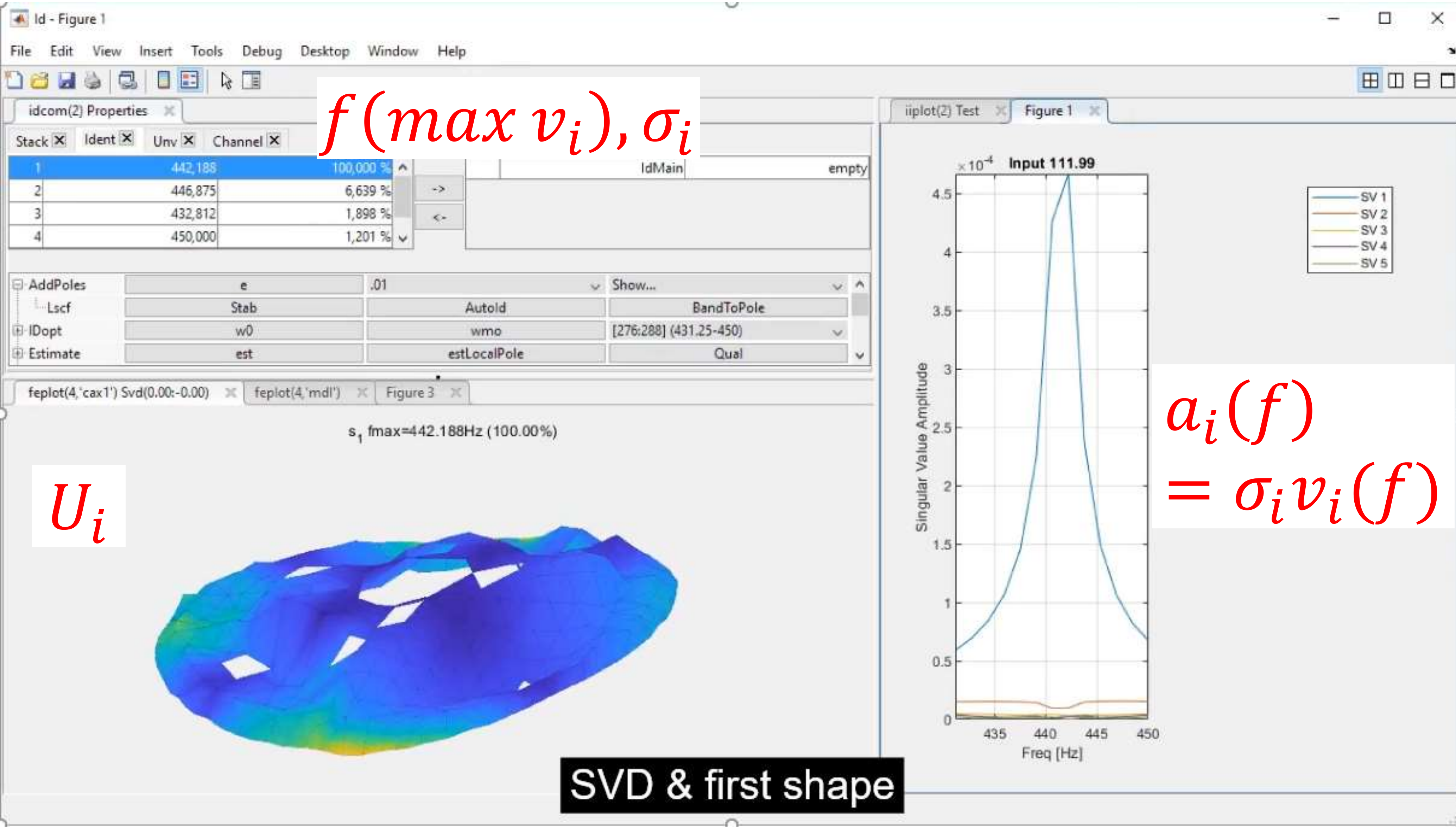


$$\begin{bmatrix} y(c, f_{\max}(t)) & \rightarrow & time \\ \downarrow & & \\ sensors & & \end{bmatrix}$$

- only 2 significant real shapes & 2 associated DOF
- shapes independent of parameter
- Parameter : difficult to control (wheel position, brake event, …) but exists

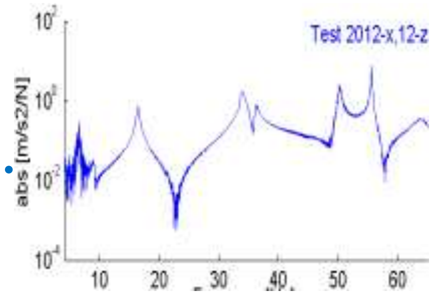
Details : G. Martin & al. ISMA 2018

SVDCur button in SDT

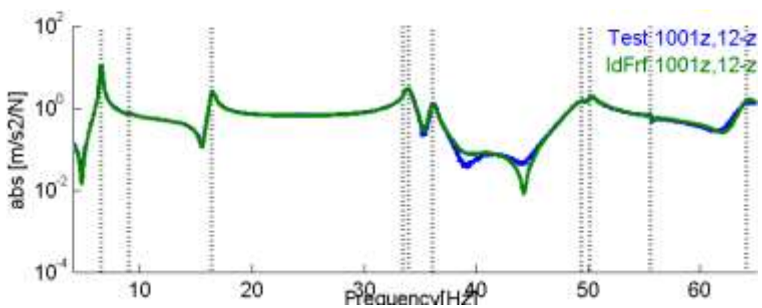


Experimental modal analysis: an inverse problem

Data
- Transfer
- Hankel mat.

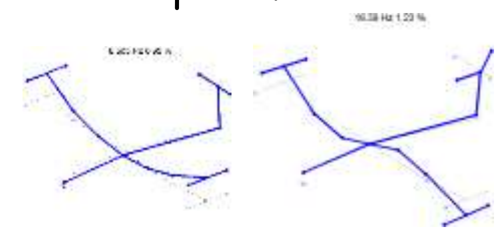


Objective H_{test}-H_{id}



Optimization

Result : parameters



Family of models

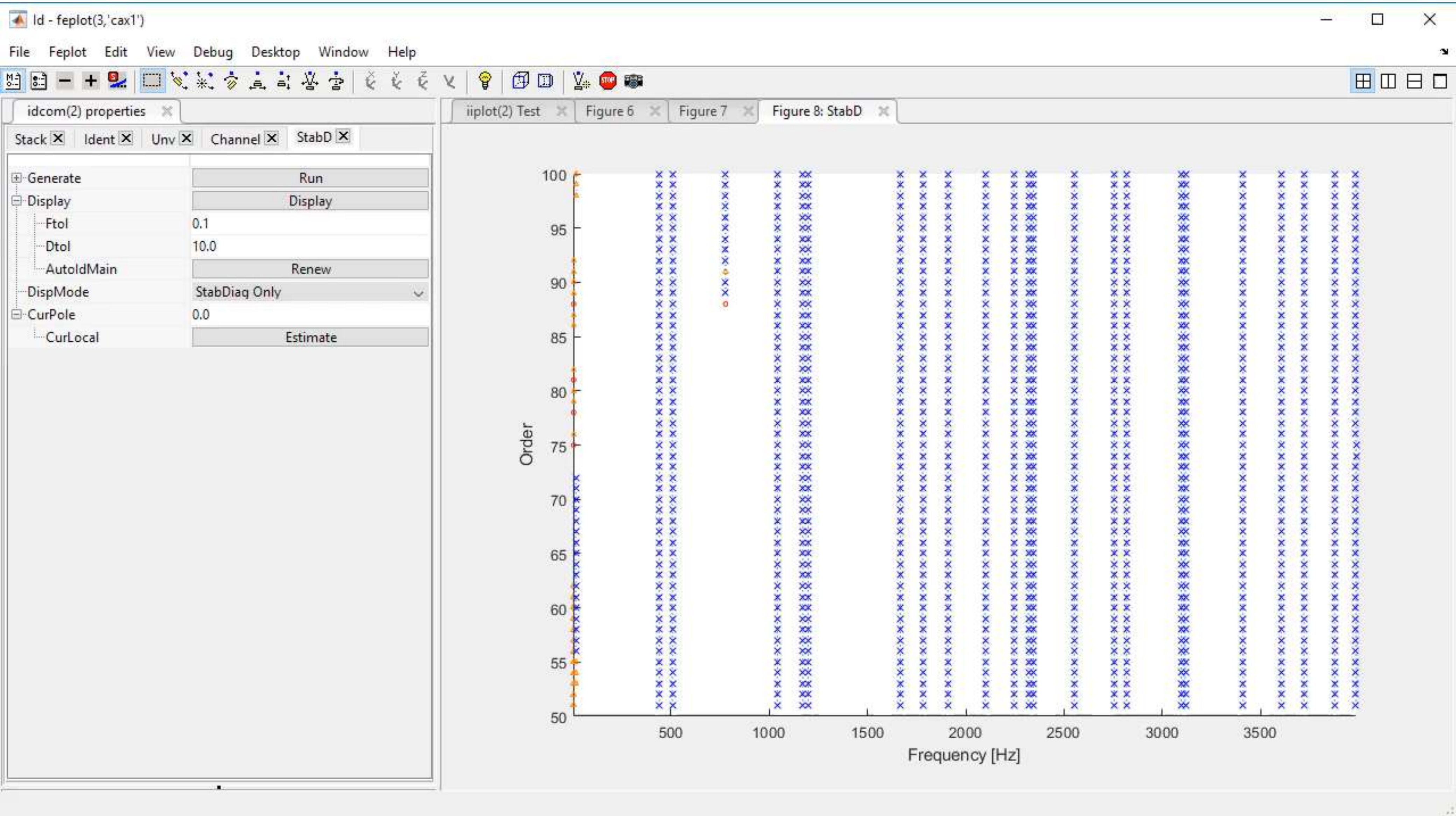
SVD $\sum_i \left\{ \begin{matrix} U_i \\ \downarrow \\ sensors \end{matrix} \right\} \{a_i(t) \quad \rightarrow \quad time\}$

State space $\{\dot{x}\} = [A]\{x\} + [B]\{u\}$

Pole residue $\sum \frac{[R_j]_{NS \times NA}}{s - \lambda_j}$

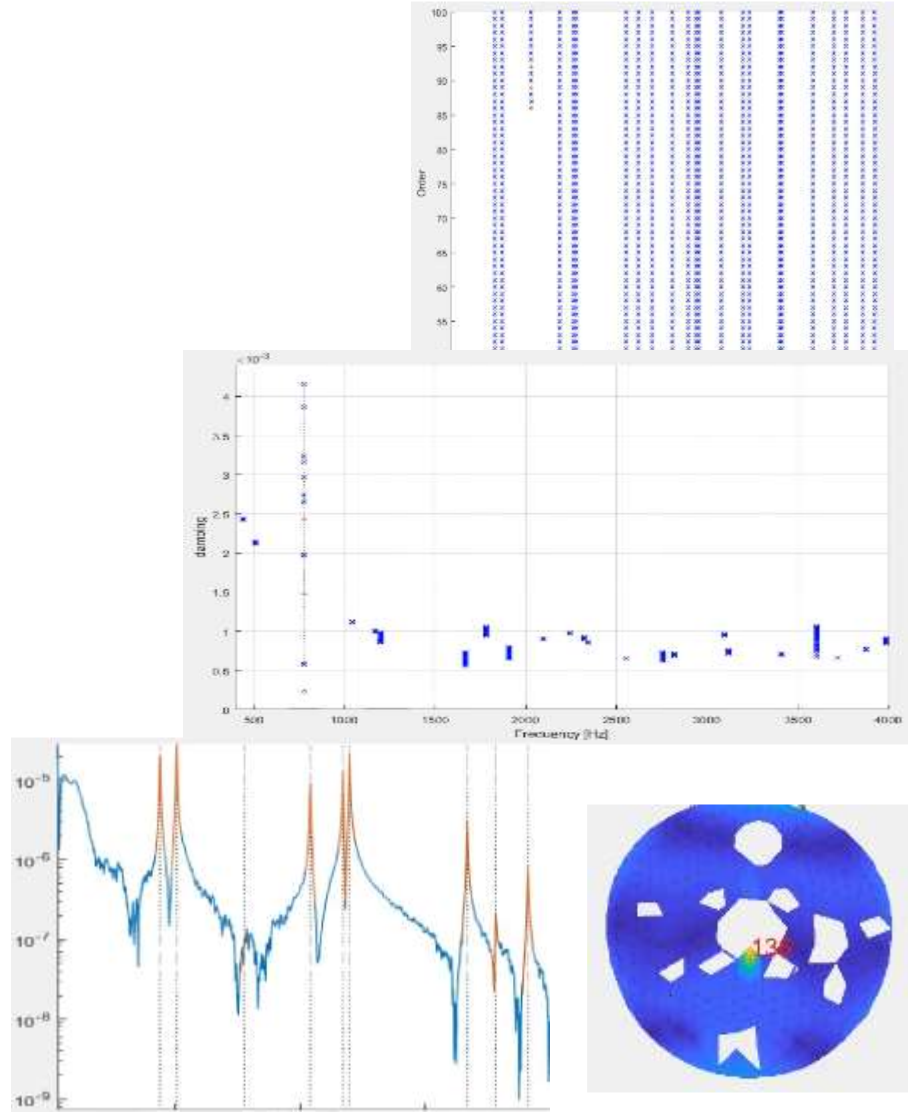
Rational fraction, modal model, second order …

Classical linear system Id



Id Phases

- Initialization :
  - Pick (stab or single pole estimate [1]) ?
  - Stabilization diagram [2]
- Estimate by band (why ? [1,3])
- How can problems be detected ? [3-4]
- Re-optimize poles (why ?)



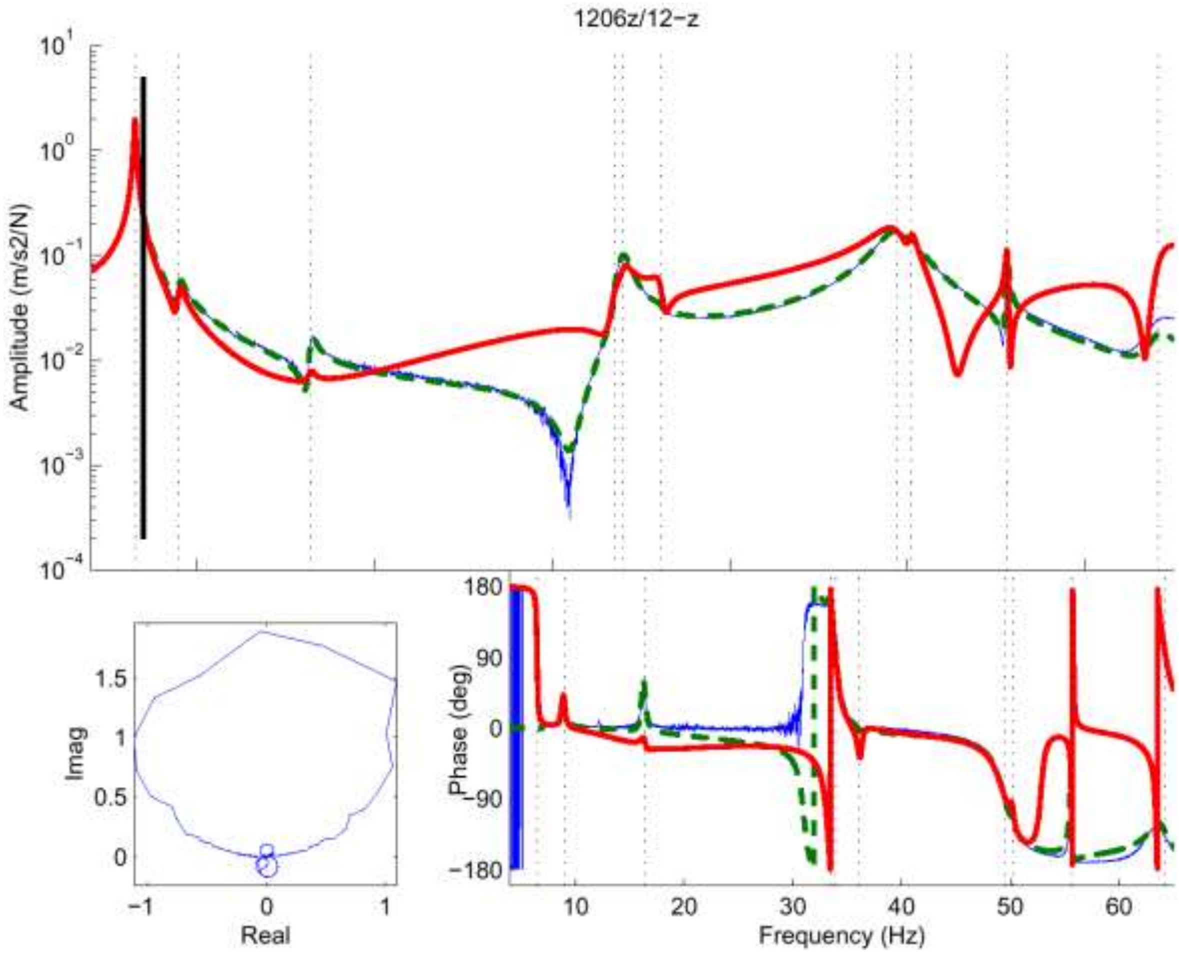
[1] E. Balmes, « Frequency domain identification of structural dynamics using the pole/residue parametrization », IMAC 1996

[2] P. Verboven, « Frequency-domain system identification for modal analysis », Ph.D. thesis, 2002.

[3] G. Martin, « Méthodes de corrélation calcul/essai pour l'analyse du crissement », Ph.D. CIFRE SDTools, Arts et Metiers ParisTech, Paris, 2017

[4] G. Martin, E. Balmes, et T. Chancelier, « Characterization of identification errors and uses in localization of poor modal correlation », MSSP 2017.

Noise induced bias

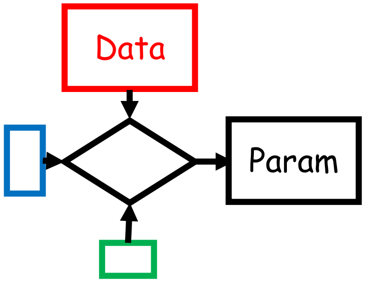


Inconsistence between data and family of models induces bias/variability

[1] M. Böswald, « Analysis of the bias in modal parameters obtained with frequency-domain rational fraction polynomial estimators », Proceedings of ISMA 2016

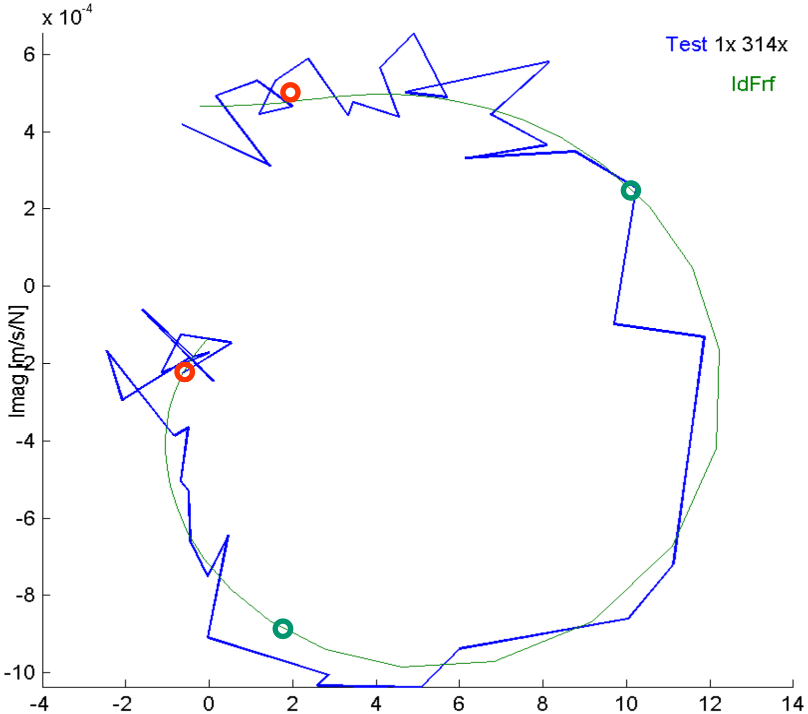
[2] El-kafafy, De Troyer, Peeters, Guillaume, « Fast maximum-likelihood identification of modal parameters with uncertainty intervals: A modal model-based formulation », MSSP, 2013

Quality : an error criterion



Is the model well identified?

- Superpose measured and identified FRF for each sensor, around each mode
- Compute error on Nyquist

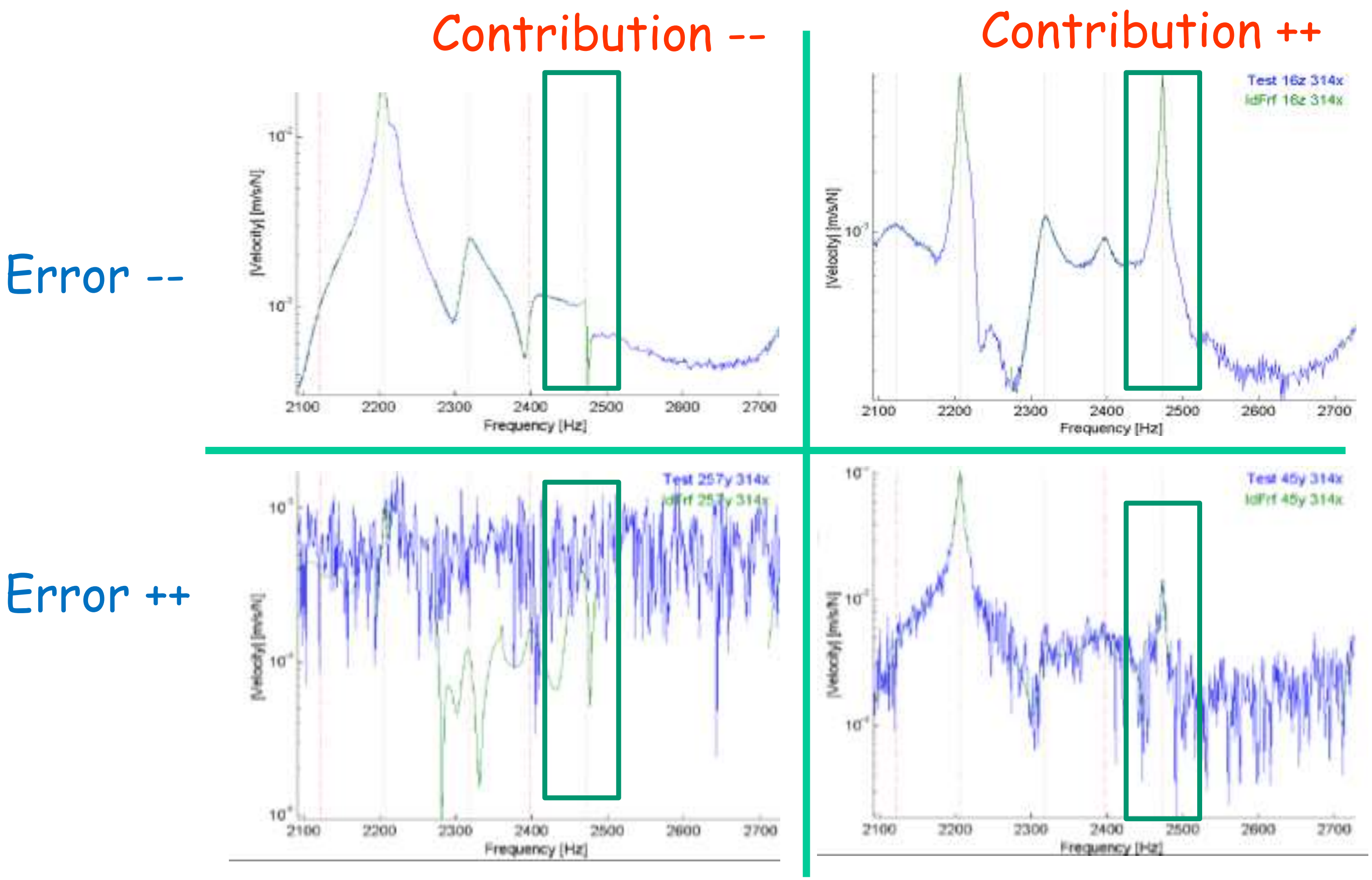


$$e_{j,c} = \frac{\int_{\omega_j(1-\zeta_j)}^{\omega_j(1+\zeta_j)} |H_{Test,c}(s) - Hid,c(s)|^2}{\int_{\omega_j(1-\zeta_j)}^{\omega_j(1+\zeta_j)} |H_{id,c}(s)|^2}$$

"Around each mode" :

- Half power : +/- $\zeta_j\omega_j$
- Visible peak : +/- $5\zeta_j\omega_j$

Test/identification error

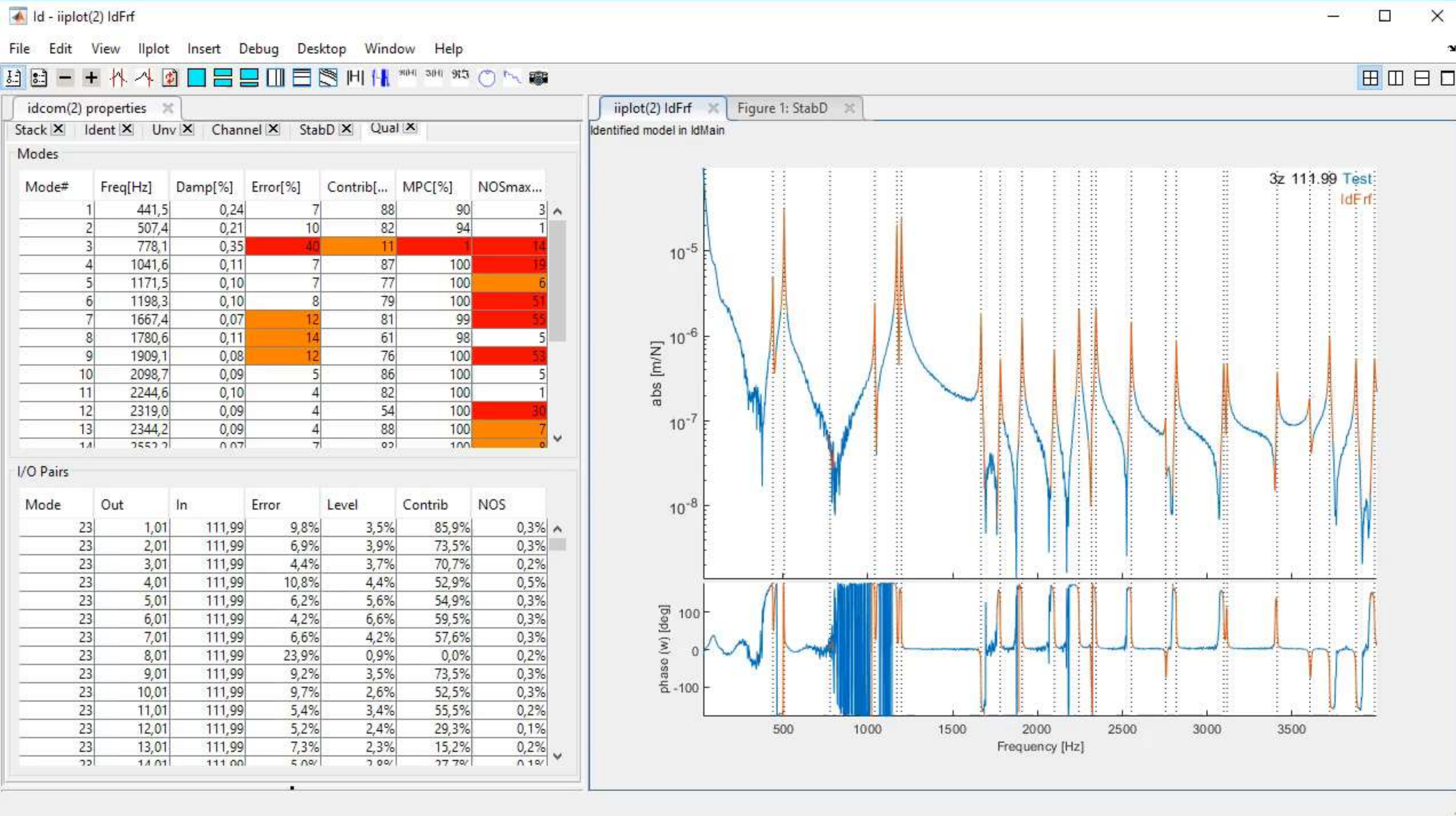


For each sensor, each mode may have strong error/noise & low contribution

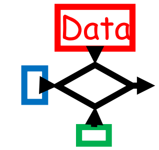
Test error per mode/sensor

[1] G. Martin, E. Balmes, et T. Chancelier, « Characterization of identification errors and uses in localization of poor modal correlation », MSSP 2017.

Evaluate quality of identification

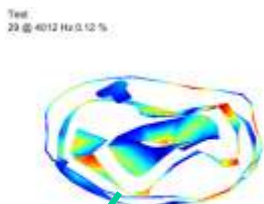


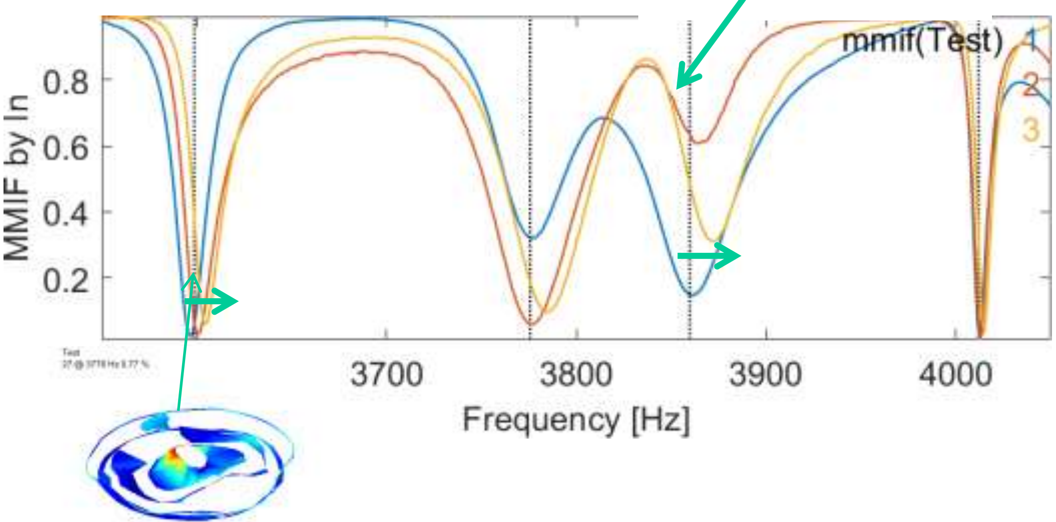
Other errors : NL ≠ one system

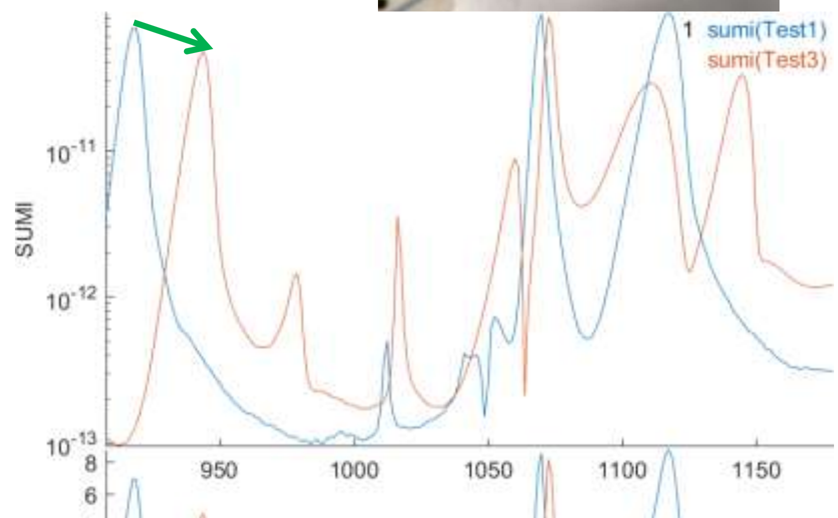


- Non linear system : resonance dependent on input point / accurate positioning

- MMIF or identification per impact location shows significant dispersion
- Multiple identification results are not perfectly coherent

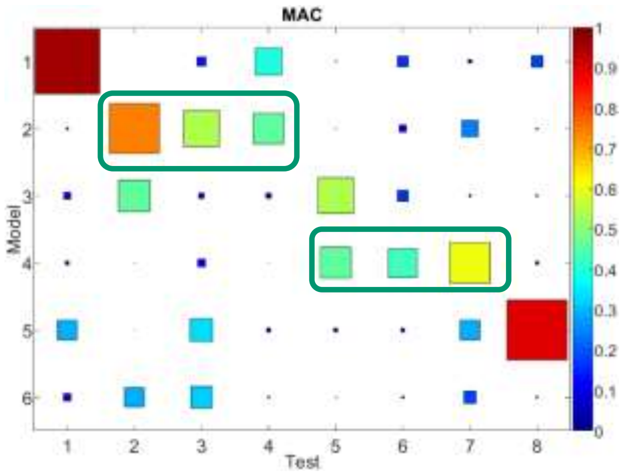




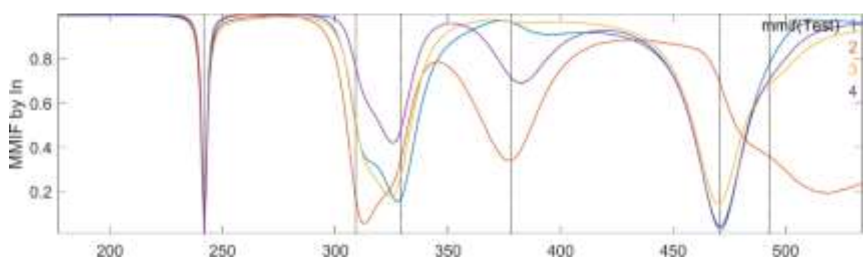


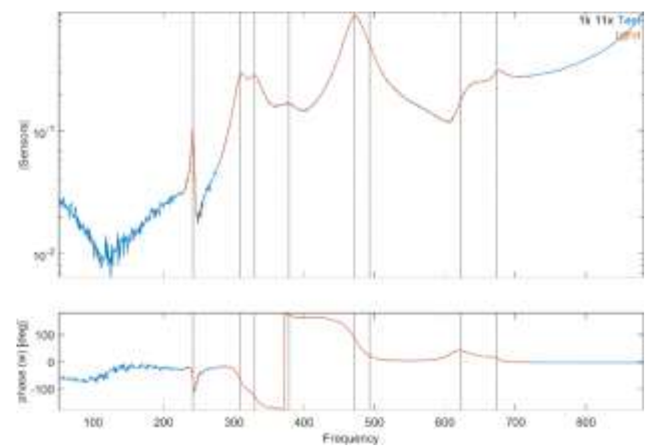
Other errors : local modes

- « Same mode » multiple times



- non structural masses generate global mode duplicates
- « small mass » ≠ « tiny peak » (if this were true proof mass dampers would not work)

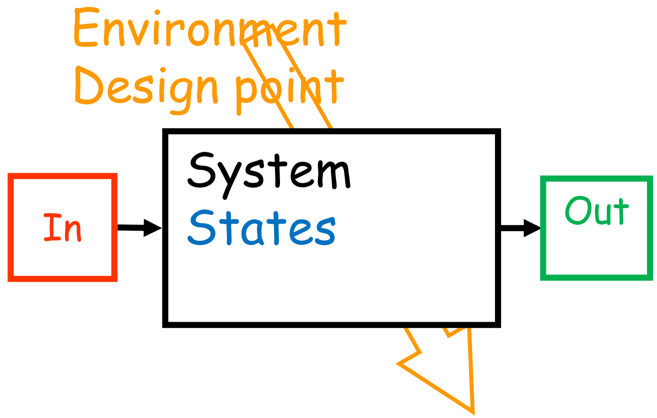


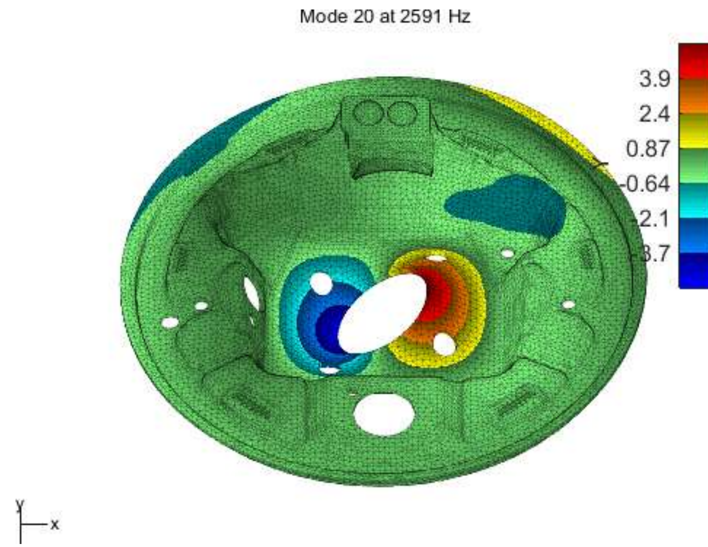


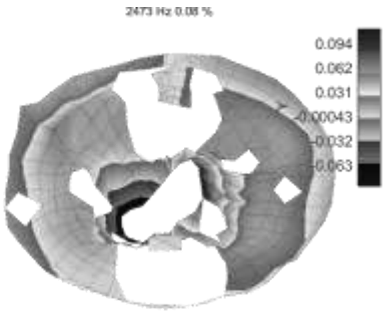
Summary : experimental modal analysis

- Space/time decomposition applies very often
- Base assumption LTI (linear time invariant) should be challenged
- Inconsistent data & model family $\Rightarrow$ bias rather than variance
- Error classification is per mode/sensor
- General handling of NL & local modes is beyond state of the art

Parametric direct problems

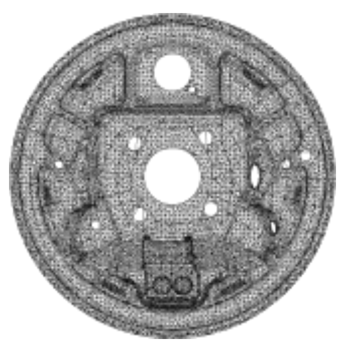






Identification error
- Noisy measurements
- Identification bias
- NL, time varying, …

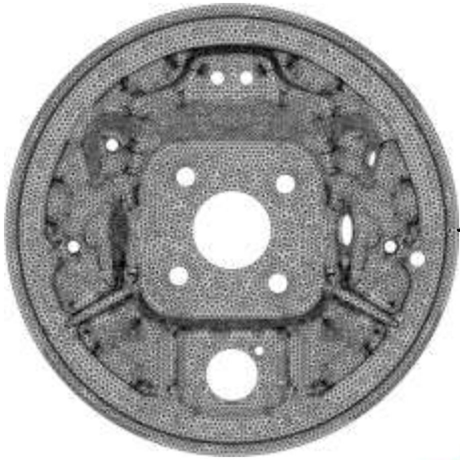
Topology errors
- sensor/act position
- matching



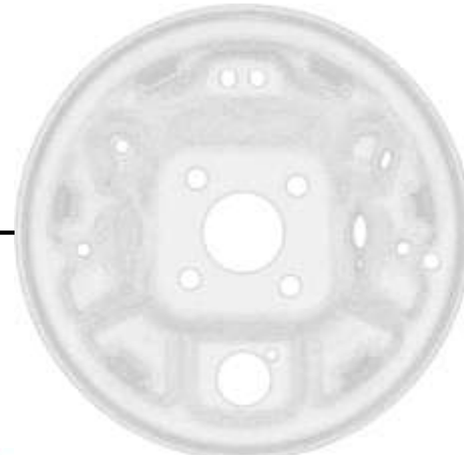
FEM error
- Geometry
- Material parameters
- Multi-scale problems & equivalent parameters
- Junction representation
- Design change & modal property tracking

Geometry updating

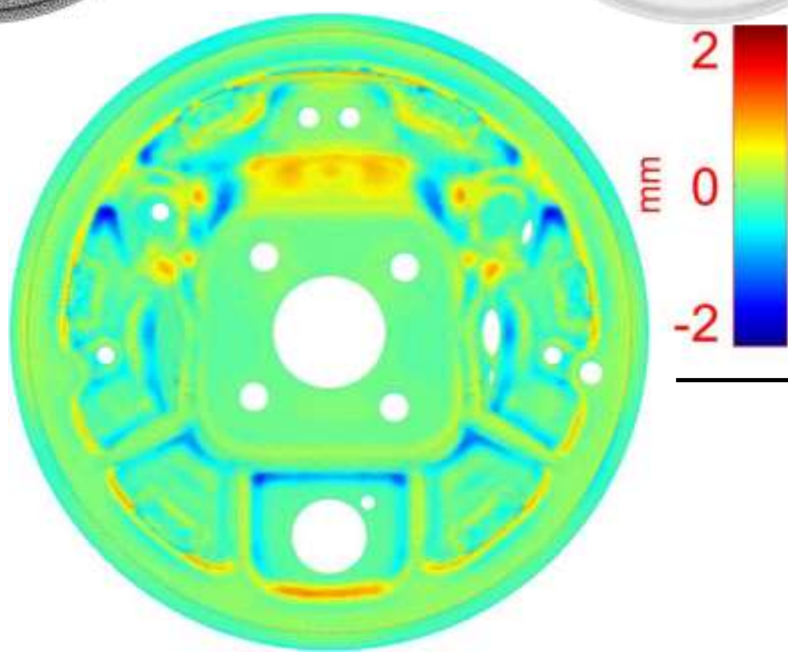
Nominal geometry



Measured geometry

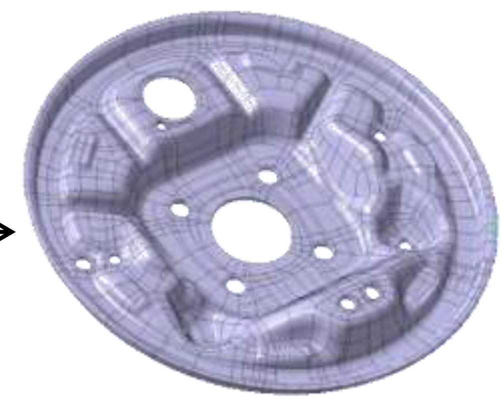


- Up to 2 mm on surface
- 11% error on volume



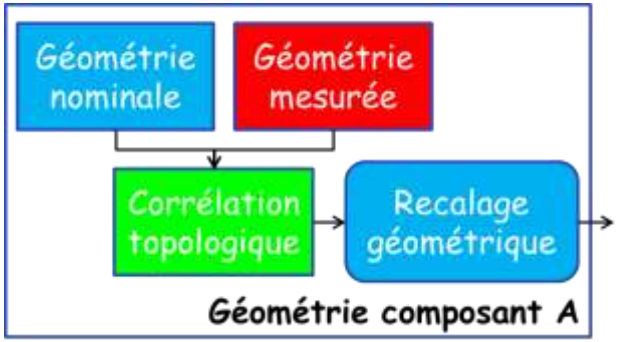
Geometry correlation

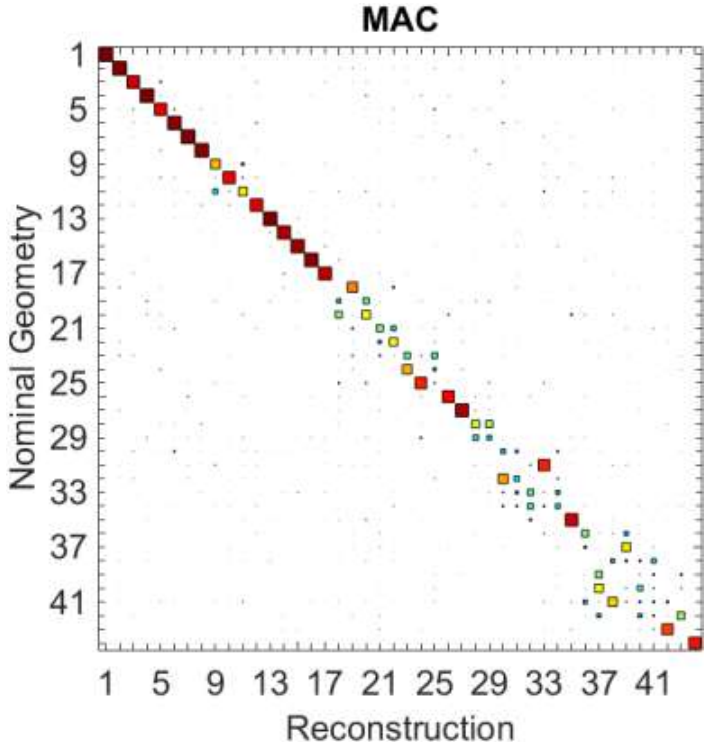
Inverse CAD
(Geomagic, …)
Morphing (SDT)

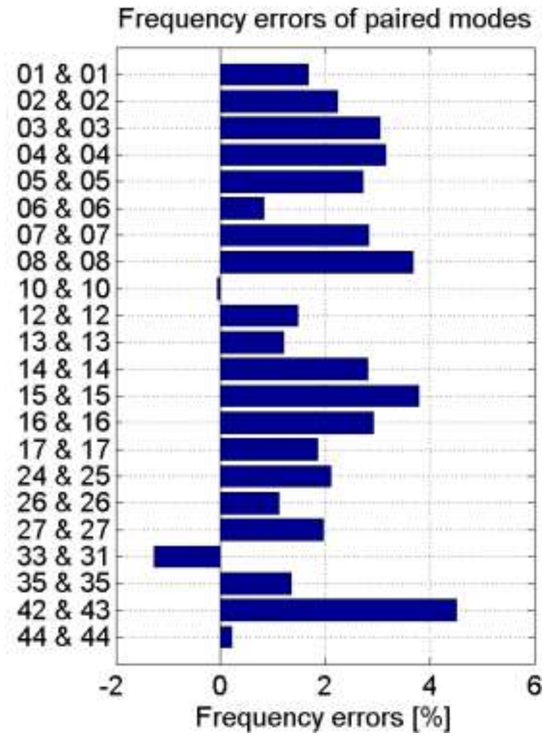


Geometry updating

- Frequency band : 0 – 6kHz
- Notable shape difference >3 kHz
- Frequency error
  - 2 % mean
  - 5 % max







Checking geometry should always be the first step

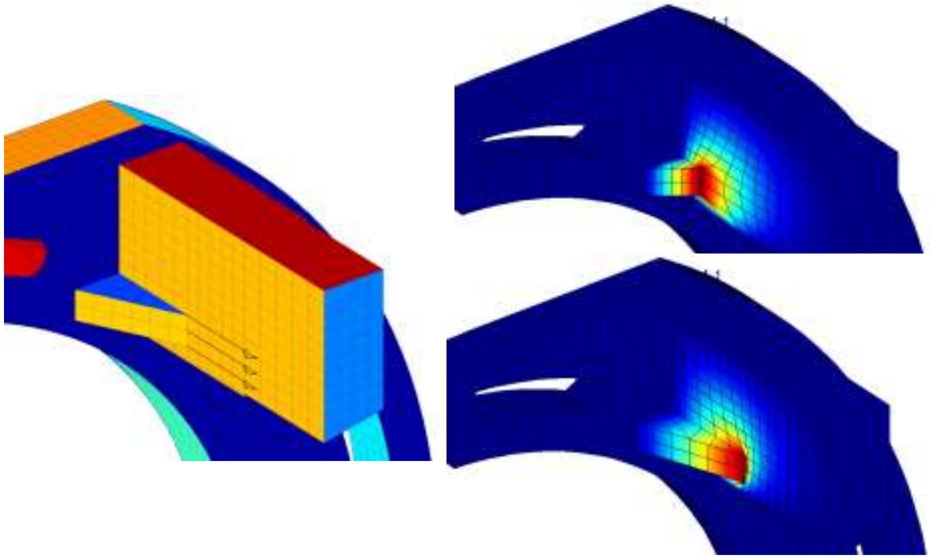
Geometry parametrization / morphing

- Shape optimization / morphing

$$P(x) = \sum_i \{p_i(x)\} q_{imaster}$$

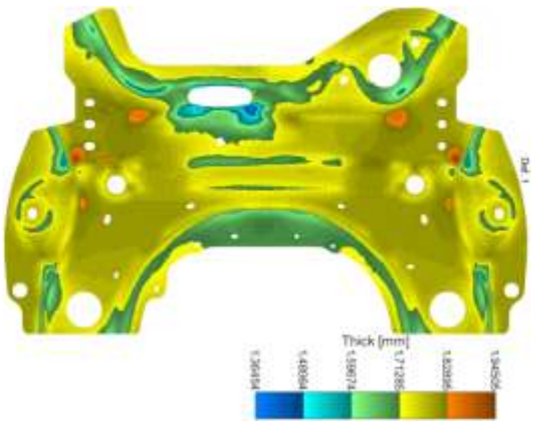
fe_shapeoptim BuildFromSel

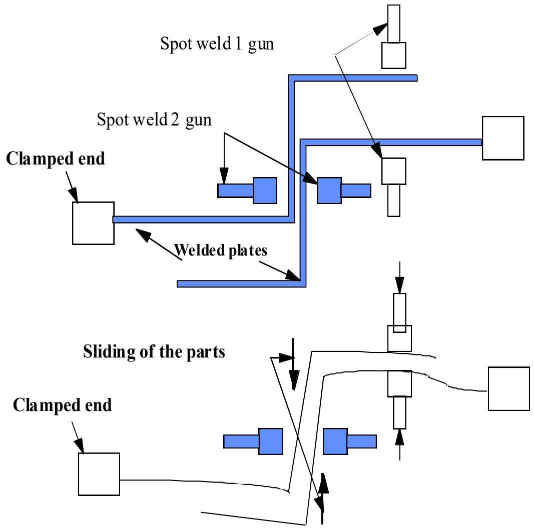
1) fix bottom face, 2) prescribe edge motion
3) deform edges (straight)
4) deform faces (flat)
5) deform interior (good elements ?)

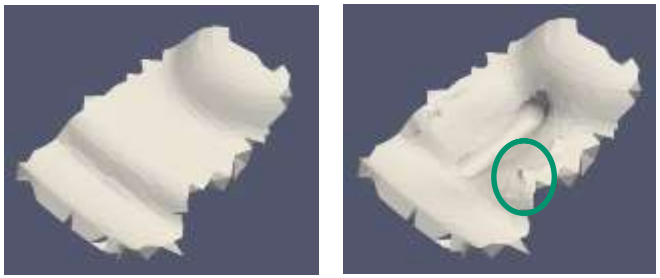


- Process simulation + field projection

fe_shapeoptim Interp







[1] de Paula, Rejdych, Chancelier, Vermot, Balmes, « On the influence of geometry updating on modal correlation of brake components. », in *Vibrations, Chocs & Bruit*, 2012.

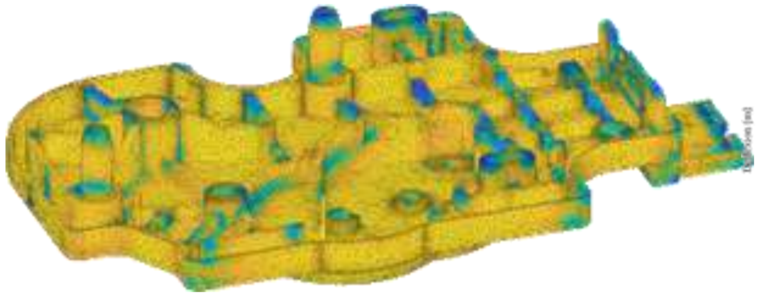
[2] G. Vermot Des Roches, E. Balmes, et S. Nacivet, « Error localization and updating of junction properties for an engine cradle model », in ISMA, Leuven, Belgium, 2016, p. ID 372.

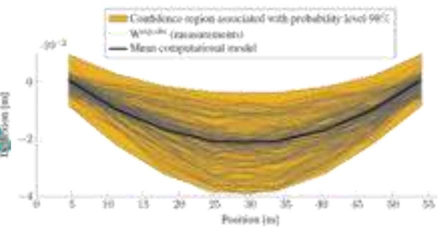
[3] E. Blain, « Etudes expérimentales et numériques de la dispersion vibratoire d'assemblages soudés par points », Ph.D. thesis, Ecole Centrale de Paris, 2000.

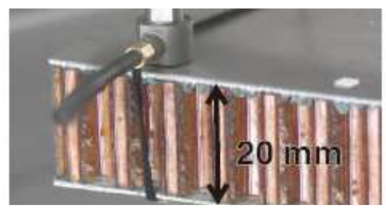
Direct problems : material parameters

- Uniform
- Field
- Equivalent
  (at certain scales)

- Geometry
- Material parameters
- Junction representation
- Equivalent parameters







- Basic parametrization tool : dependence on constitutive parameters C_{ij}

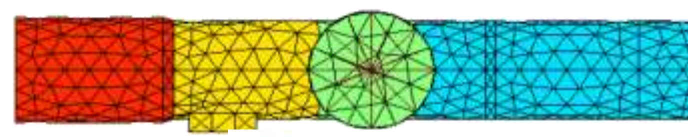
$$K=\int_{\Omega}\{\epsilon\}^T[C_{ij}]\{\epsilon\}=\sum_g B^T[C_{ij}]B\,w_g=\sum_{ij}C_{ij}\left[\sum_g B^T[C_{ij}^u]Bw_g\right]=\sum_{ij}C_{ij}[K_{ij}]$$

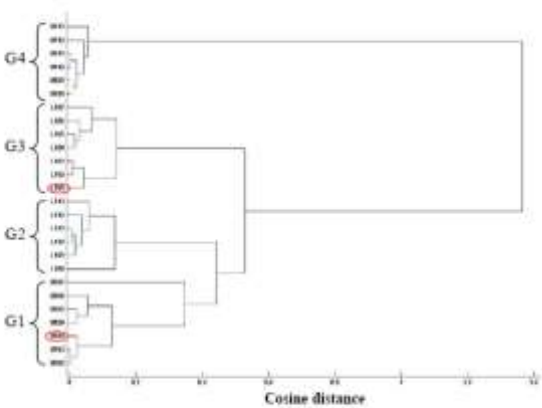
Element/model weights

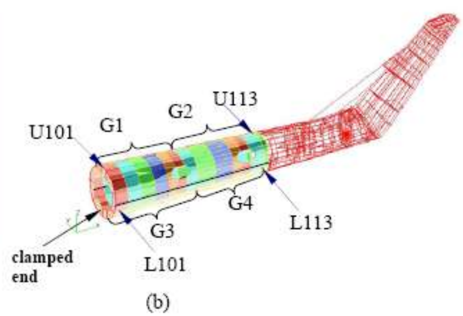
$$K(p) = \sum_e \alpha_e(p)[K^e]$$

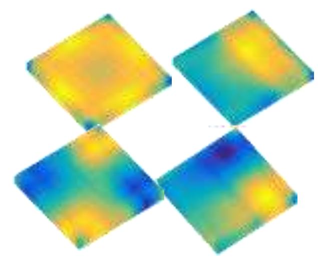
Weighted element matrices = standard

- Element-wise (topology optimization)
- Field/groupwise
  - parameter groups …
  - solution of eigenvalue problem (polynomial chaos, Ghanem, Soize, …)
  - clustering









Clustering of Parameter Sensitivities: Examples from a Helicopter Airframe Model Updating Exercise. Shahverdi, Mottershead & All

SDT : upcom / zCoef

- Element wise $K(p) = \sum_e \alpha_k^e [K^e]$ www.sdtools.com/help/upcom.html

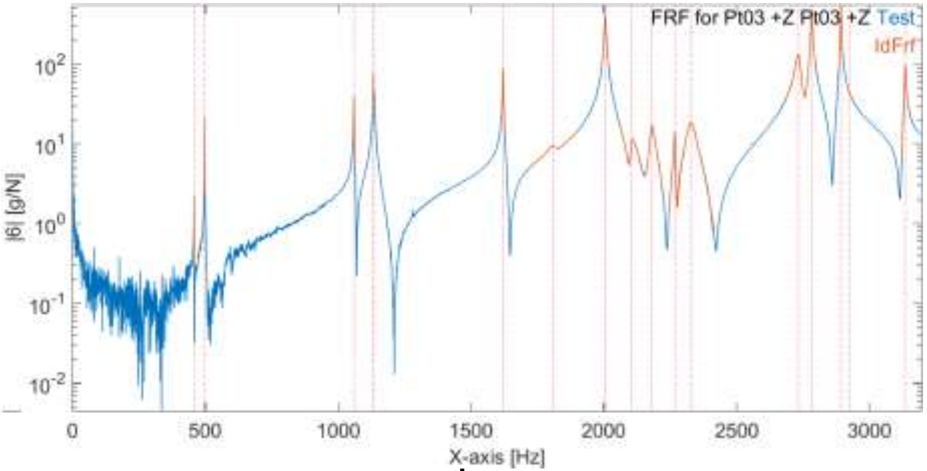
$$\text{mind} = \begin{bmatrix} M_s & M_e & K_s & K_e & \alpha_m & \alpha_k \\ \vdots & & & & & \\ elt & & & & & \end{bmatrix}$$

- Group wise $K(p) = \sum_g \alpha_g \, [K^g]$ www.sdtools.com/help/zCoef.html

```
zCoef={'Klab','mCoef','zCoef0','zCoefFcn';
       'M'    1       0        '-w.^2';
       'Ke'   0       1        1+i*fe_def('DefEta',[]);
       'Kv'   0       1        'par(1)'};
```

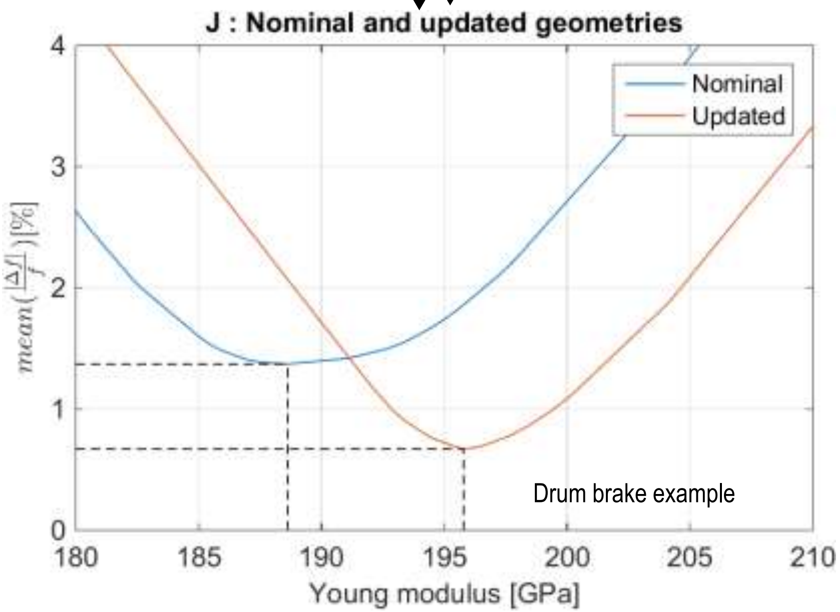
Updating of material properties

Modal frequencies



Weight

- Weight / volume ⇒ density
- Frequencies ⇒ modulus
- Test/FEM distance below physical dispersion (df/f≈2%)



Frequency correlation

$$J = \min_E \left(\sum_i \frac{|\Delta f_i|}{f_i} \right)$$

Wrong volume = bias

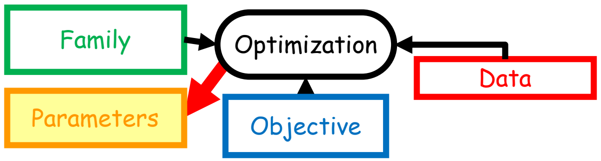
Equivalent materials / homogeneization

Family
- Micro : cell walls, glue, face-sheet, viscoelastic material
- Macro : shell/ orthotropic volume/ shell

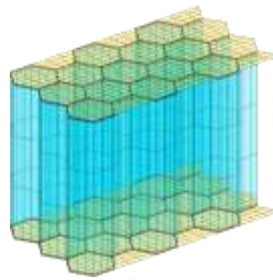
Objective
- Equivalence: waves/modes

Parameters
- orthotropic law

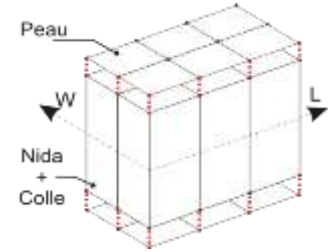
- Scale separation

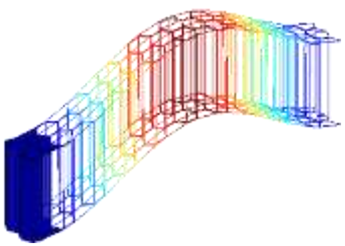


Detailed 3D honeycomb

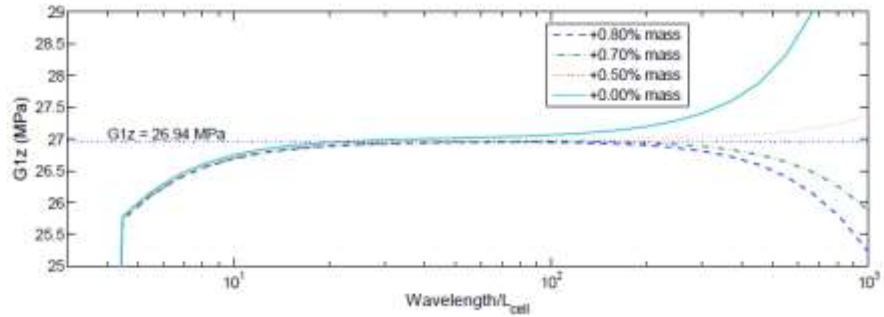


Shell/volume/shell

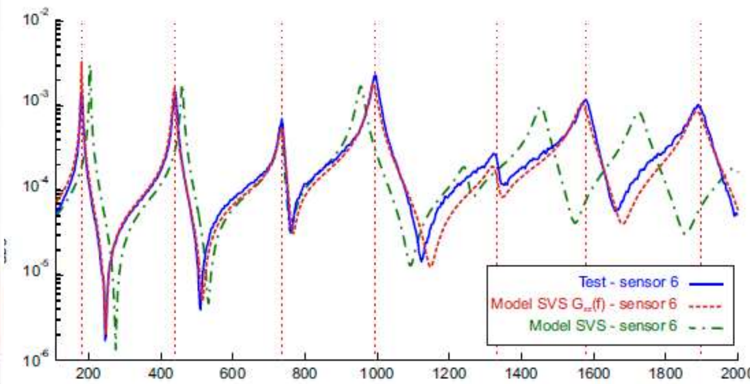




Numerical homogeneization



Updating from test



C. Florens, « Modeling of the viscoelastic honeycomb panel equipped with piezoelectric patches in view of vibroacoustic active control design », ECP, 2009.

Direct problems : visco junction parameters

- Geometry, material
- Junction representation
- Equivalent parameters

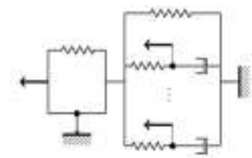
Test of visco mount

$F_x = f(x, \dot{x})$

- Measure disp/load
- Macro : results mix material and geometry

Identification **(p)**

0D / macro / meta-model



Behavior

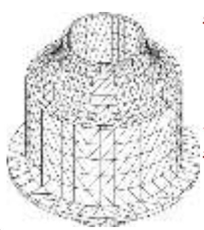
- material & structure
- $F_x = f(x, \dot{x}, x_i, \boldsymbol{p})$

Identification **(p)**

Reduction [1] & Hyper-reduction [2]

ient simulation

3D physical / micro



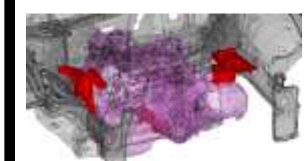
Knowledge

- FEM structure
- Implicit/explicit time

Behavior

- $\{\sigma\} = f(\{\varepsilon\}, \{\dot{\varepsilon}\}, \{\varepsilon_i\}, p)$
- Hyper-visco-hyster. mat

Multi-body / system simulation



Use cases

- Engine suspension
- Drive line
- Comfort, endurance

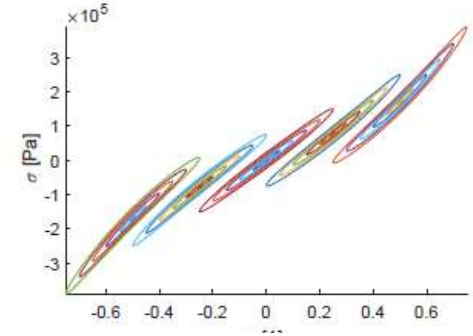
[1] Hammami, Balmes, Guskov, « Numerical design and test on an assembled structure of a bolted joint with viscoelastic damping », *MSSP*, v70, p.714–724, 2015

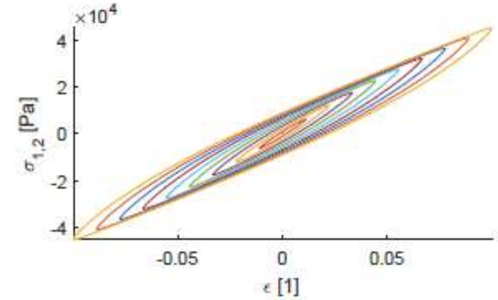
[2] Farhat, Avery, Chapman, Cortial, « Dimensional reduction of nonlinear finite element dynamic models with finite rotations and energy-based mesh sampling and weighting for computational efficiency», *IJNME*, vol. 98, nº 9, p. 625-662, 2014.

Visco junctions a few keywords

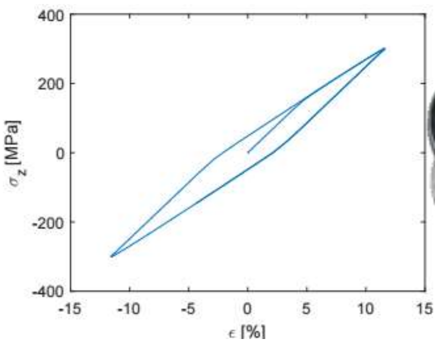
Material
- 3D finite strain (visco/hysteretic)
- Effects : Mullins, Payne, Madelung, Masing, …

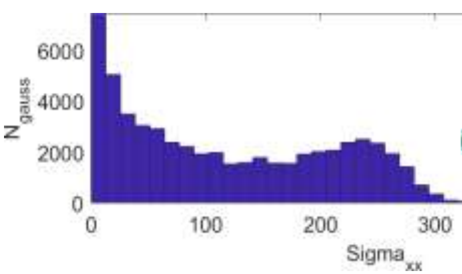
Structure
- Micro/macro transition
- Reduction / hyper-reduction (macro-Gauss points)

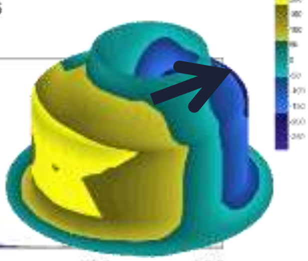




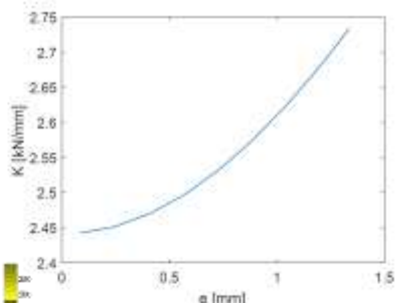
Micro

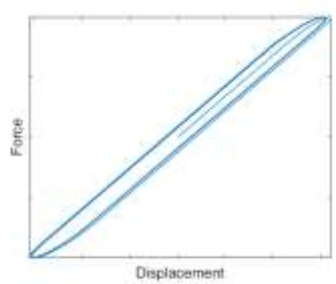




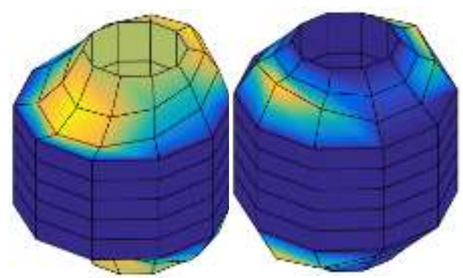


Macro

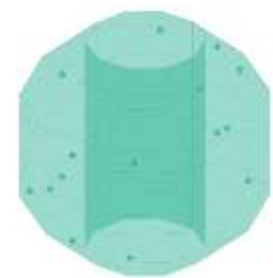




Reduction

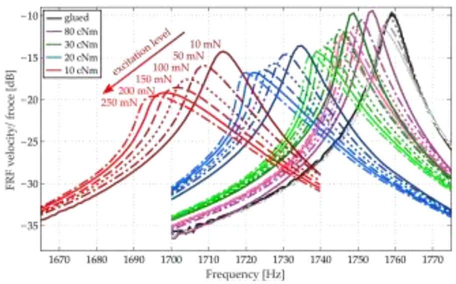


Macro-gauss points



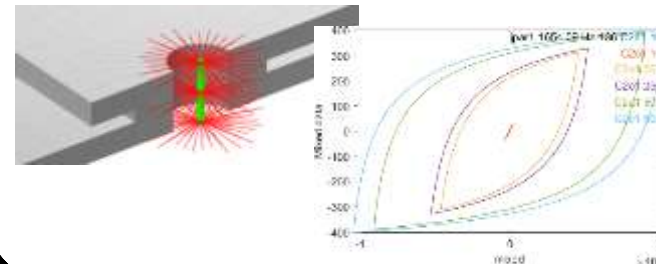
Direct : contact/friction junction parameters

Test



Identification **(p)**

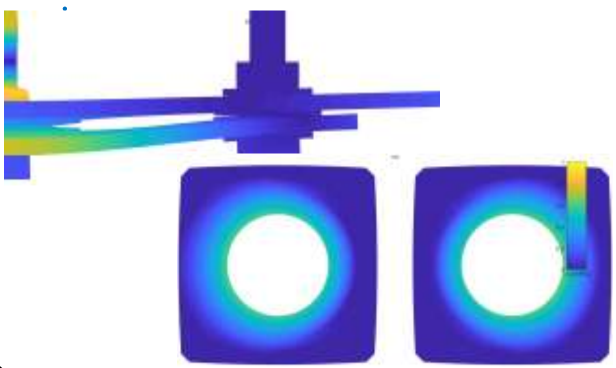
0D / macro / meta-model



Identification **(p)**

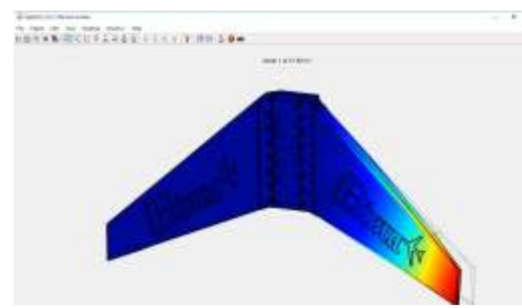
Reduction & Hyper-réduction

2D physical /



Transient simulation

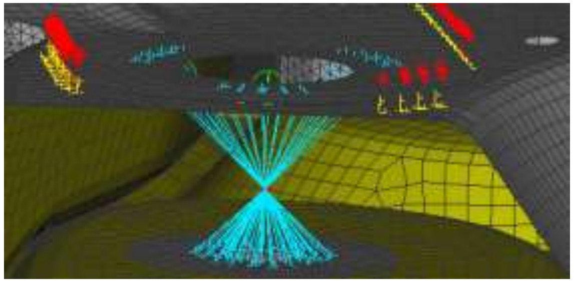
System simulation

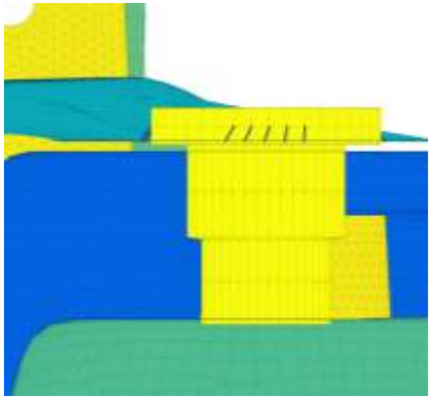


[1] FUI CLIMA (Conception de LIaisons Mécaniques Amortissantes)
[2] Thèse Marco Rosatello. Supméca 2019.

Junctions : contact / 0D strategies

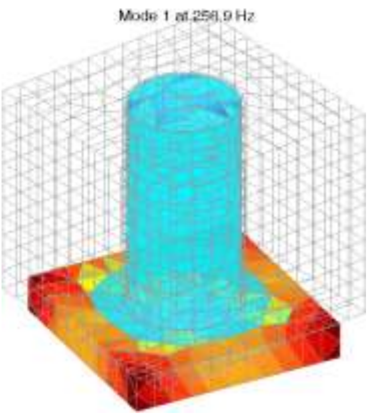
Typical simplification : mass & stiffness

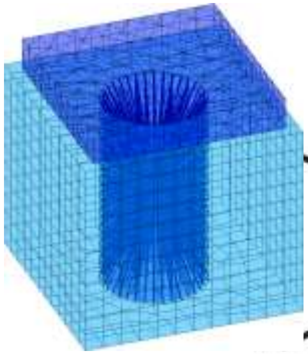


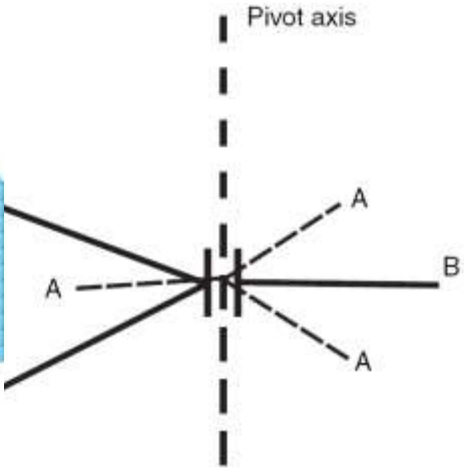


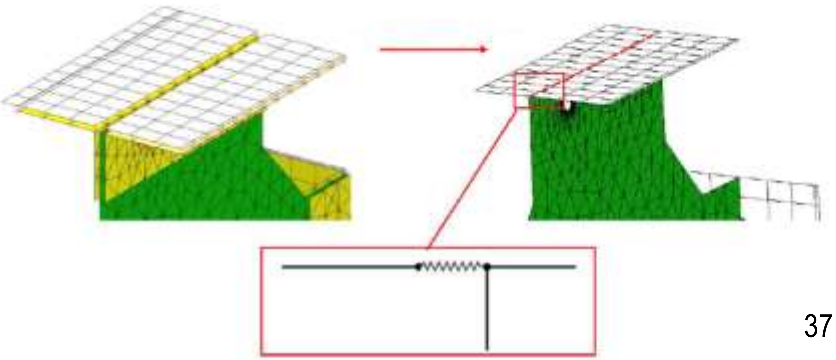
Kinematic junctions (pivots, ...)
often difficult with 2D

Equivalent spring build may require complex numeric/test identification







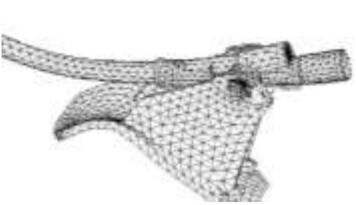


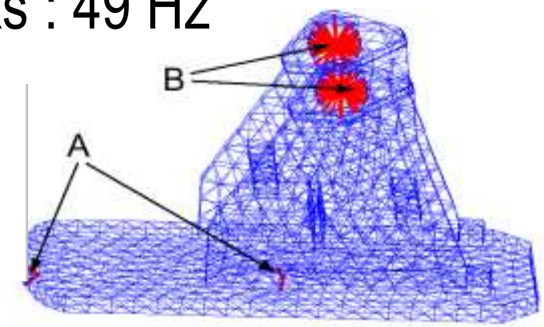
Contact : pivot mode at 256 Hz

Sample junction problems

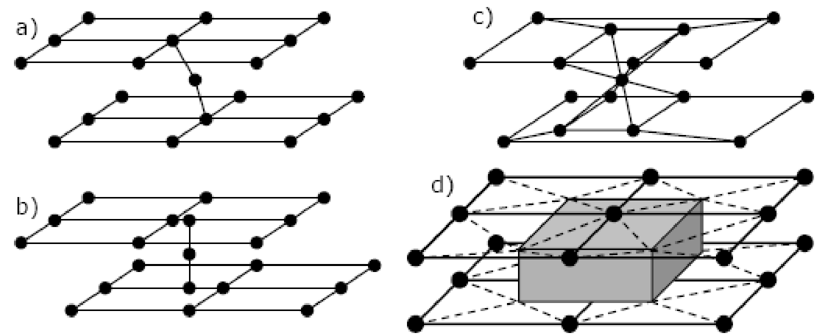
- Master nodes for multi-body

Type B links : 82 Hz
Type A links : 49 Hz





- Weld spots



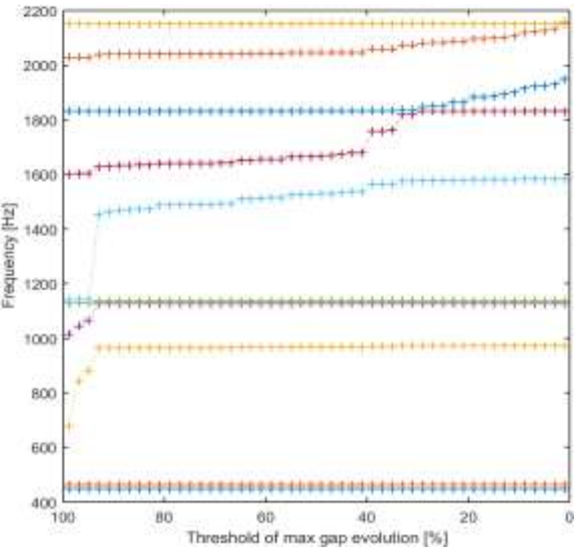
Lardeur & All .: Spot weld modelling techniques and performances of finite element models for the vibrational behavior of automotive structures, ISMA 2000.

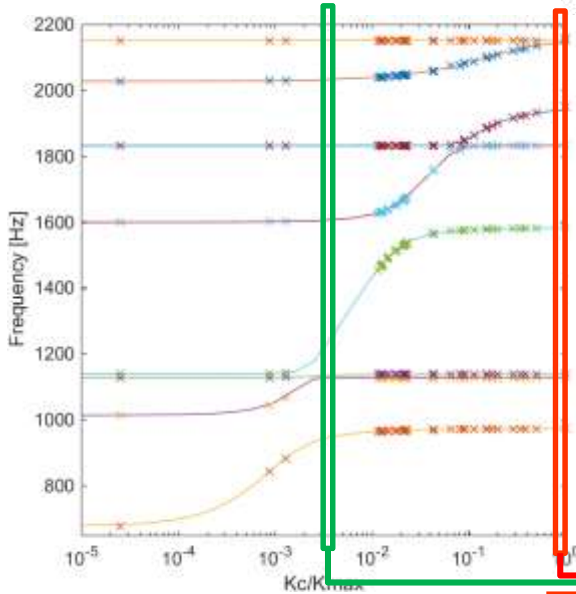
- Contact around WS

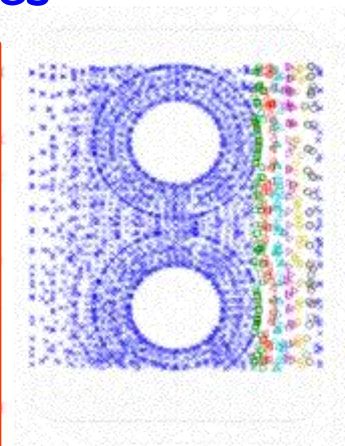
G. Vermot Des Roches, E. Balmes, et S. Nacivet, « Error localization and updating of junction properties for an engine cradle model », in ISMA, Leuven, Belgium, 2016, p. ID 372.

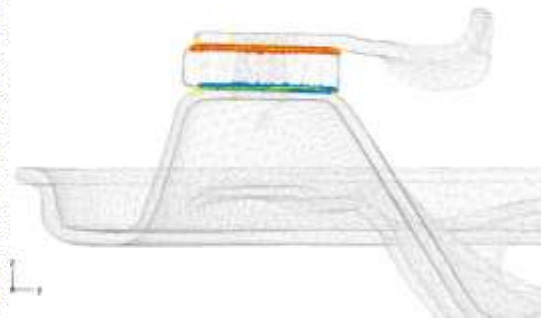
Junction parameters : contact surface/stiffness

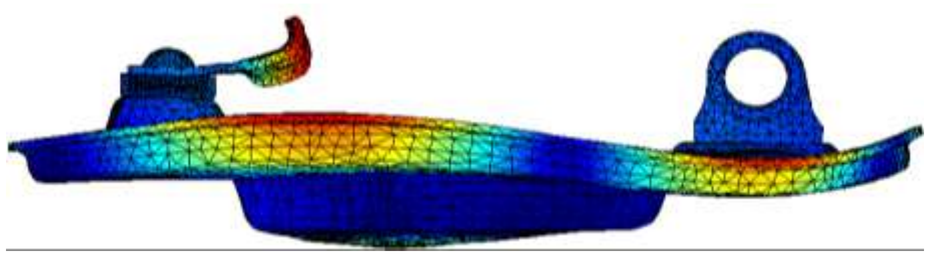
- Variable contact surface and stiffness
- Comparable impact frequencies and shapes

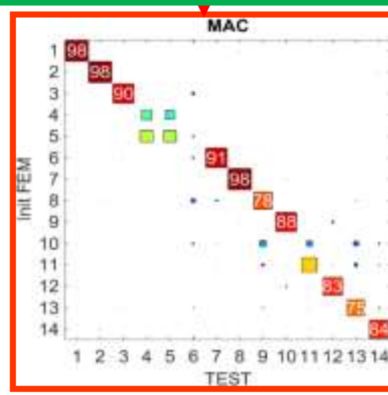


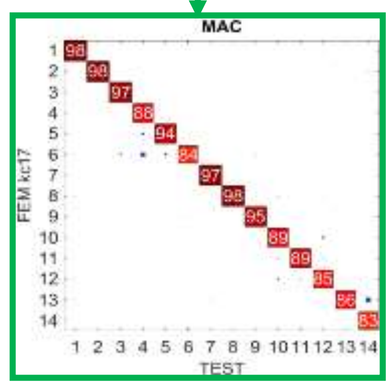








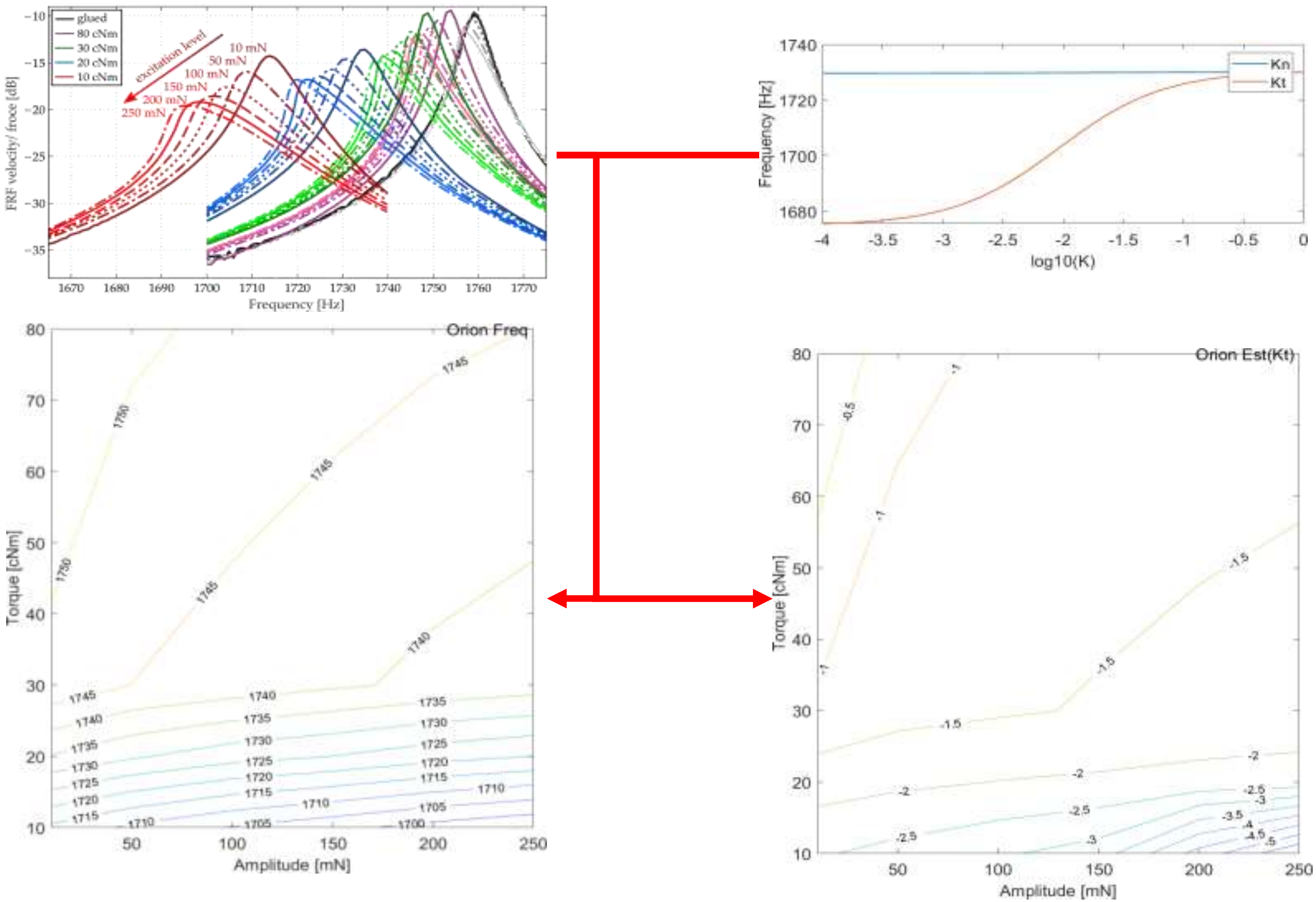




[1] G. Martin, E. Balmes, T. Chancelier, « Review of model updating processes used for brake components », in Eurobrake, 2015
[2] Y. Goth, H. Reynaud, « A bolt assembly parametric model », in ISMA, Leuven, 2016.

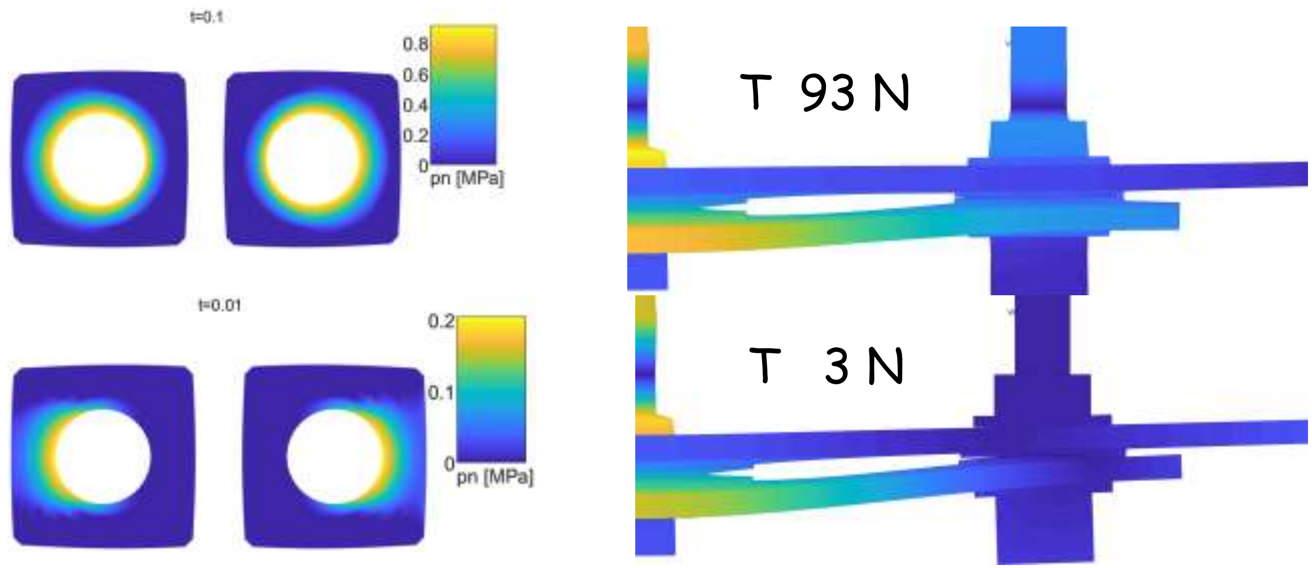
Equivalent stiffness : frequency/stiffness map

- Sensitivity to prestress/amplitude -> tangent stiffness map

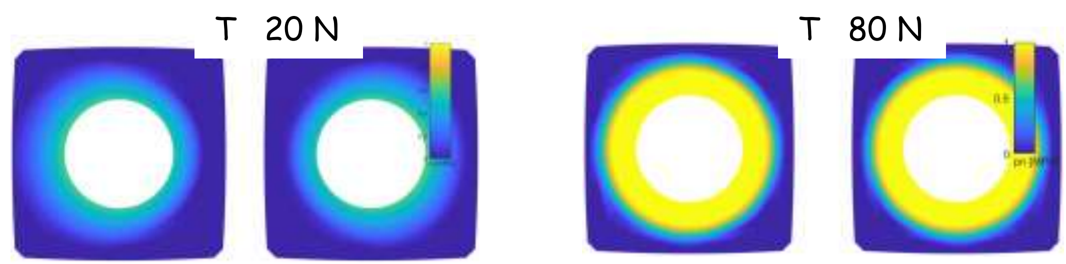


Numerical methologies useful/necessary to treat 2D

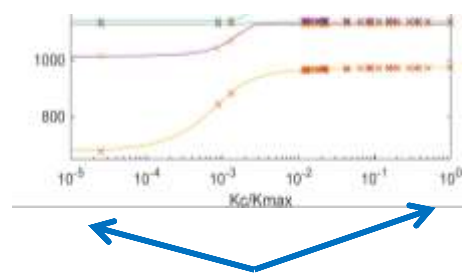
- StaticStatus
  iterate on nodes either
  bilateral or no contact



- StaticModeTraj [1]
  series of statics
  with $[M]\{\phi_j\}$ load



- Multi-model reduction [2]
  Ritz analysis on a few
  learning points

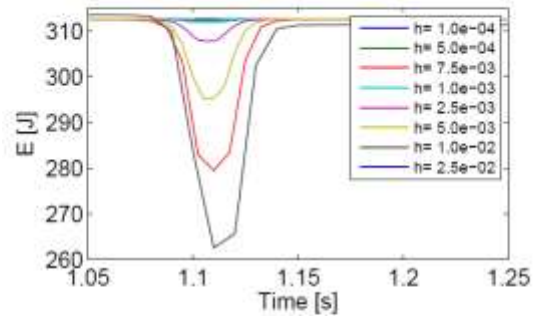


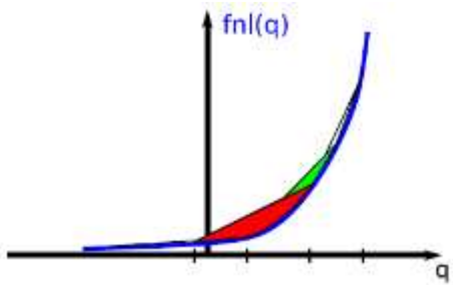
[1] H. Festjens, G. Chevallier, et J. Dion, « A numerical tool for the design of assembled structures under dynamic loads », IJMS, vol. 75, no 0, p. 170-177, 201
[2] Hammami, Balmes, Guskov, « Numerical design and test on an assembled structure of a bolted joint with viscoelastic damping », MSSP 2015.

On the need for equivalent laws

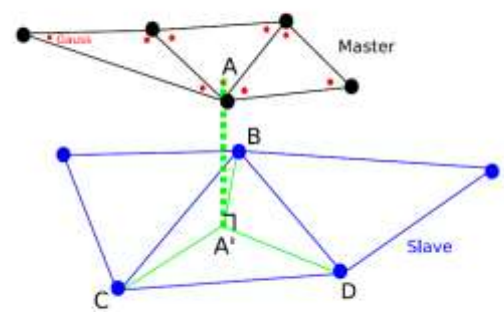
Physical contact laws are sensitive to accuracy on

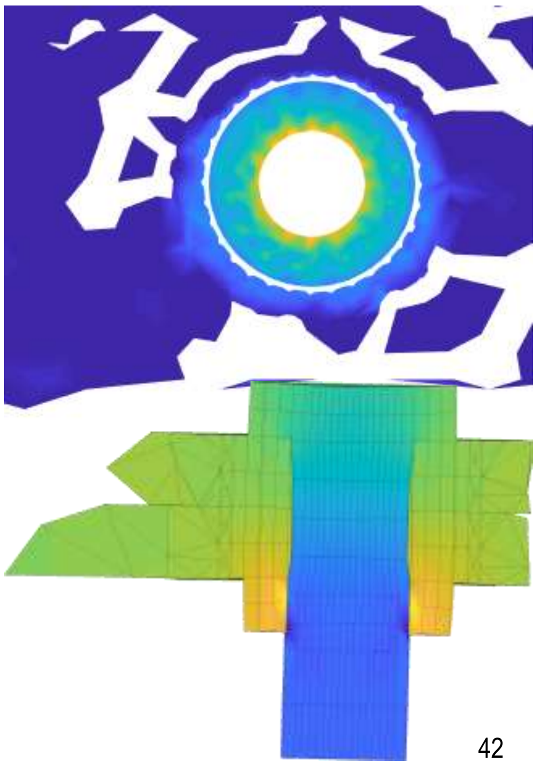
- Time events





- Spatial events





Equivalent 2D law needed to approximate both

Design change & pole tracking

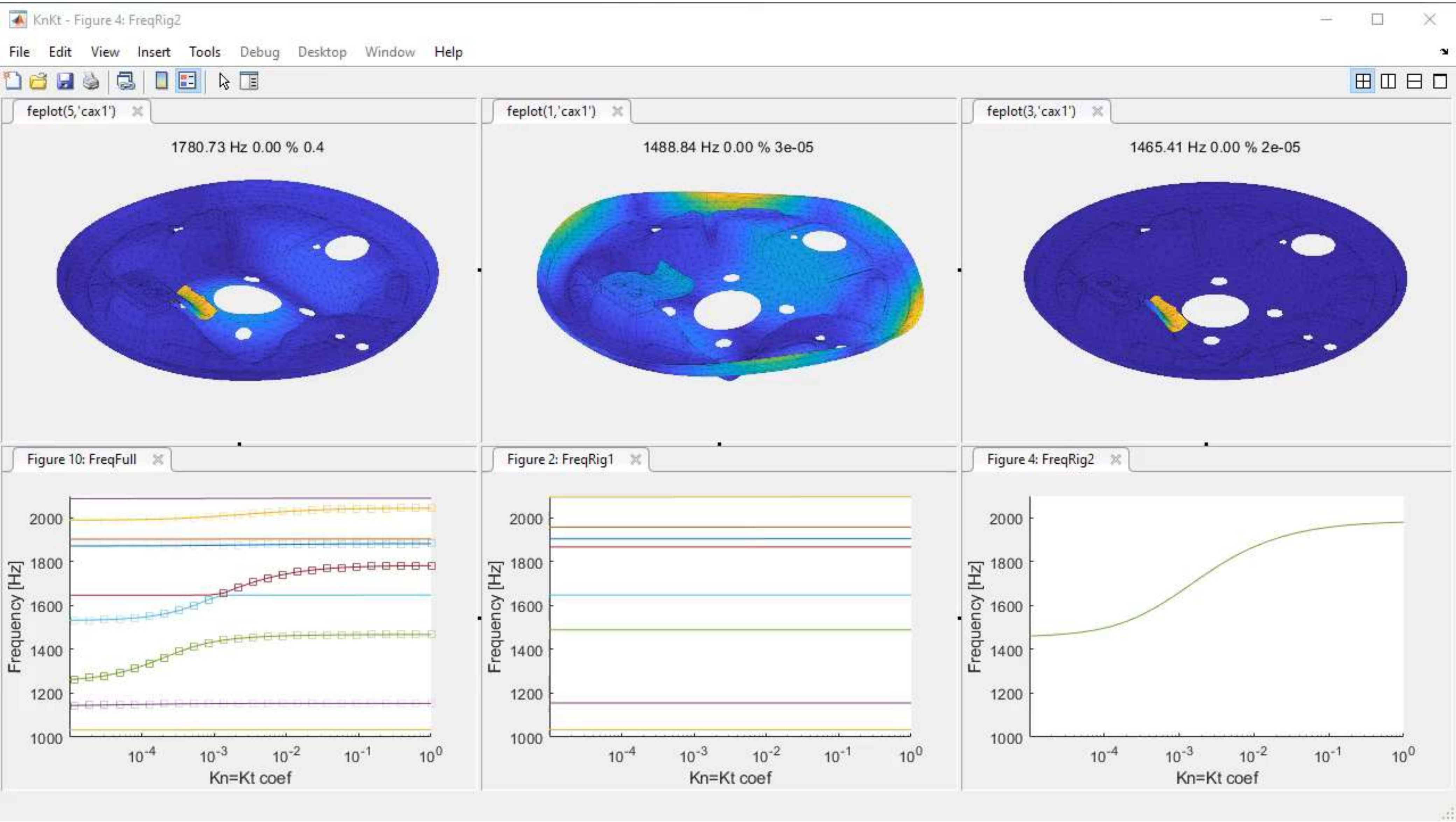
- Choose a reference $Z(p) = (K - \omega^2 M) = Z_0 + (Z(p) - Z_0)$
- Compute reference modes $\{\phi_j(p_0)\}, \omega_j(p_0)$
- Use generalized coordinates/energies [1] rather than MAC

$$\alpha_j(t) = \{\phi_j^0\}^T [M(p_0)] \{q(t)\}$$
$$\{e_j(t)\} = \dot{\alpha}_j^2 + \omega_j^2 \ \alpha_j^2$$

- Possibly use component modes
  - CMT [2] : free +global $\begin{bmatrix} \phi_A^0 & \phi_{global}^0\big|_A & & \\ & & \phi_B^0 & \phi_{global}^0\big|_B \end{bmatrix}_\perp$
  - Flexible with rigid global [3] in subspace $\begin{bmatrix} I_A & \\ & \phi_{rig}\big|_B \end{bmatrix}_\perp$

[1] Bianchi, Balmes, Vermot, Bobillot, « Using modal damping for full model transient analysis. Application to pantograph/catenary vibration », ISMA, 2010
[2] G. Vermot Des Roches, « Frequency and time simulation of squeal instabilities. Application to the design of industrial automotive brakes », Ph.D ECP 2011
[3] G. Martin, « Méthodes de corrélation calcul/essai pour l'analyse du crissement », Ph.D. CIFRE SDTools, Arts et Metiers ParisTech, Paris, 2017

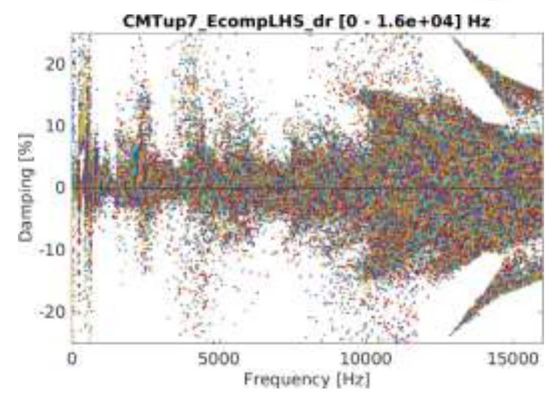
KnKt study (SDT contact)



https://youtu.be/A9V2MtKRcS8

Summary : parametric direct problems

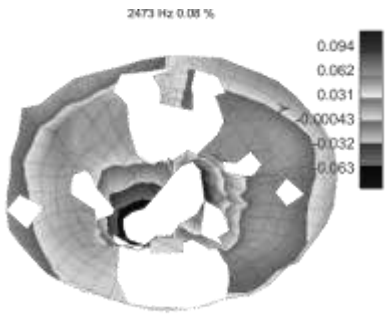
- Geometry updating key : inverse CAD or morphing
- Material parameter classics
  - grouping/clustering/field generation
  - homogeneization & scale separation
- Junction parameters
  - 0D or 2/3D laws
  - Equivalent 2D for contact (time/space constraints)
- Parametric studies
  - Reduction is key for CPU time AND memory
  - Tracking tools needed to make sense of data



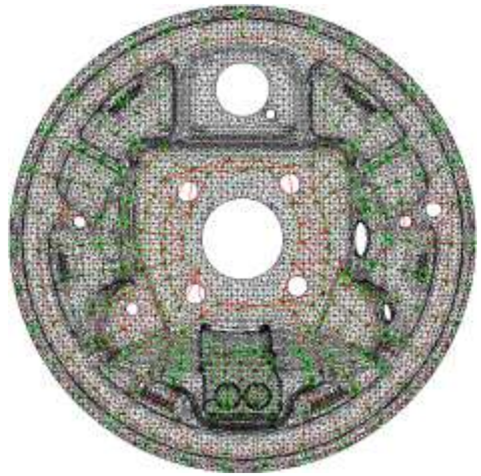
Design exploration 1000 points

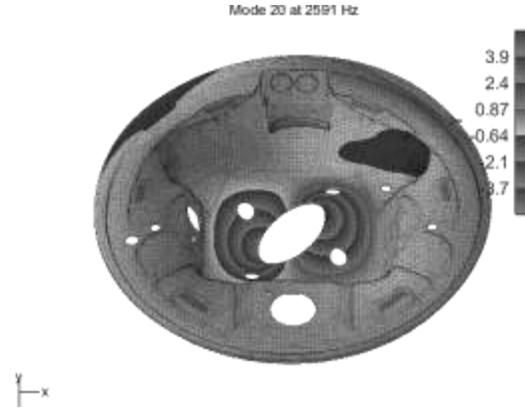
- Full 80 days CPU, 22 TB
- CMT a few hours off-line learning, <1h exploration, 10 GB

Hybrid test & FEM



Identification error
- Noisy measurements
- Identification bias
- NL, time varying, …





FEM error
- Geometry
- Material paramet
…

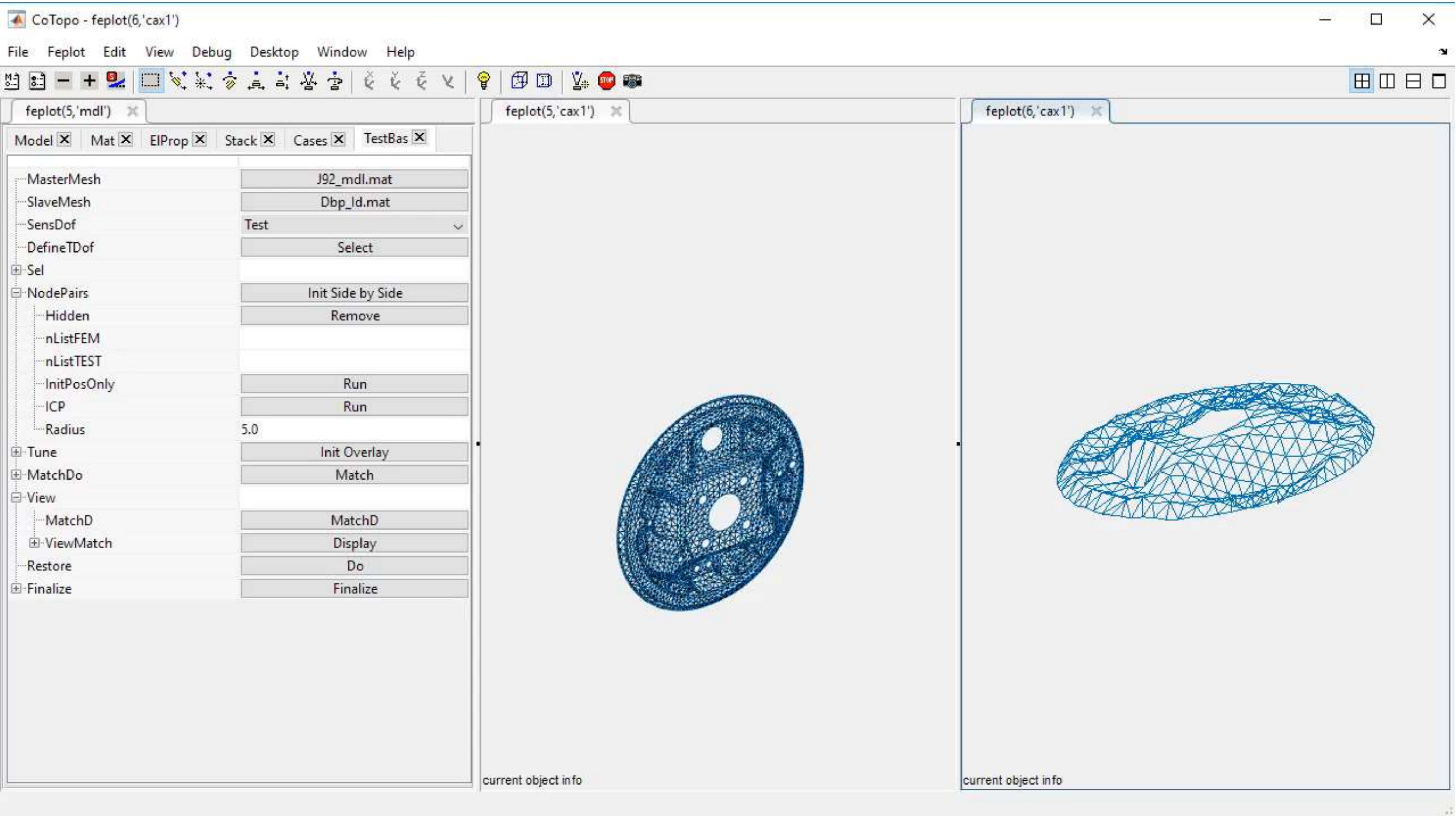
Topology correlation= observe FEM @ sensors

Correlation = distance between test & FEM

Updating = estimate parameters using correlation

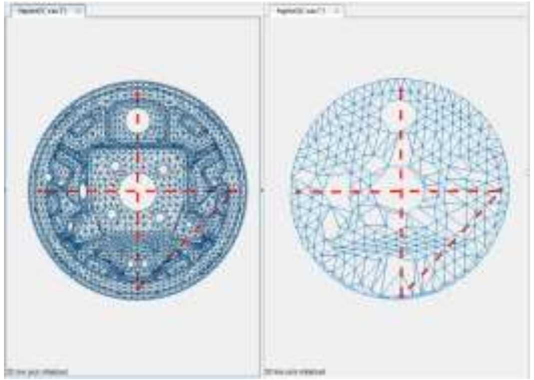
Expansion = estimate all DOF knowing test and model

Basic topology correlation process

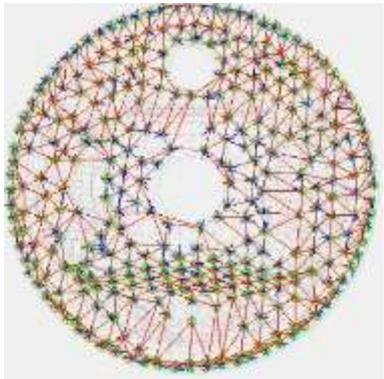


Topology correlation process

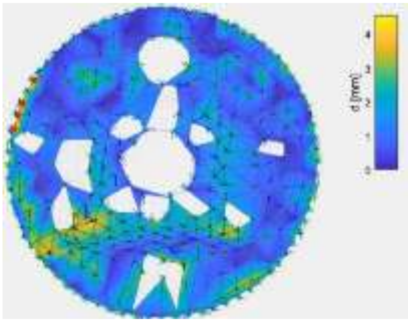
- Initial orient (coarse)
- Give reference points & reposition



- Optimize orientation to minimize distance

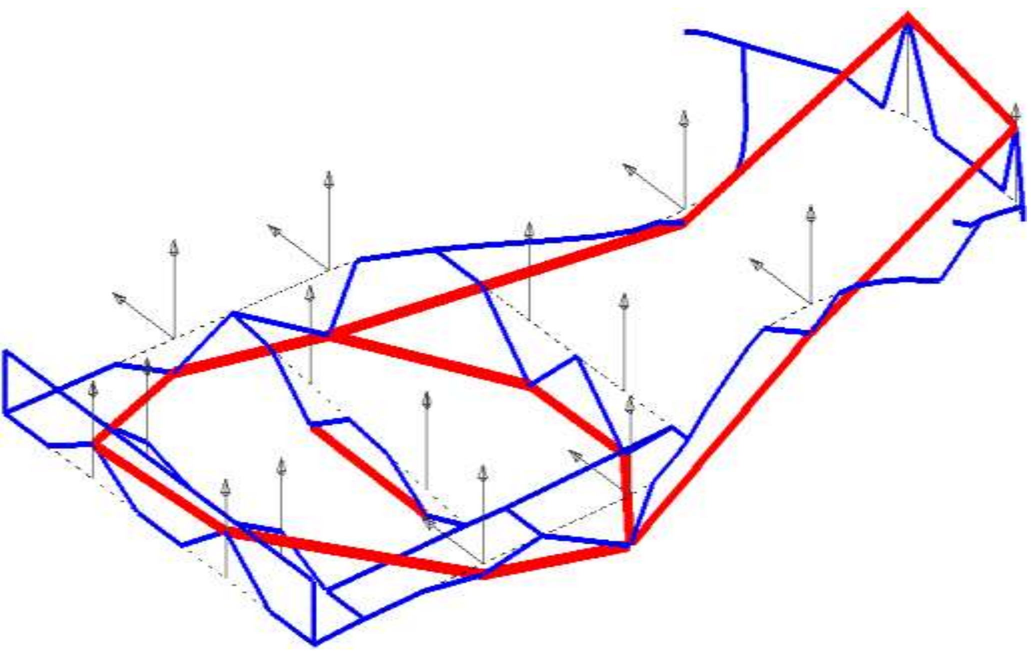


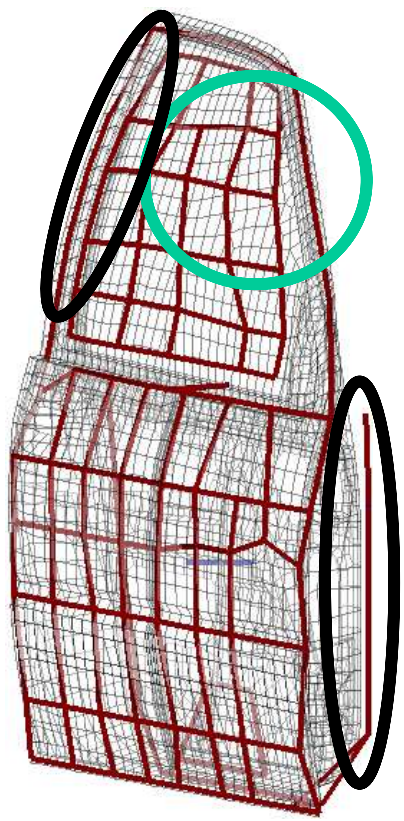
- Analyze errors

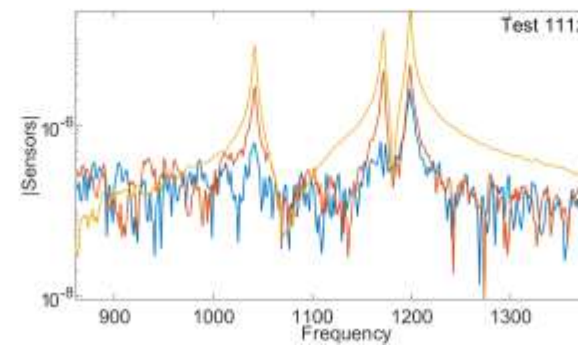


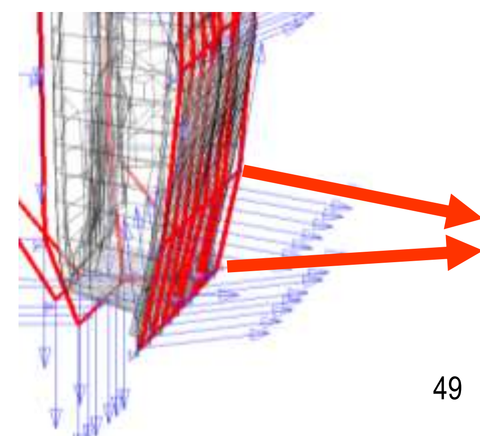
Test topology objectives : view & correlate

- Connect to allow viewing
- Use regularity to avoid distortion
- Use exact topology
- Triax measurement is not the universal answer (⇒ expand)

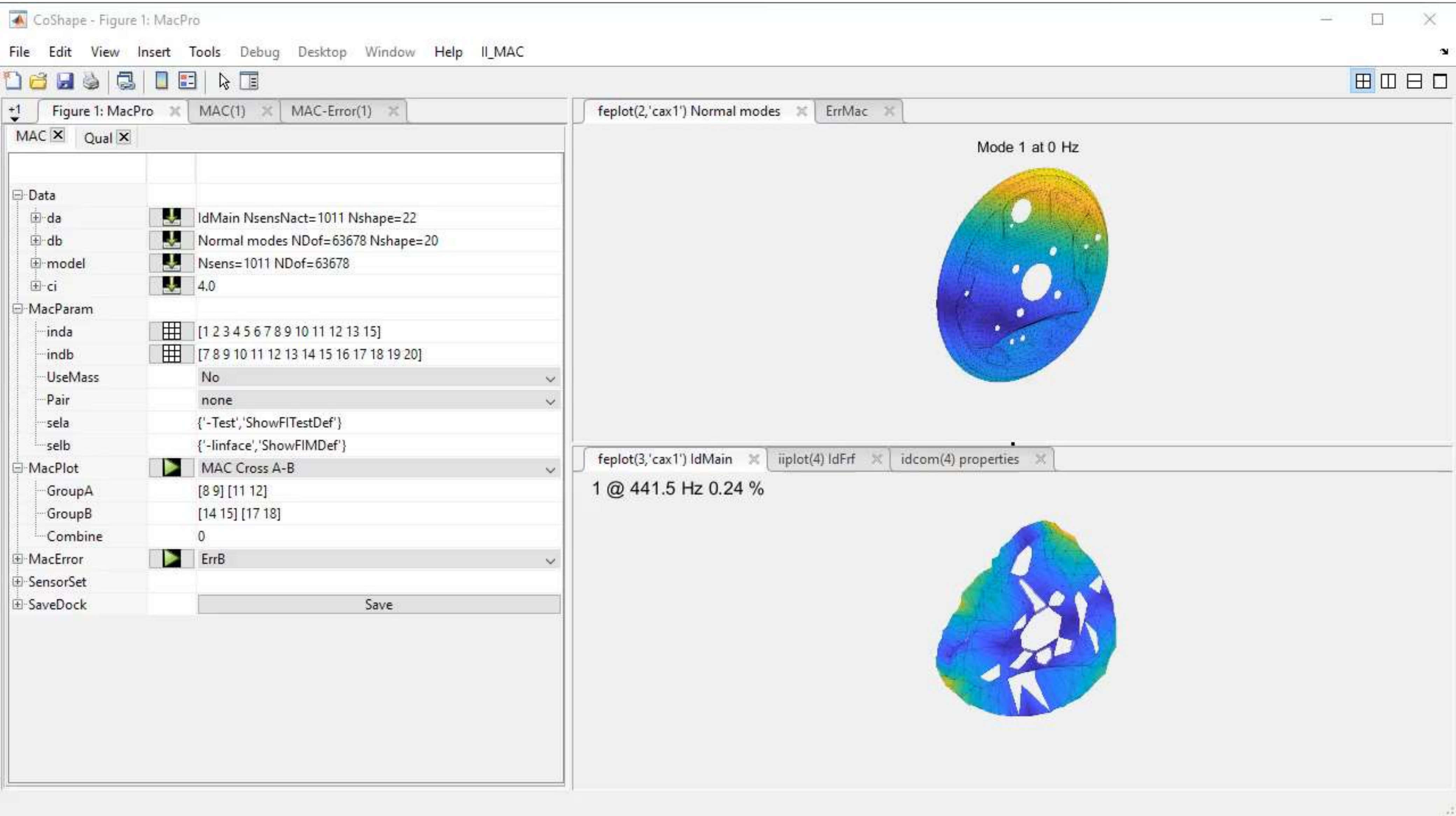








Basic shape correlation

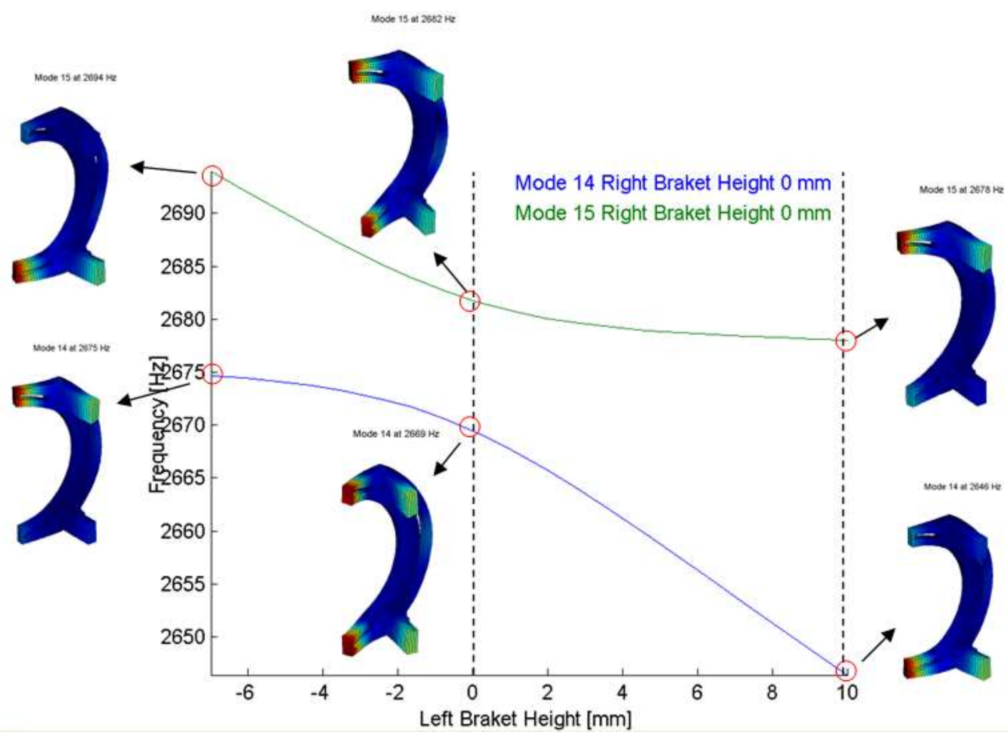


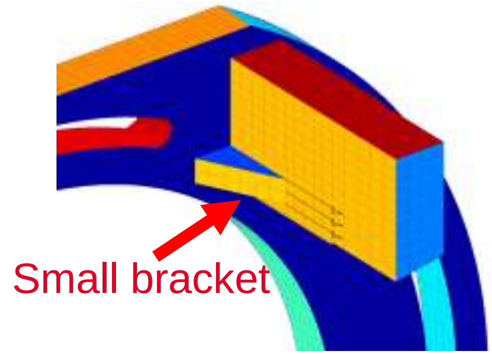
https://youtu.be/tVV-R2r-3M8

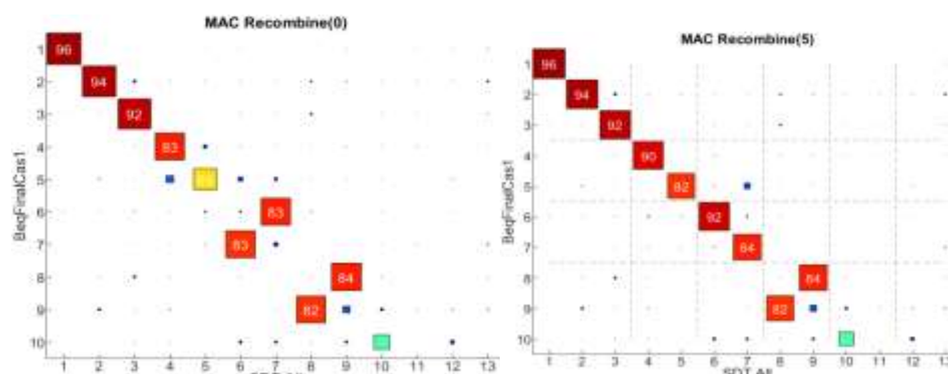
Correlation : mode crossing & MAC combine

- High sensitivity for close modes associated with mode crossing
- MAC can drop to 0.5
- MAC combine linear combination of close modes = stays at 1.
- Orthonormal combination of modes in a group

$$A = argmin\left\|[\phi_{test}]_{gA} - [c\phi_{FEM}]_{gB}[A]\right\|$$

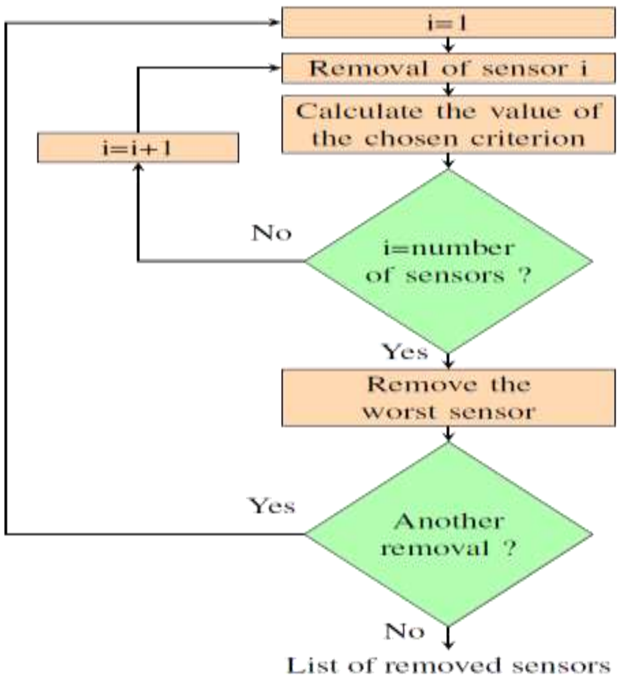






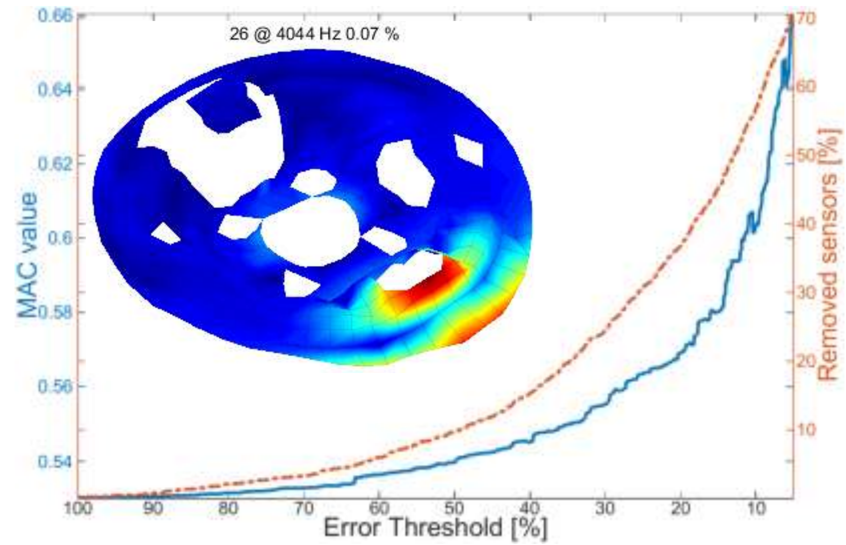
MAC & sensors : historical methods

- MAC on sensor sets
  - ☺ Localize area of poor correlation
  - ☹ No automated set definition
- COMAC : is a specific sensor bad ?
  - correlation of multiple modes at one sensor
  - eCOMAC : allow incorrect modeshape scaling
  - Not per mode / sensor
- MACCo : does correlation improve with less sensors ?
  - ☺ Easy to implement
  - ☺ Sensor sets per mode or global
  - ☹ No understanding of why each sensor is removed.



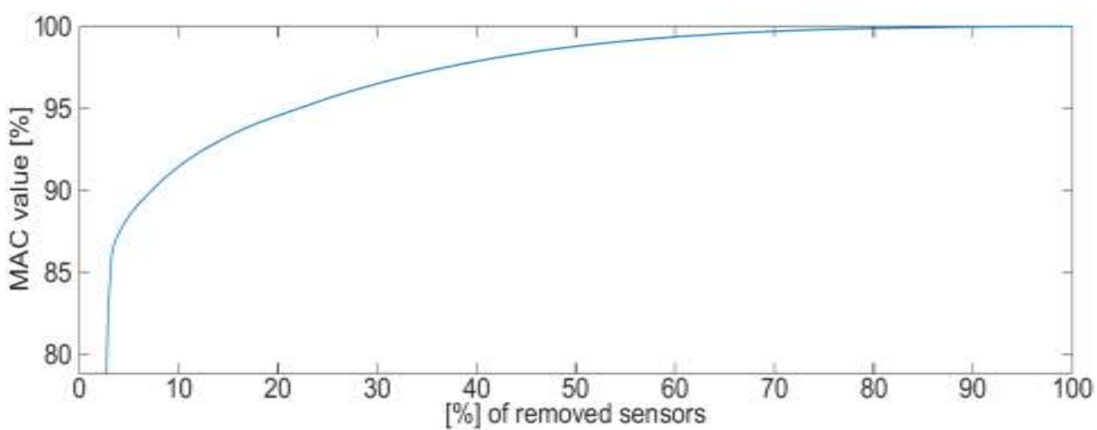
[1] N. A. J. Lieven, D. J. Ewins, « Spatial Correlation of Modeshapes, The Coordinate Modal Assurance Criterion (COMAC) », IMAC, 1988.
[2] Brughmans, Leuridan, Blauwkamp, « The application of FEM-EMA correlation and validation techniques on a body-in-white », IMAC, 1993
[3] Martin, Balmes, Chancelier, « Characterization of identification errors and uses in localization of poor modal correlation », MSSP, vol. 88, p. 62-80, may 2017

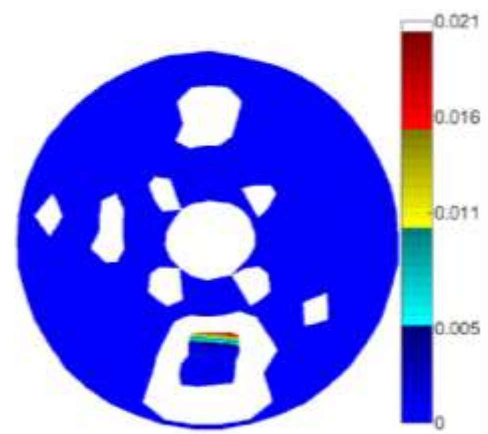
MAC & sensors : MACCo variants



MACErr : Ignore sensors with bad identification

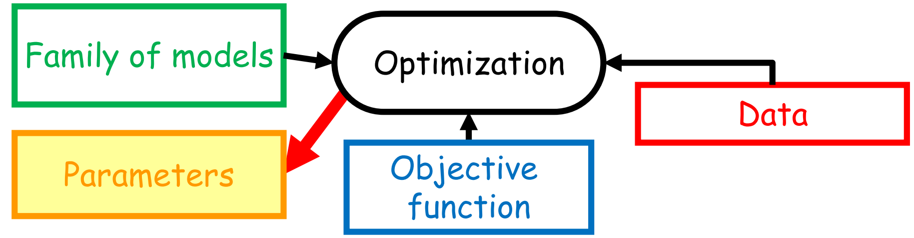
MACCo : sort sensors by impact on MAC
Removed sensors may indicate
- Bad measurements
- Model errors





Updating : freq-MAC objective

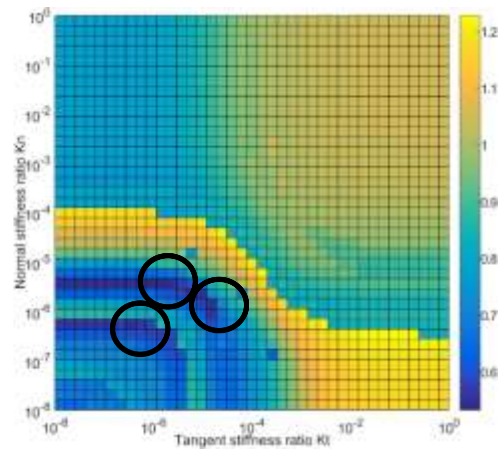
Family : FEM with updated geom.
Parameters : mat/junction
Data : test modes
Objective function $J_{freq\text{-}MAC}$

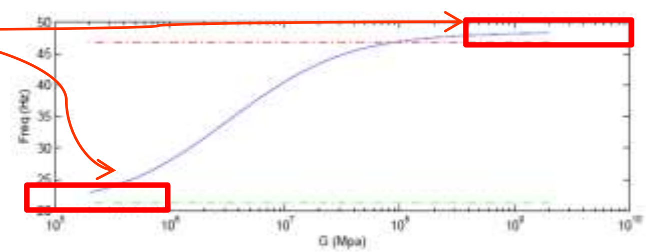


$$R_j(p) = \left\{ \begin{array}{c} \frac{f_{EF,i} - f_{Test,j}}{f_{Test,j}} \\ \boxed{\beta}(1 - MAC_{i,j}) \end{array} \right\} \qquad J_{freq-MAC}(p) = \sqrt{\sum_{j=1}^{NM} \|R_j(p)\|^2}$$

Associated problems

- Mixed quantity (scaling)
- Jumps on mode pairing
- Parameter choice
- Sensitive range



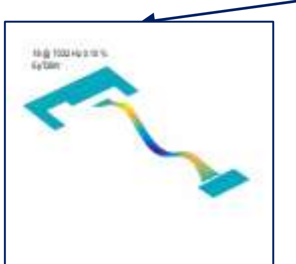


Parameter equivalence / conditionning

Tests

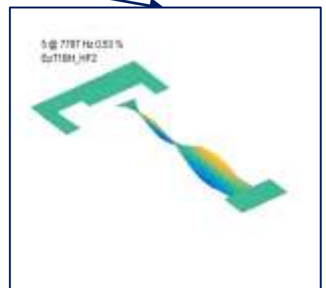
Observable modes

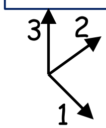
Vertical bending



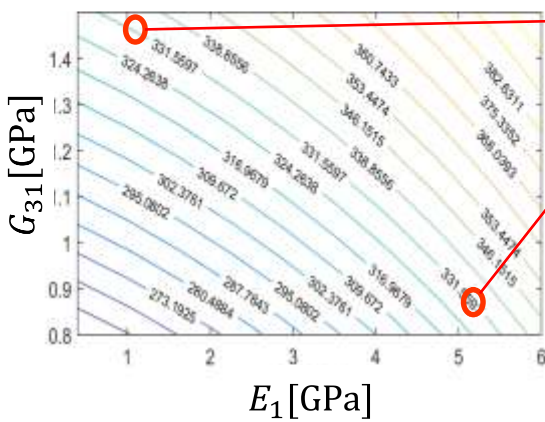
Identification

Torsions

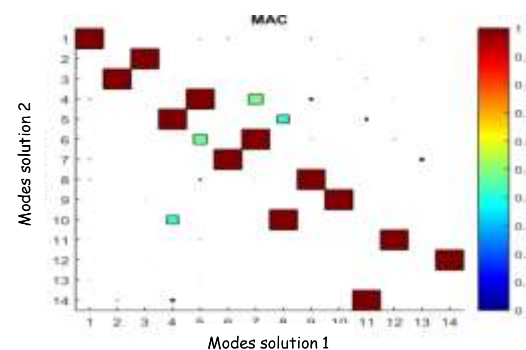




$$C^{-1} = S = \begin{pmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{pmatrix}$$



Solution is not unique
Mode-shape very similar



Assumption must be made to improve conditioning

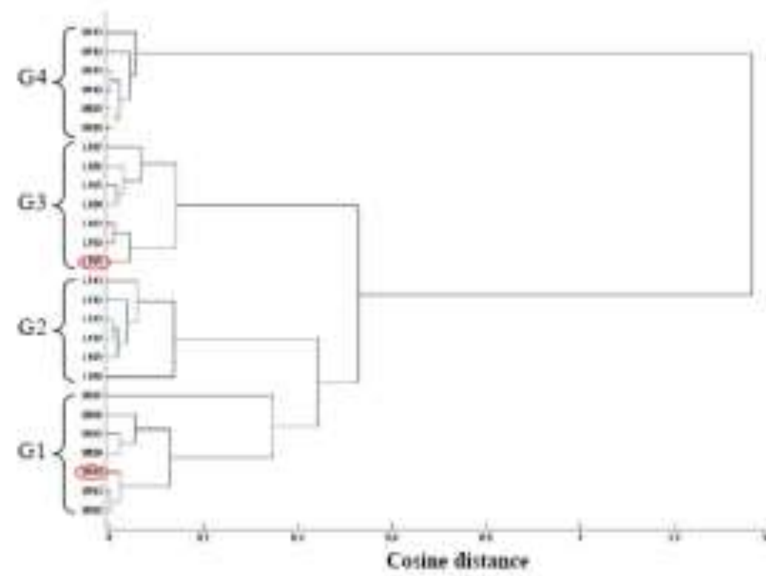
- G_{31}/E_1 constant ?
- E_1 traction ?

Sample clustering method

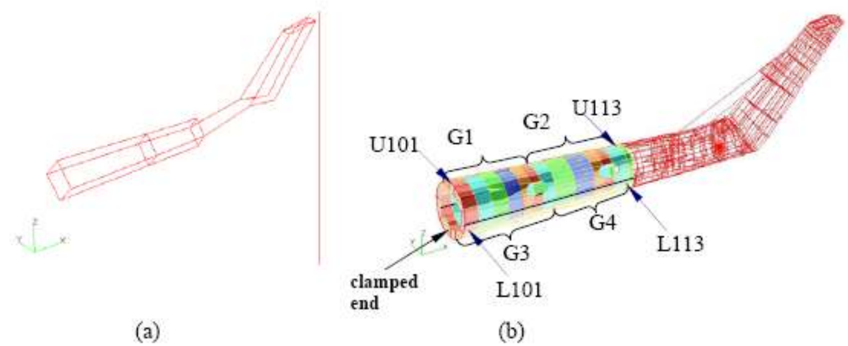
- Compute cosine distance

$$\cos^{-1}\left(max\,\frac{\langle u|u\rangle}{\|u\|\|v\|}\right)$$

A,B -> U_A,U_B -> $\sigma_{max}\left(U_B - U_A(U_A^H U_B)\right)$



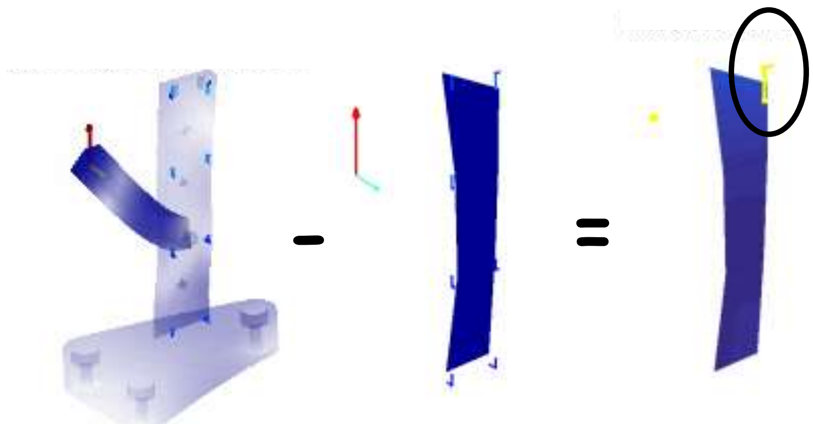
- Recursively group elements with smallest distance



Shahverdi, Mottershead & All, Clustering of Parameter Sensitivities: Examples from a Helicopter Airframe Model Updating Exercise.

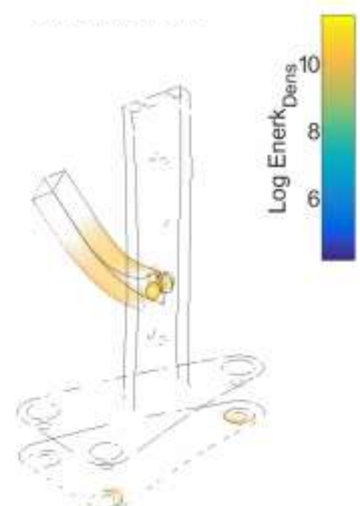
Expansion / hybrid models

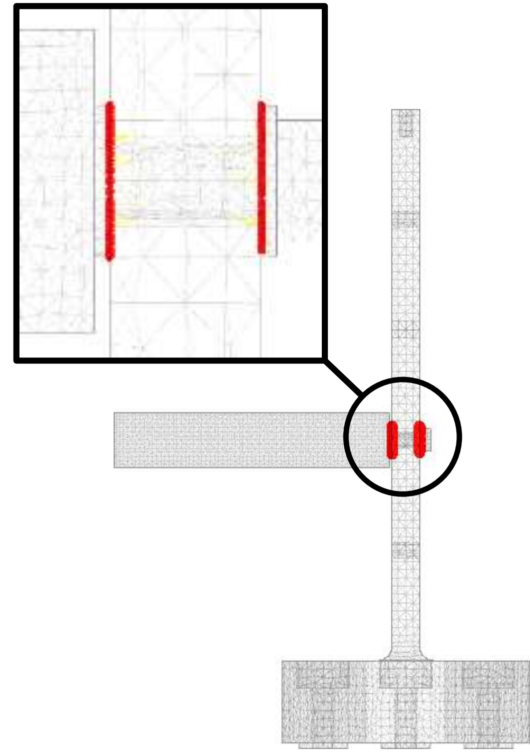
Test error:
Reverse sensor direction



$$[c]\{\hat{\phi}\} \quad - \quad \{y_{Test}\} \quad = \quad \epsilon_{Test}$$

Model error:
coupling stiffness





$$\{R_L\} = \left[K - \omega_j^2 M\right]\{\hat{\phi}\} \neq 0$$ Load residual

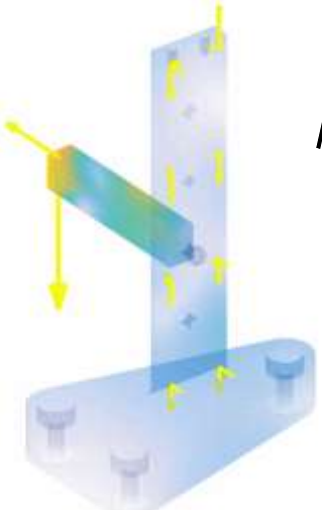
$$\{R_D\} = [K]^{-1}\{R_L\}$$ Displacement residual

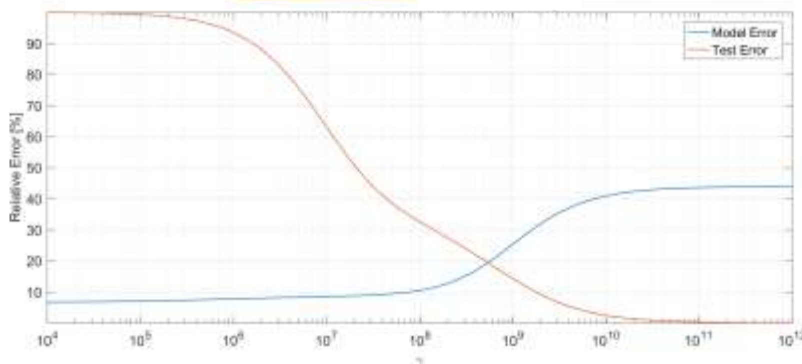
Residuals for test and model error

MDRE : multi-objective cost function

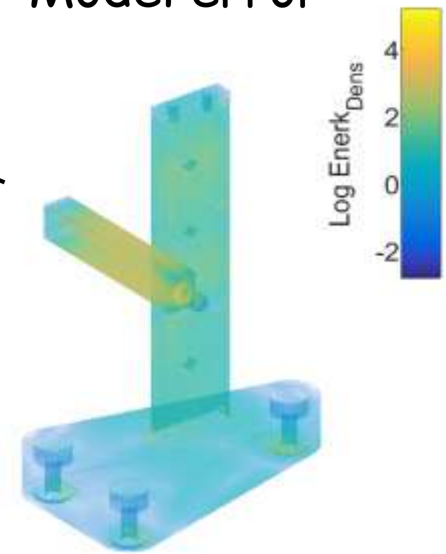
$J(\phi_{exp}) = \left\|\epsilon_{Test}(\phi_{exp})\right\|\gamma + \left\|\epsilon_{Mod}(\phi_{exp})\right\|$ multi objective cost function

Test error





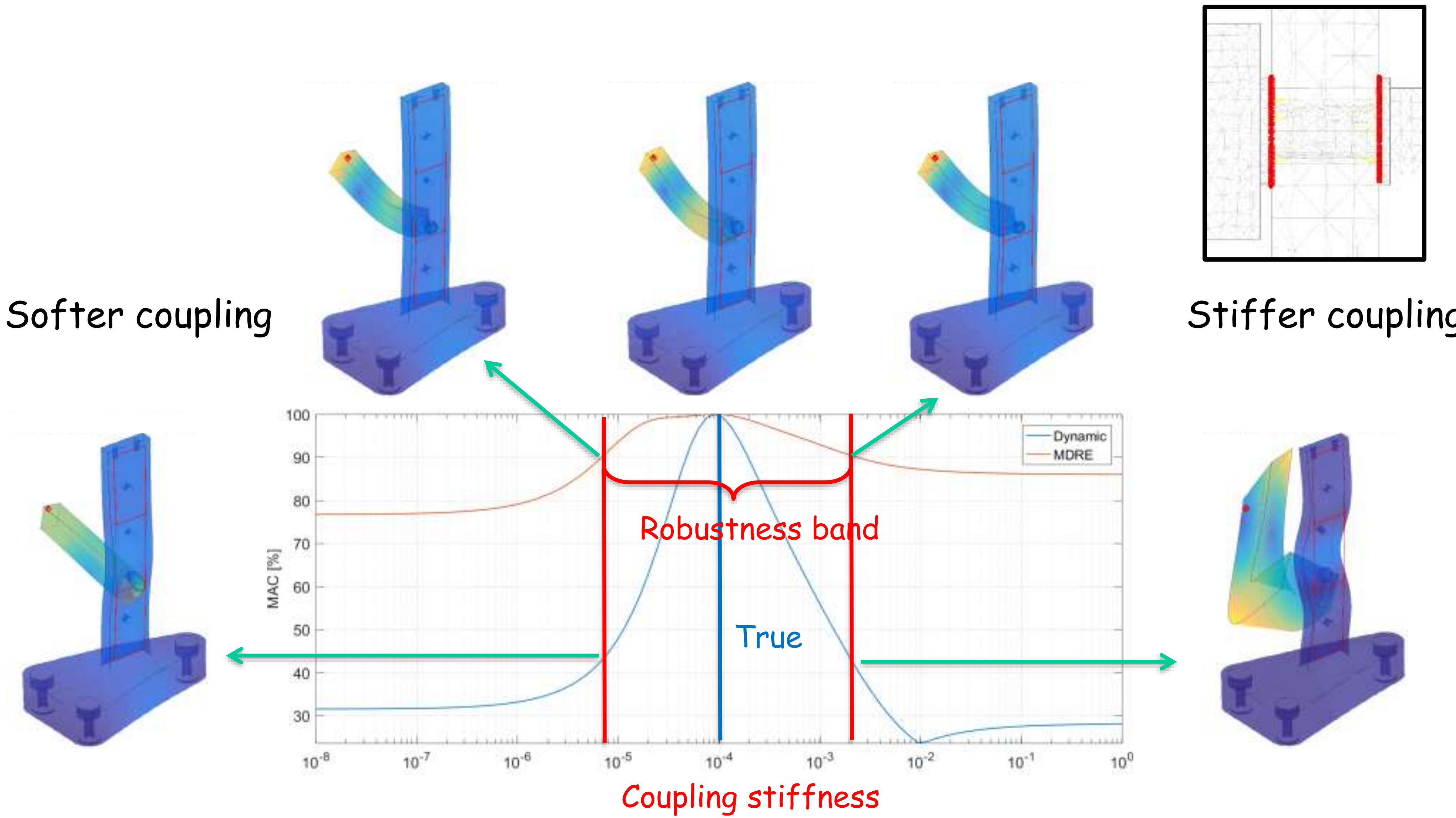
Model error



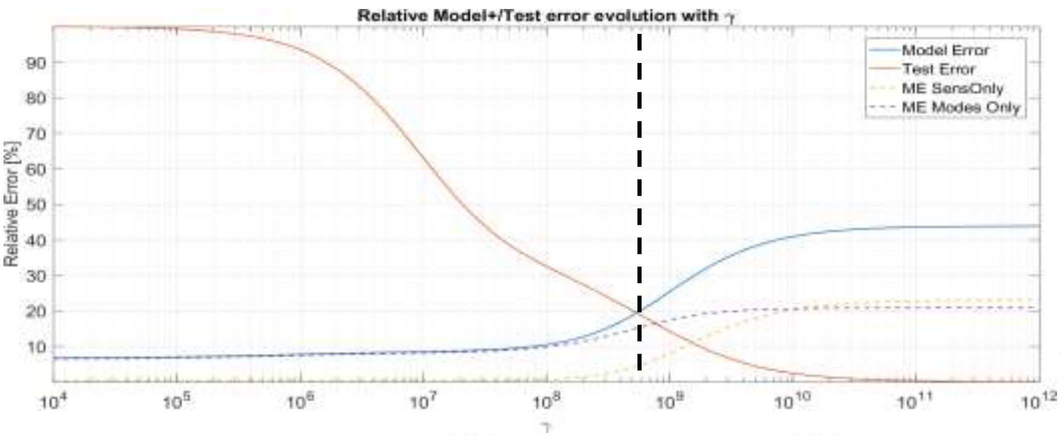
ϵ_{Test}
$= \left\|[c]\{\phi_{exp}\} - \{y_{Test}\}\right\|$

$\{R_L\} = \left[K - \omega_j^2 M\right]\{\phi_{exp}\} \neq 0$
$\epsilon_{Mod} = \|[K]^{-1}\{R_L\}\|_K$

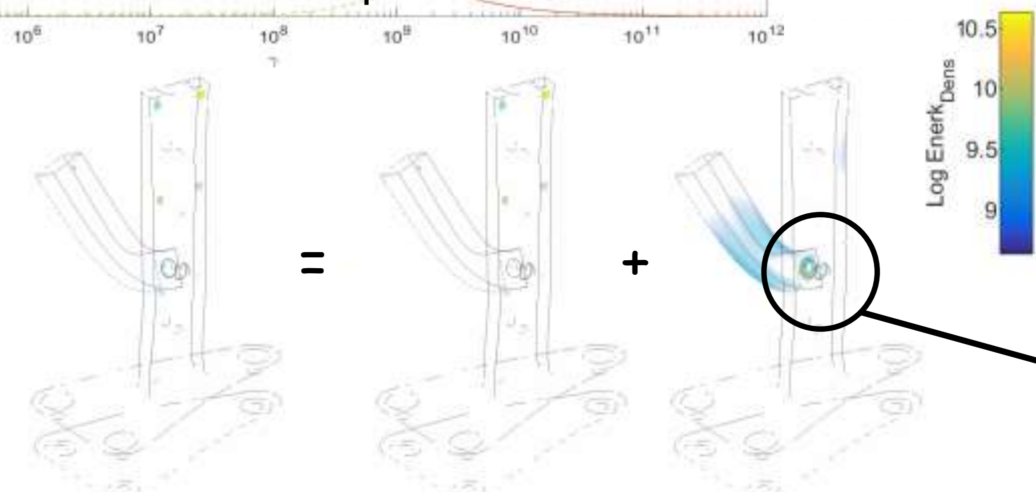
Expansion : robust to model error



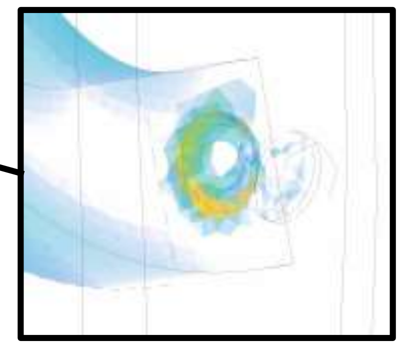
Model error = clues for parameterization



Model error concentration at sensors
Decoupling model error on
- Model modes
- Enrichment



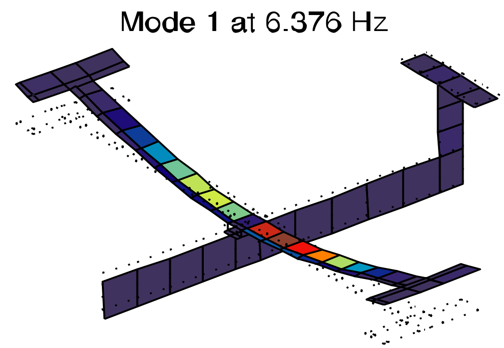
Concentration of model error close to the coupling



Model error : guides model parameterization
Test error : can be used to locate poor sensors

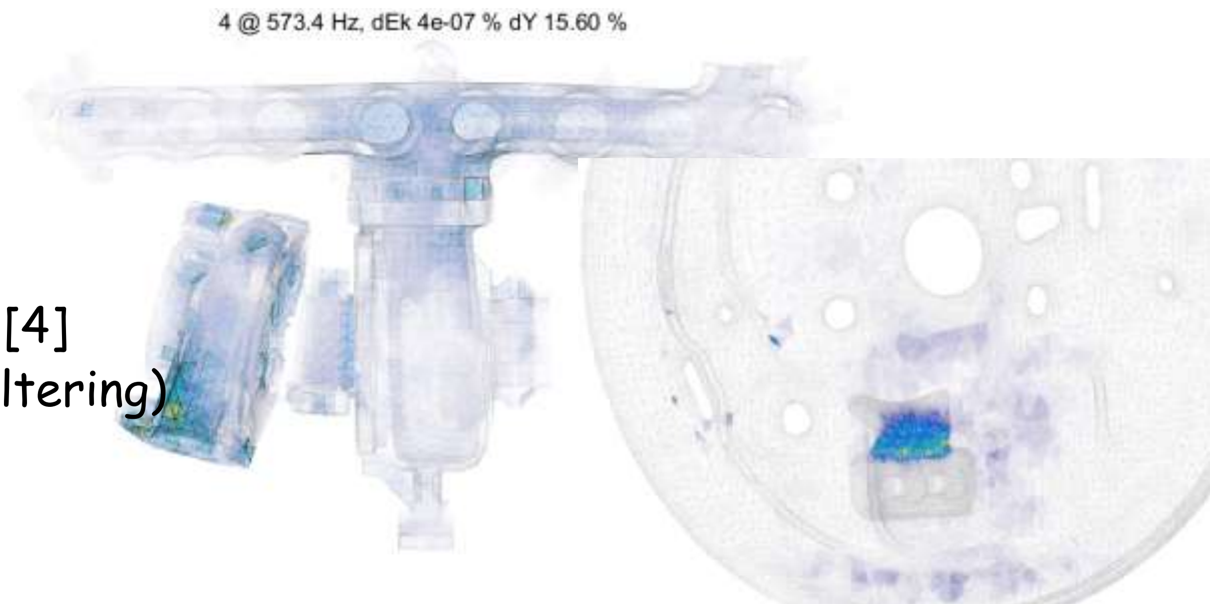
Expansion : an old method with renewed interest

Early work : Ladeveze 72 -> [1] present
Independent start : Balmes [2] 00 -> present
Not much used for 40 years due to computation challenge (see [3])



Renewed interest at SDTools (last 3 years) due to reduction

- Modes / multi-model + sens $[K]^{-1}[b]$
- A few thousand DOF, a few seconds per point



Potential uses

- Updating
- Frequency domain
  structural dynamics modification [4]
- Time domain (extended Kalman Filtering)

[1] Deraemaecker, A., Ladevèze, P., Leconte, P. Reduced bases for model updating in structural dynamics based on constitutive relation error. computer methods in applied mechanics and engineering. CMAME 2002

[2] E. Balmes, « Review and Evaluation of Shape Expansion Methods », IMAC, p. 555-561, 2000.

[3] N. Nifa, « Solveurs performants pour l'optimisation sous contraintes en identification de paramètres », Ph.D. thesis, Université Paris-Saclay, 2017

[4] M. Corus, « Amélioration de méthodes de modification structurale par utilisation de techniques d'expansion et de réduction de modèle. », ECP, 2003.

Summary : hybrid test/FEM

- With enough sensors (extreme case field measurement)
- Reasonable match between model and reality (LTI, no local modes, …)

… updating (=parametric identification) works

If it does not, don't throw the model away

- Lots of things will never be measured : hidden faces, internal loads, …
- Many can be estimated even with somewhat wrong model
- Hybrid models have used efficiently for decades
- Reduction extends usability to vibration problems

"Digital twin" is the current hype keyword for this old idea

Acknowledgements

- Engineers @ SDTools
  Guillaume Martin, Guillaume Vermot des Roches, Jean Philippe Bianchi, Jean -Michel Leclère
- Ph.D. students @ ECP & ENSAM
  M. Corus, A. Sternchuss, C. Florens, T. Thénint, C. Hammami, E. Arlaud, O. Vo Van
- Companies & funding agencies for work shown