Shapes & DOF : on the use of modal concepts in the context of parametric, non-linear studies

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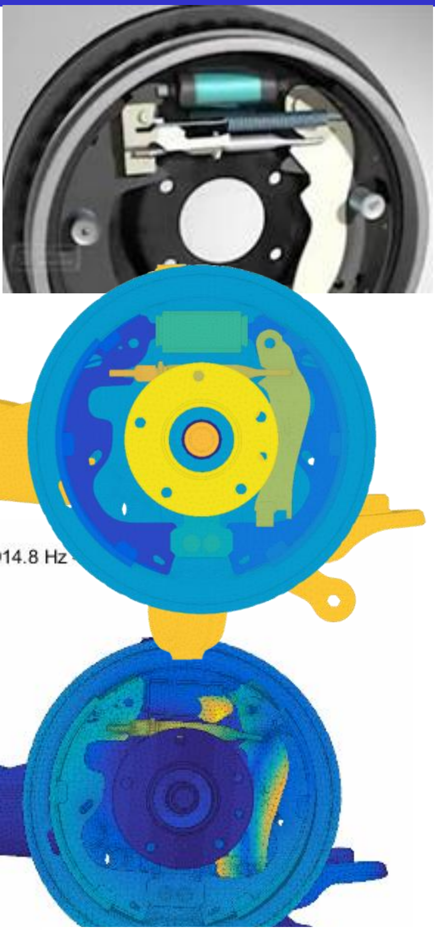
Modal analysis : the starting point

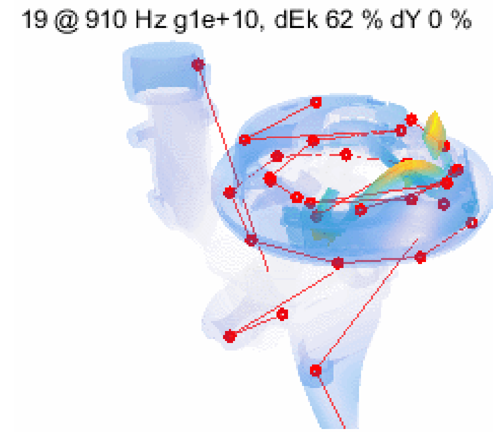


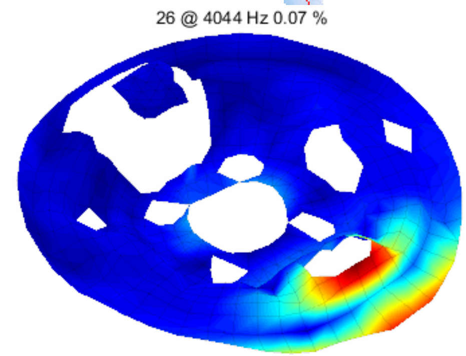
Direct (numerical) :

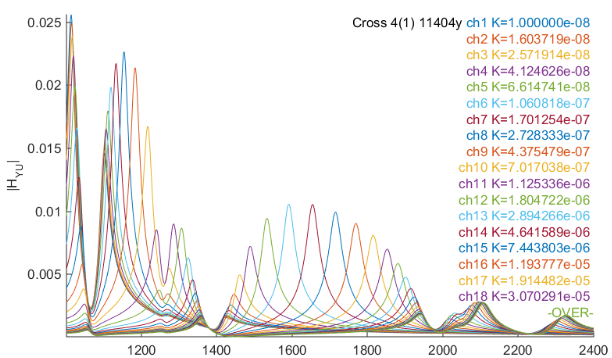
- CAD
- FEM
- Modes
- Observation
- Transfers
- Time simul.

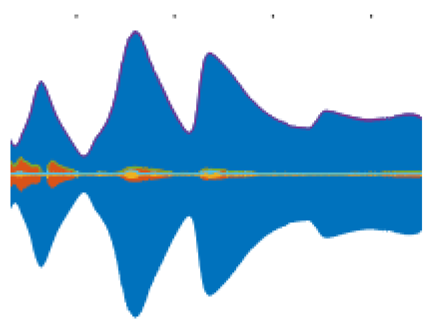
Inverse (test) :

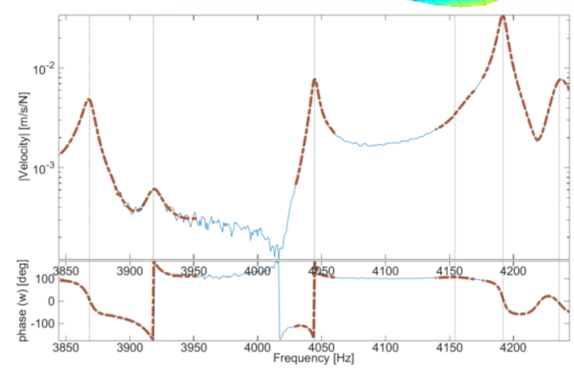
- Expansion
- Correlation
- Modes @ sensors
- Transfers
- Time acquisition



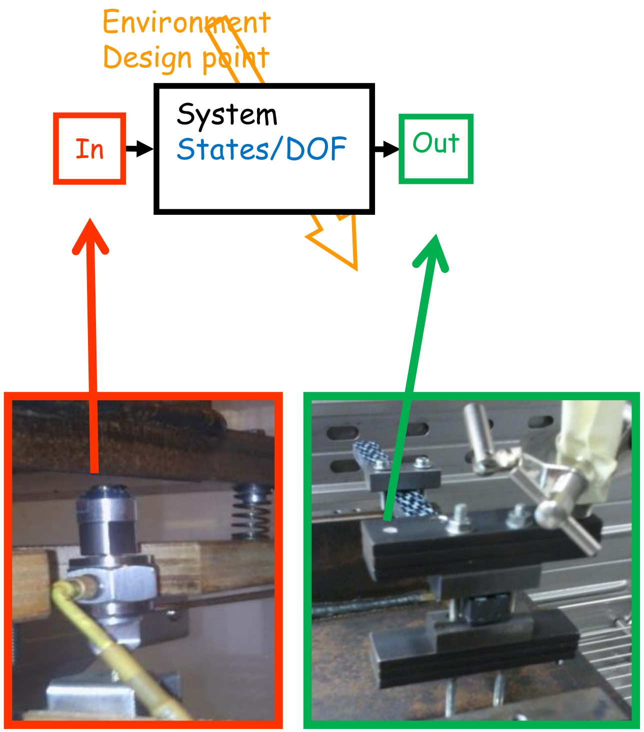








System modeling : the other key perspective

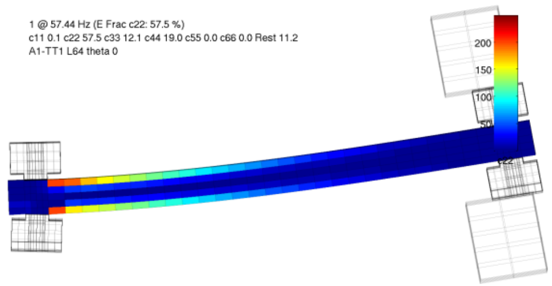


- **Inputs $u(t)$**
- **Outputs $y(t)$**
- **Environment variables p**
  - Dimensions, test piece (design point)
  - Temperature (value of constitutive law or state of thermo-viscoelastic)

- **State/DOF $x(t)$**
  - Displacement & velocity field as function of time

$$\{\dot{x}(t)\} = f(x(t), u(t), p, t) \qquad \text{evolution}$$

$$\{y(t)\} = g(x(t), u(t), p, t) \qquad \text{observation}$$



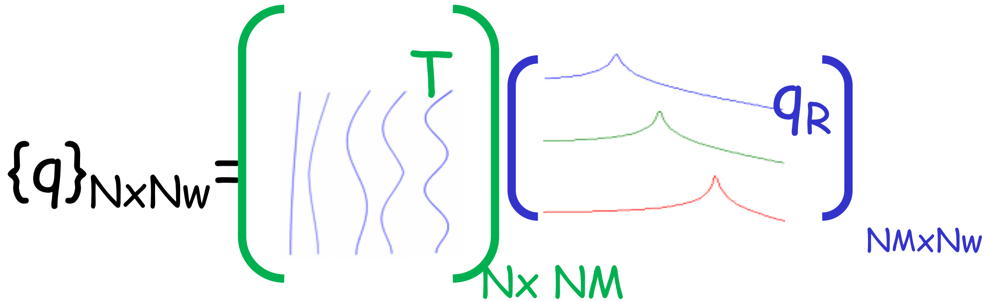
First bending =
mass on a spring = 1 DOF

Strain energy prediction $\Rightarrow$ FEM field many DOF

Simple example : modified Oberst test for 3D weaved composite test

Linear modal analysis : assumptions

- Space/freq (time) decomposition



$$[H(s)] = \sum_{j=1}^{NM} \frac{\{c\psi_j\}_{NS}\{\psi_j^T b\}_{NA}}{s-\lambda_j}$$

Shapes DOF

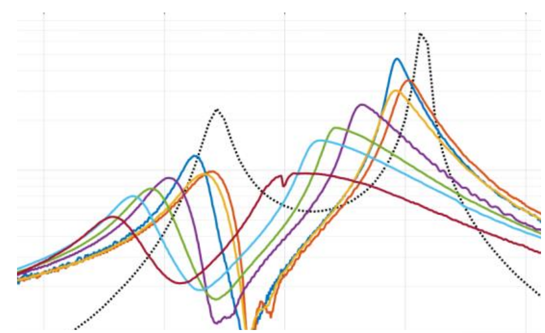
Modeshapes Modal amp.

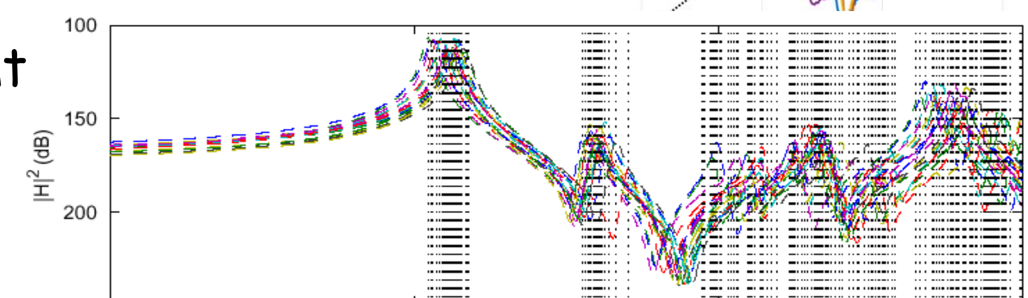
Works perfectly if

- Linear, time invariant, single design

But we want

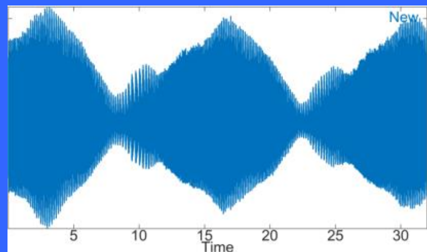
- Non-linear, stochastic
- Parametric design & environment
- Error control, …





Learning shapes in squeal event

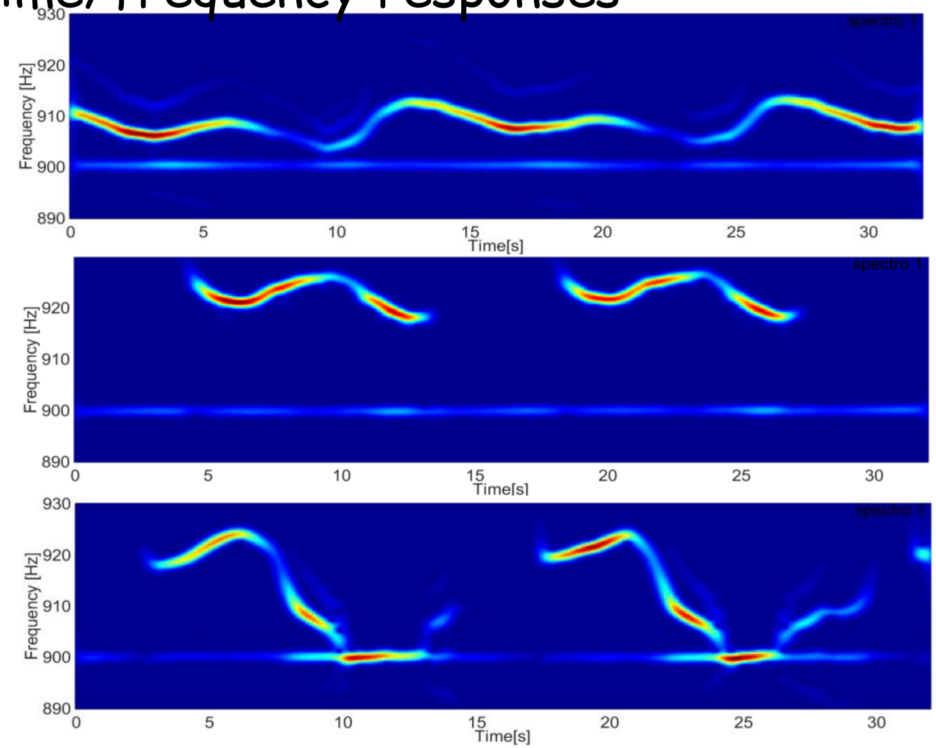
Time measurement during squeal



Variability
- influence of wheel angle

Reproductibility
- Multiple events

Sample time/frequency responses



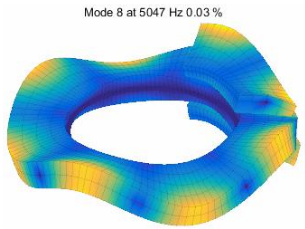
Braking event 1

Braking event 2

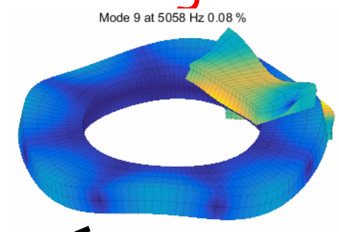
Braking event 3

What is expected from theory?

Real mode with
normal contribution

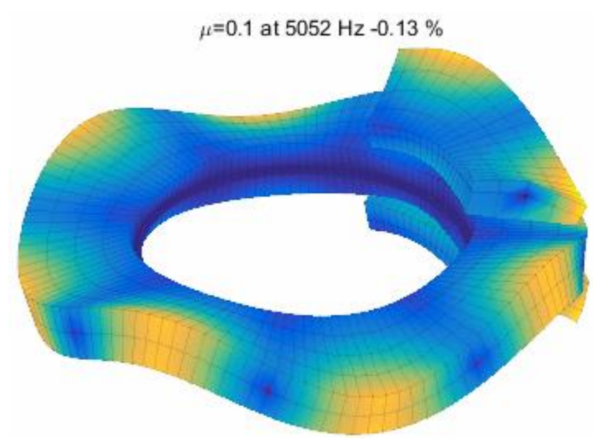


Real mode with
tangential contribution



Parameter : friction = non symmetric coupling
normal displacement ⇒ tangential load

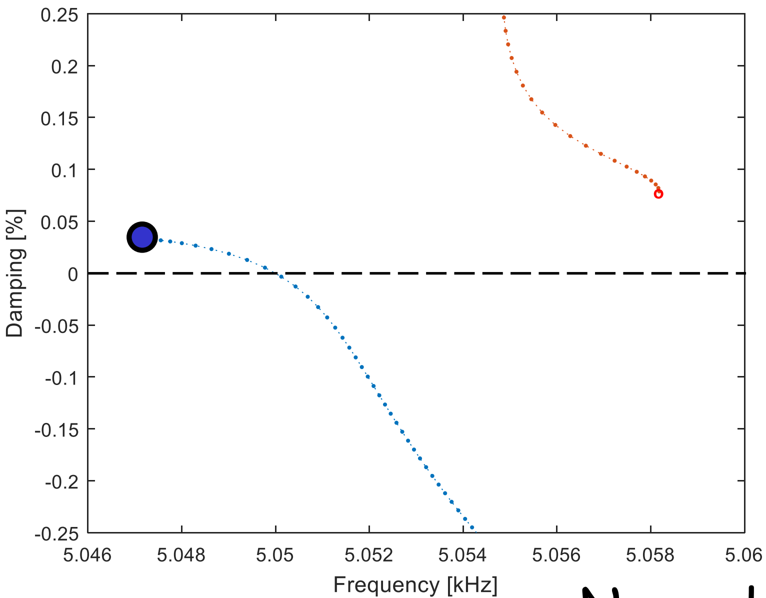
$$[K_u] = [\mathbf{b}_{TAN}]\left[\ {}^{\backslash}\mu w_j J \frac{\partial p}{\partial q}{}_{\backslash}\ \right][\mathbf{c}_{NOR}]$$



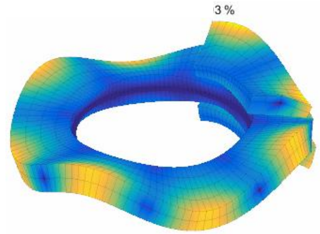
PhD. Vermot, 2011

Shapes, DOF, transition

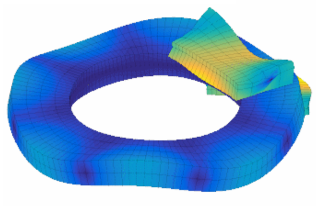
- Start $\mu = 0$ small damping
- increase μ : coupling and transition towards instability

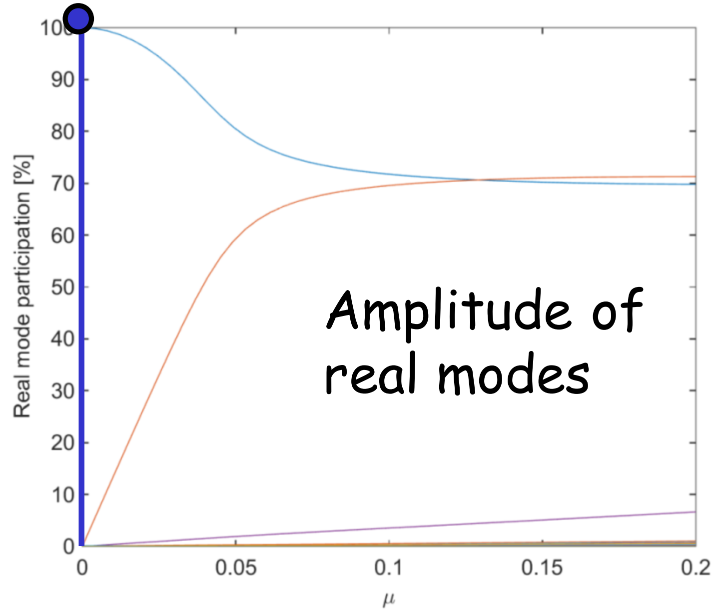


Normal

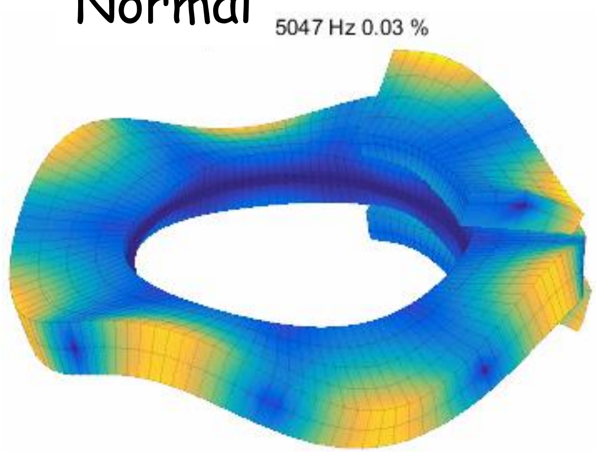


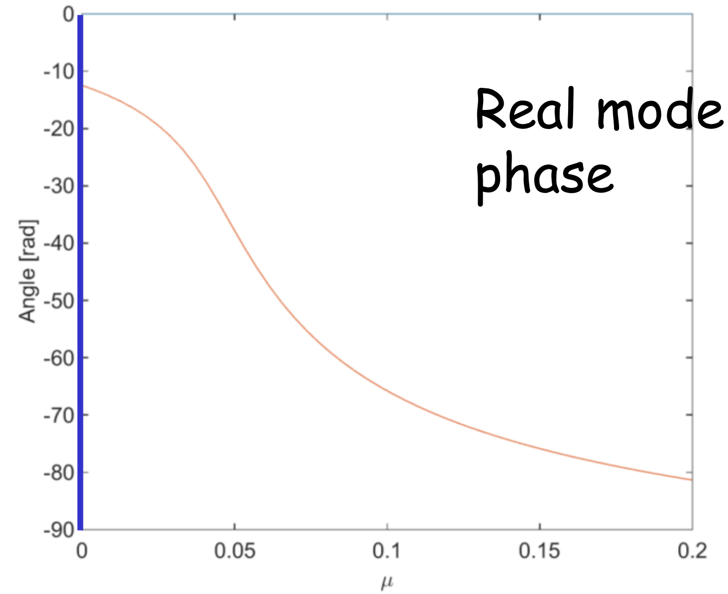
Tangent



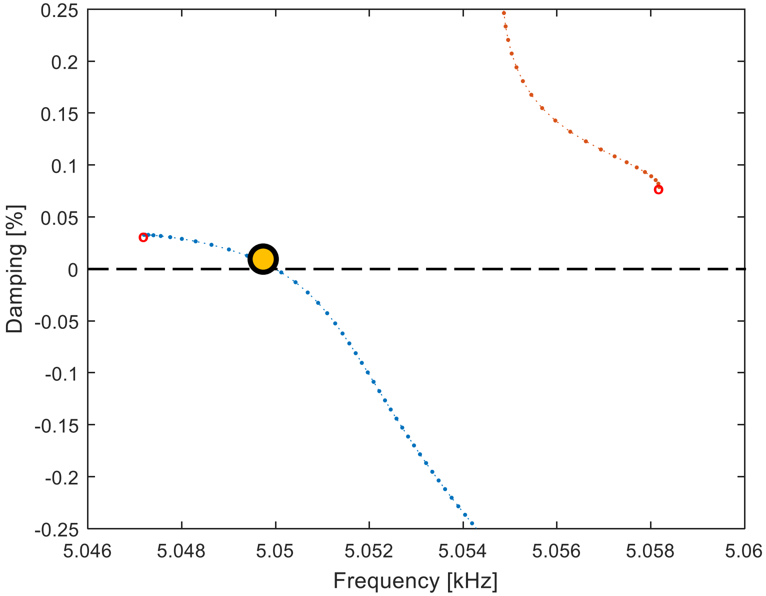


Normal

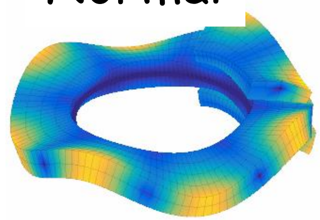




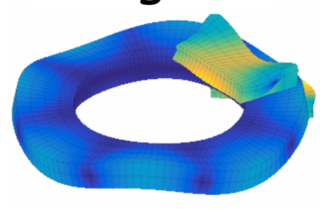
Participation of real shapes to complex modes

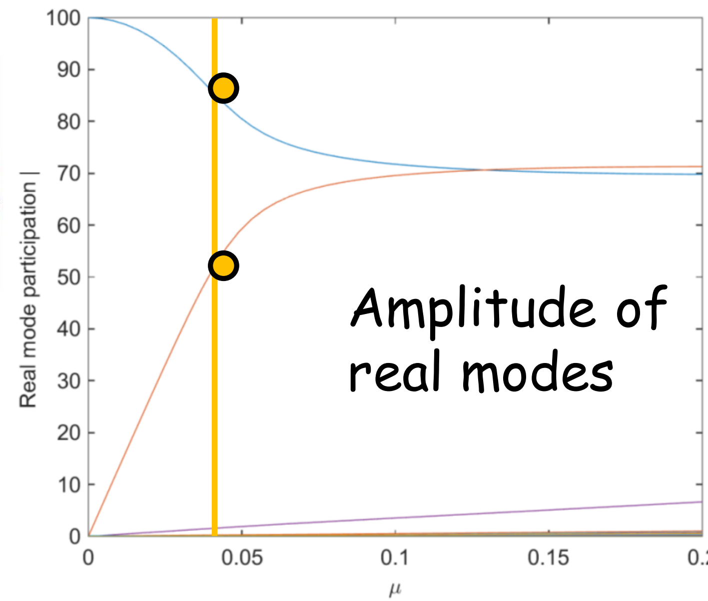


Normal

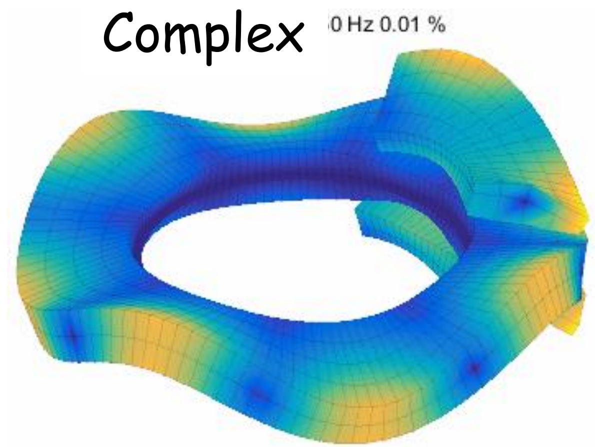


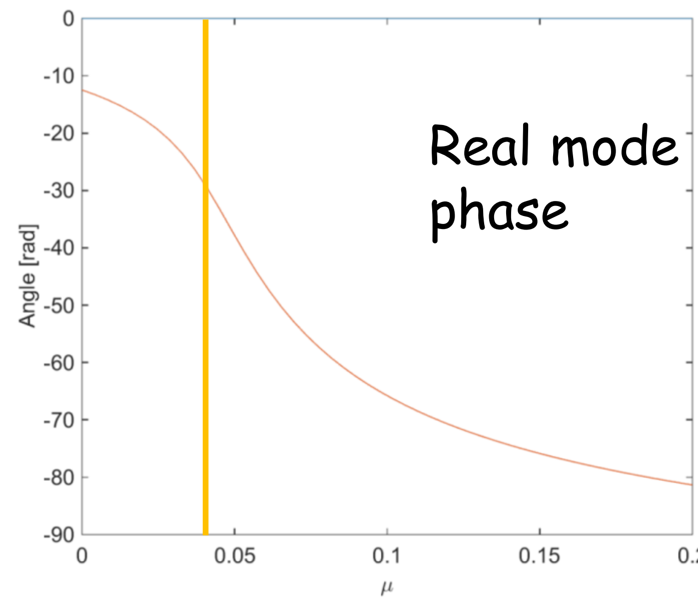
Tangent



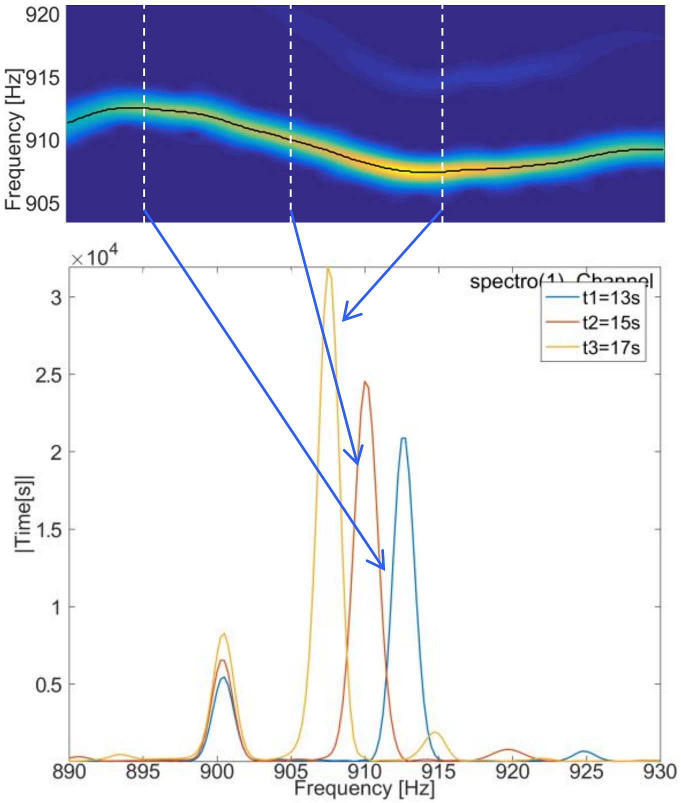


Complex



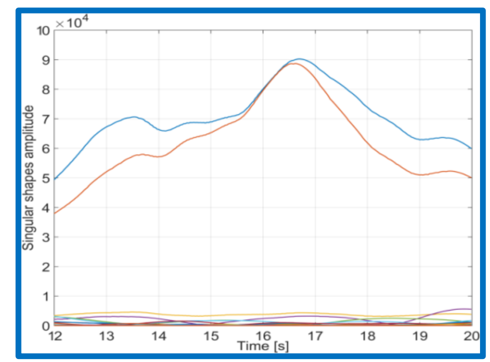


Subspace learning & basis selection



$y(f_{max}(t),t)$ Time varying Complex shape	=	**Space :** Principal real shapes	**Time :** Complex amplitudes

SVD $$\sum_i \left\{ \begin{matrix} U_i \\ \downarrow \\ sensors \end{matrix} \right\} \{a_i(t) \quad \rightarrow \quad time\}$$



$$\begin{bmatrix} y(c, f_{\max}(t)) & \rightarrow & time \\ \downarrow & & \\ sensors & & \end{bmatrix}$$

- only 2 significant real shapes & 2 associated DOF
- shapes independent of parameter
- Parameter : difficult to control (wheel position, brake event, …) but exists

More details : G. Martin session SD1 Monday 17:10

Outline

System models, shapes & DOF

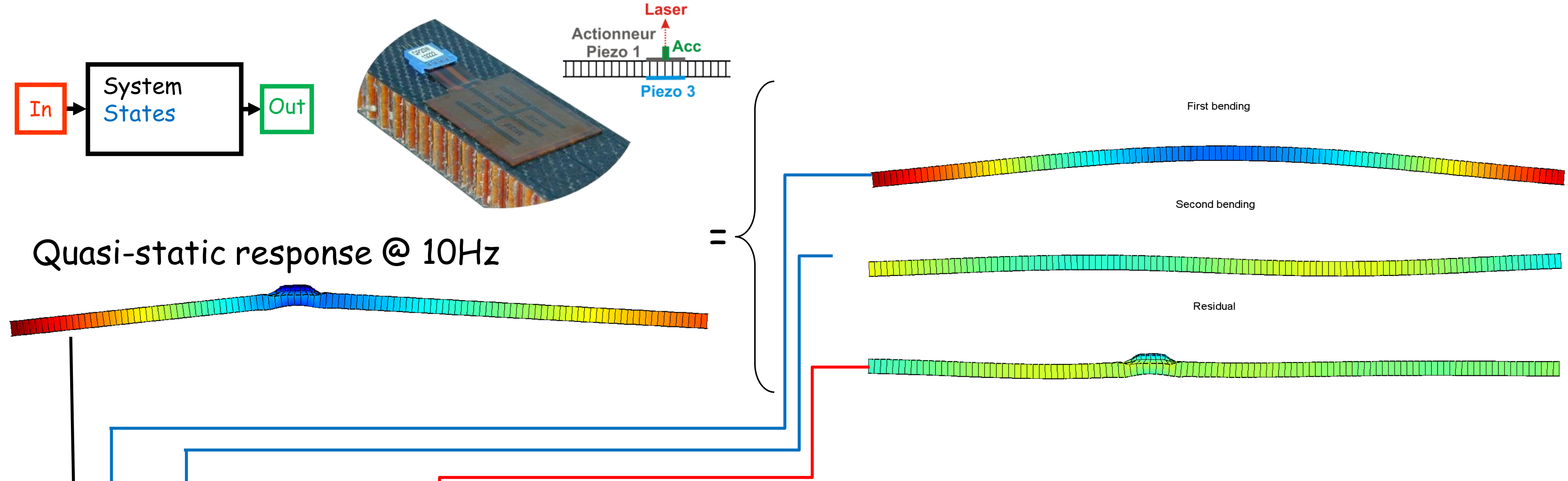
- Subspace learning = extract shapes & DOF
- Subspace for dynamics = modes & residuals
- Parameter loads & subspace, error control
- Basis generation (DOF selection) & objectives (complexity)
- Sparsity

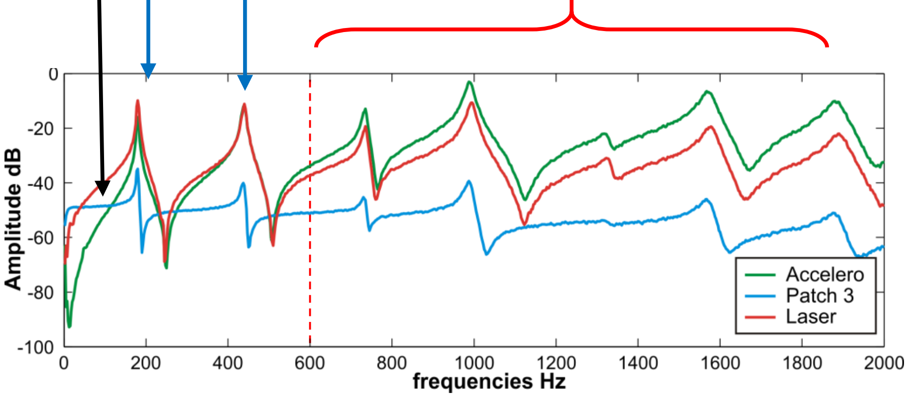
Analyzing parametric models

- Coupling = mechanical root locus, S shape frequency evolution
- The parametric NL equivalence
- Modal amplitudes & shape tracking
- Local modes & damping

- Component modes, interfaces & design

Measured transfers : what subspace is needed ?





- Modes : high energy, load independent (no blister shape)
- Static response (influence of input=blister), important away from resonance

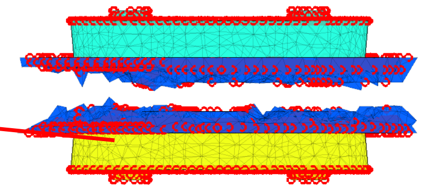
- Experimental modal analysis subspace :
  - Modes within bandwidth
  - Upper residual (residual flexibility, static correction, state-space D term)
  - Lower residual (rigid body inertia, …)

Nearby modes = poor representation of static

FEM subspace : CMS modes+static

McNeal : free modes + residual flexibility to applied loads

$$T = [\phi_{1:NM} \quad K^{-1}b]$$

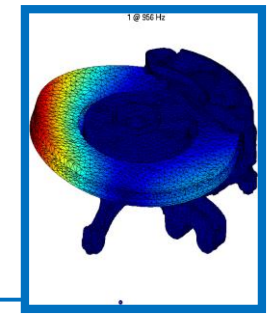


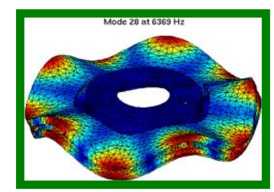
Craig-Bampton : applied disp. + fixed interface modes

$$T = \begin{bmatrix} I & 0 \\ -K_{cc}^{-1}K_{cb} & \phi_{c,1:NM} \end{bmatrix}$$

Any « new » combination : static + trace of system modes

$$T = \left[\begin{matrix} I \\ -K_{cc}^{-1}K_{cb} \end{matrix} \quad \phi_{System}\big|_c \quad \boldsymbol{\phi_{component}}\right]$$





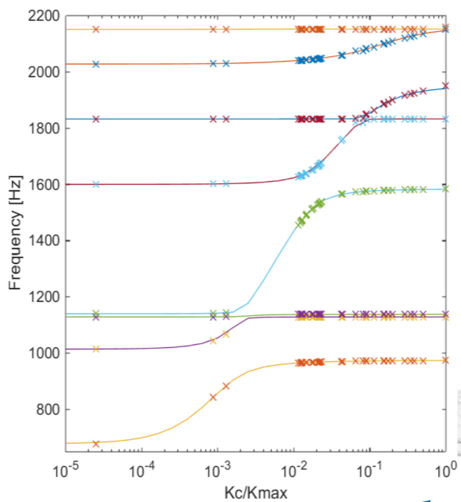
Issues :

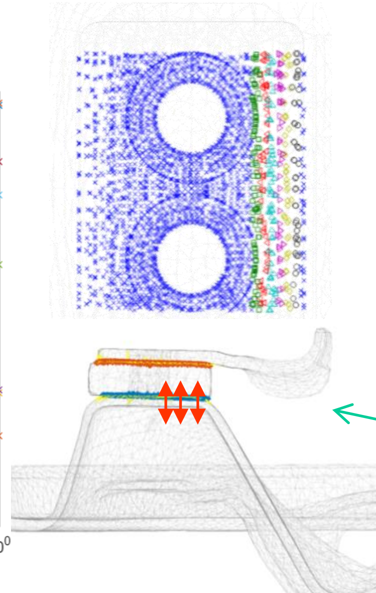
- What are "applied loads" b
- Set of vectors ≠ basis
  - Cost of generating vector set (component/system)
  - Conditioning $K^{-1}b$ (two very close loads generate identical shape)

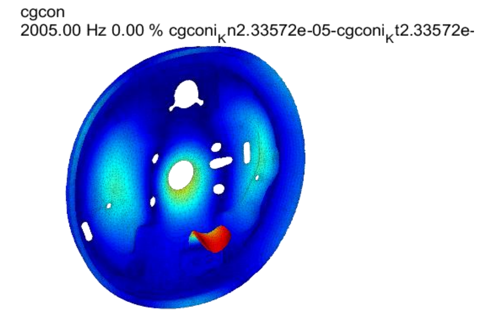
Component mode tuning (PhD Vermot des Roches 2010)

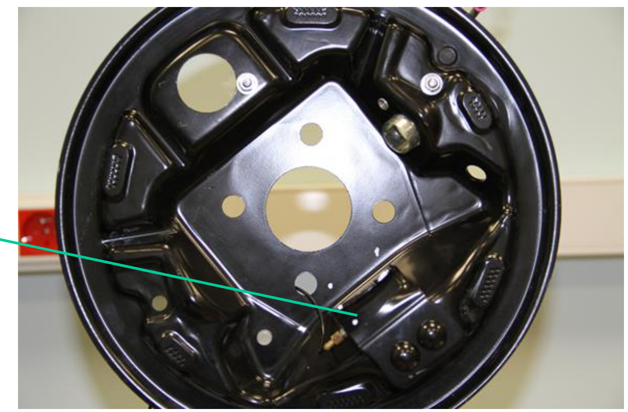
Parametric loads

- Nominal model $[Ms^2 + Cs + K(p)]\{q(s)\} = [b_{ext}]\{u_{ext}\}$
- Variable contact surface, contact, sliding









- Choose nominal elastic model & rewrite using parametric load f_p

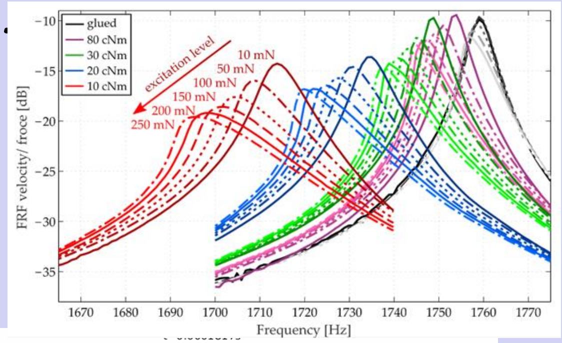
$$[M_0 s^2 + K_0]\{q\} = [b_{ext}]\{u_{ext}\} + \{f_p(q(s), p)\}$$

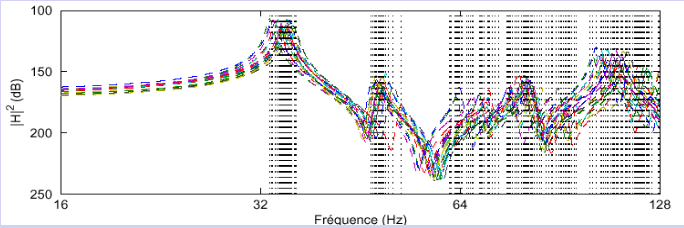
- Space time decomposition parametric load : $[b_{contact}]\{p(t)\}$

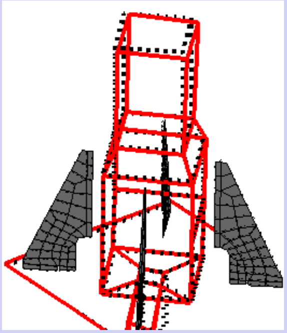
LTI + parametric/NL loads

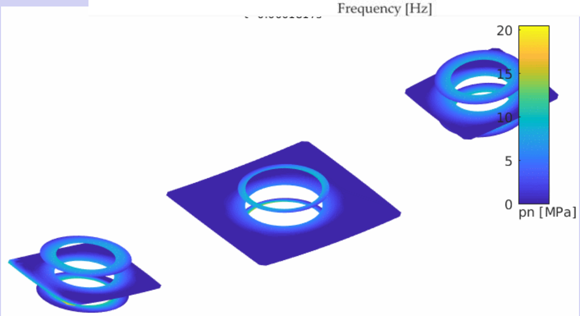
In → LTI Reduced model → Sensors

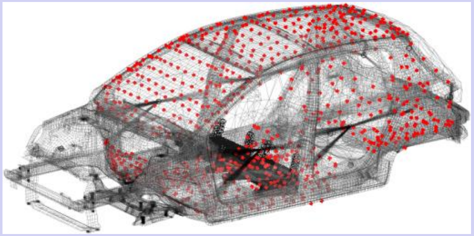
- Internal coupling : NL junctions, damping, …
- System coupling : test/FEM, multi-body, fluid/structure, active control, …
- Environment/design/uncertainty

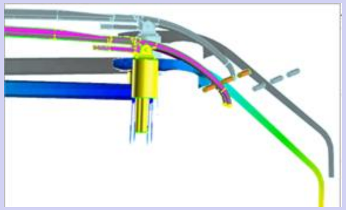




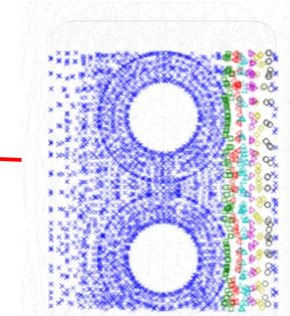






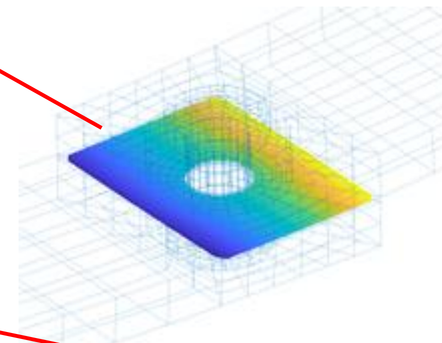


Parametric loads & reduction



Space/time decomposition of load $[b_{Contact}]_{N\times \mathrm{Ng}}\{p(t)\}$

- Know nothing about $\{p(t)\}_{Ng}$ too large

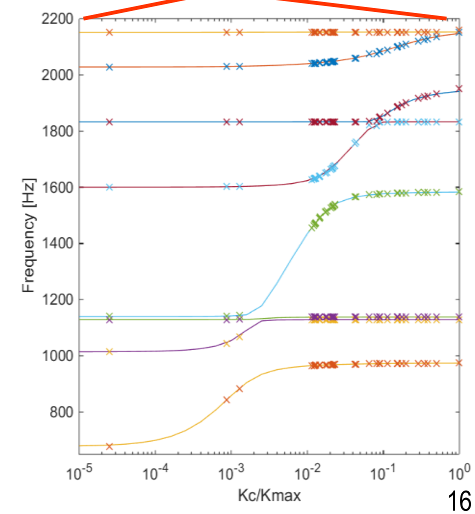


- $\{p(t)\}$ associated with initial modes = $\left[[c_{NOR}][\phi_{1:NM}]\right]_{N\times \mathrm{NM}}\{q_r(t)\}$

  Static correction for pressure load of elastic normal modes

$$T = [\phi(p_0)\ \ K^{-1}[b_c c_{NOR}\phi(p_0)]_{N\times \mathrm{NM}}]_\perp$$

- Multi-model learning $T = [\phi(p_1)\ \phi(p_2)_{N\times NM}]_\perp$



- Error control (residue iteration)

$$R_d = K_0^{-1}\left\{[M_0 s^2 + K_0]\{Tq_R\} - [b_{ext}]\{u_{ext}\} + \{f_p(Tq_R,p)\}\right\}$$

PhD A. Bobillot 2002

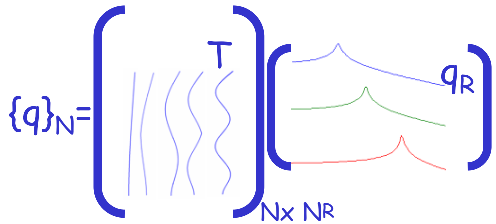
Vector set => basis => reduced model

1. Initial (off-line) subspace learning phase

- Test : modes in band, shape due to input, shape change due to NL
- Computation : CMS= static, eigenvalue (series of static+orthog.), parametric learning/error control, Snapshot (NL transient)

2. Basis generation. Objectives

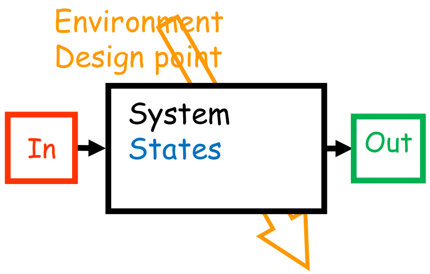
- **Size (truncation)**
- **Sparsity**



3. Model reduction/modal synthesis/Ritz-Galerkin $\{q(x,t)\} = [T(x)]\{q_R(t)\}$

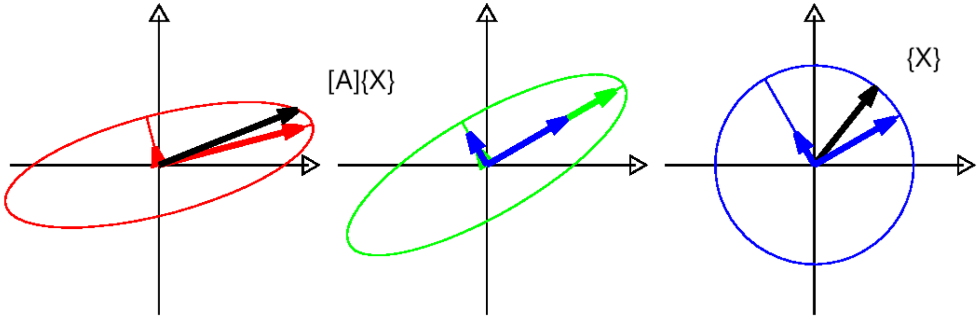
$$Z_R(\omega,p) = T^T[Z(\omega,p)]T$$

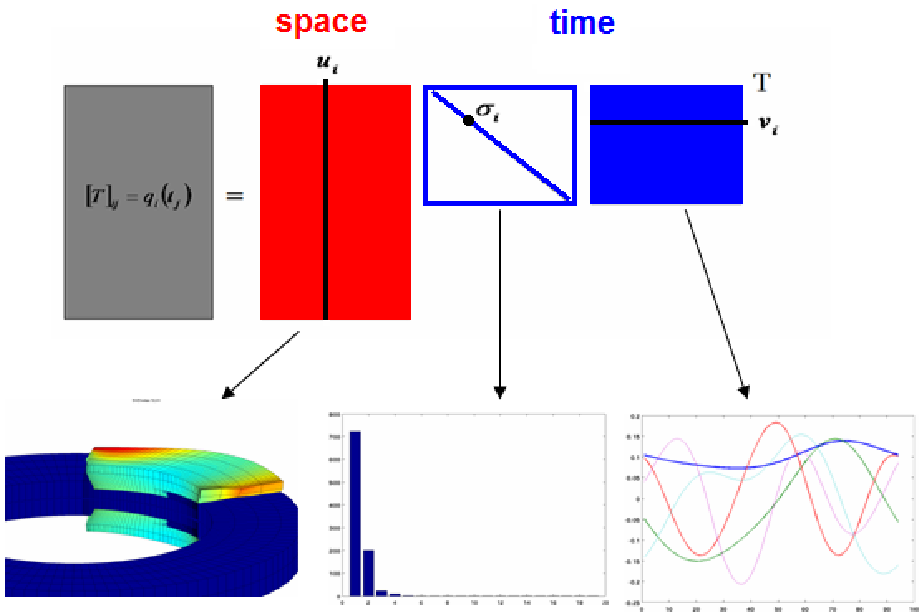
T independent of p = parametric reduction



SVD & variants

A = U S V^H





shape DOF

SVD

- {X} on sphere in input space transformed in {Y}=[A]{X} ellipsoid
- Series of rank one contributions

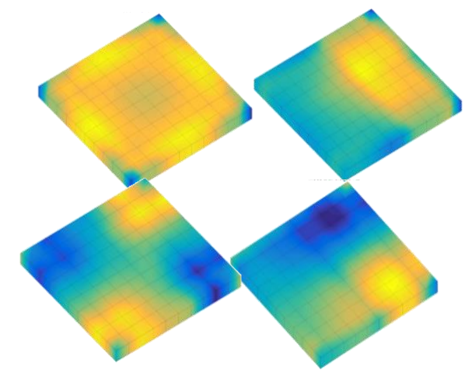
Mode

- $\{\phi\}$ on unit strain energy sphere output is kinetic energy
- Singular value $\frac{1}{\omega_j^2} = \frac{\phi_j^T M \phi_j}{\phi_j^T K \phi_j}$ = 1/Rayleigh quotient

AIAA Journal, Balmes, 1996

SVD, variants, related

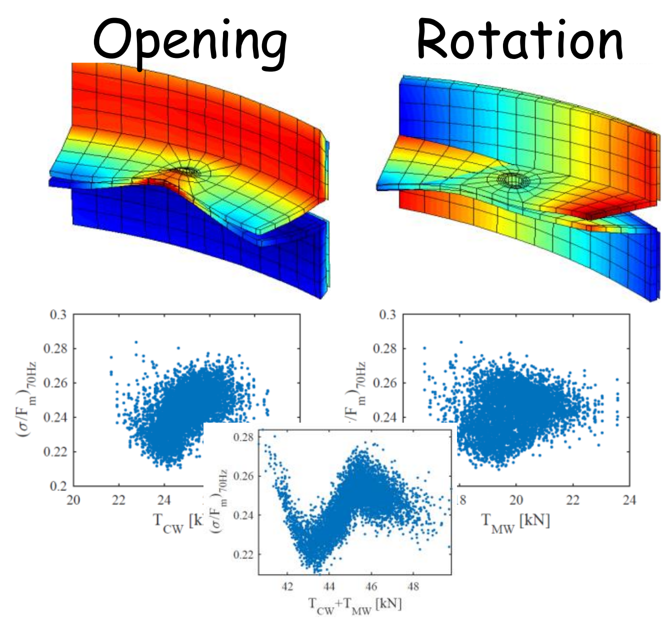
Random fields Karhunen-Loeve :
- input-norm I for all DOFs
- output norm spatial correlation
  $C = \exp[-(|x_1 - x_2| + |y_1 - y_2|)]$



PCA Principal Component Analysis
POD based on snapshot-reduction :
- input-norm I on snapshot vectors
- output norm I

Chung, Gutiérrez, & all, “stochastic finite element models,” IJME, 2005.
Kershen & al. “POD”, Nonlinear dynamics , 2005
Balmes, Vermot, “Colloque assemblages 2015”, + Bendhia 1-epsilon compatibility EJCM 2010
Ph.D. Olivier Vo Van 2016

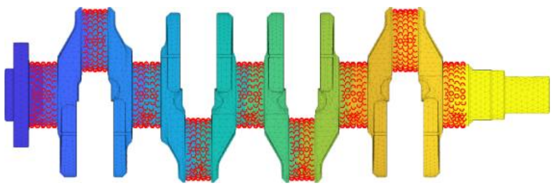
Junction modes
- input-norm I for modes
  or contact stiffness
- output norm local stiffness

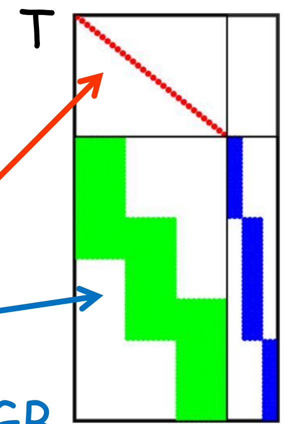


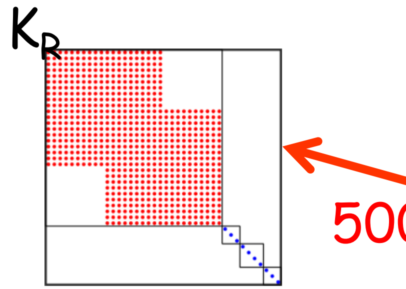
Non-linear dimensionality reduction (manifold)
- More complex relation between parameters

Interface reduction / model size / sparsity

- Craig-Bampton often sub-performant because of interfaces



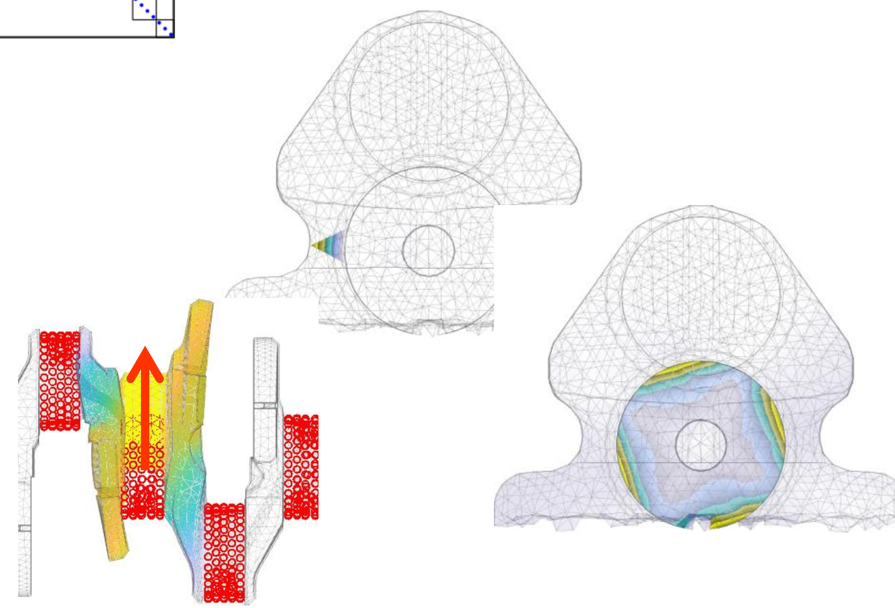




2e6 rest x 5000 Int = 74GB

5000^2 = 200 MB

- Unit motion can be redefined : interface modes
  Fourier, analytic polynomials, local eigenvalue
  5000 -> 500 interface DOFs.
- Disjoint internal DOF subsets



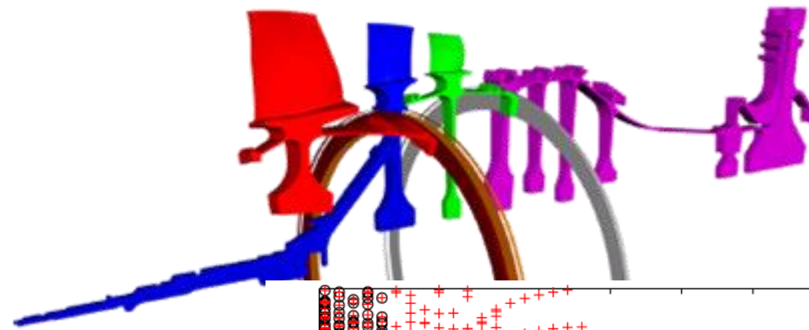
Separate requirements for learning shapes :

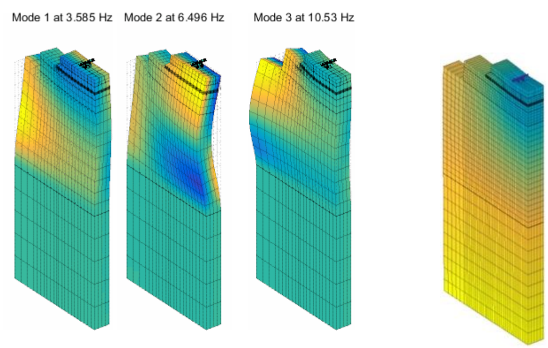
bandwidth, inputs external & parameter
truncation, sparsity

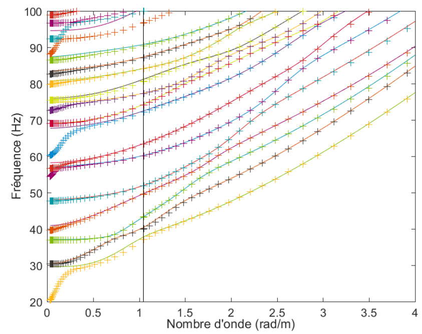
Interface reduction : wave/cyclic

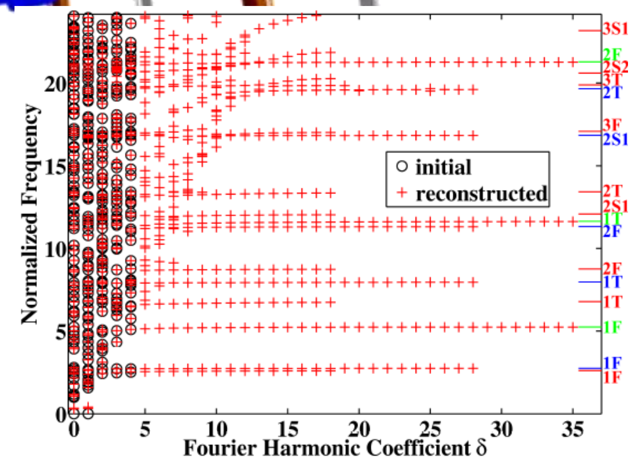
Best interface reduction = learn from full system modes

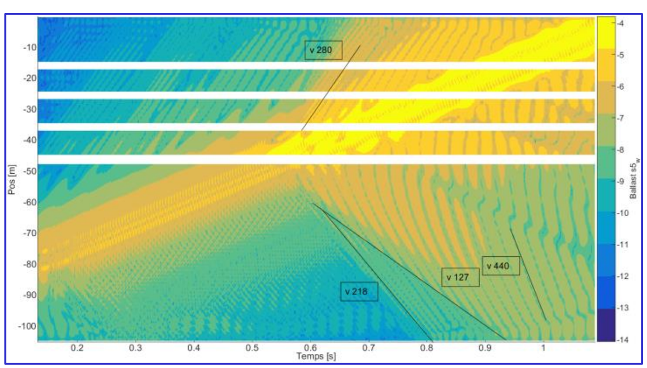
1. Learn using wave (Floquet)/cyclic solutions
2. Build basis with left/right compatibility
3. Assemble reduced model

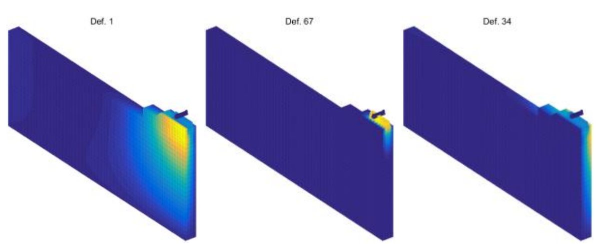




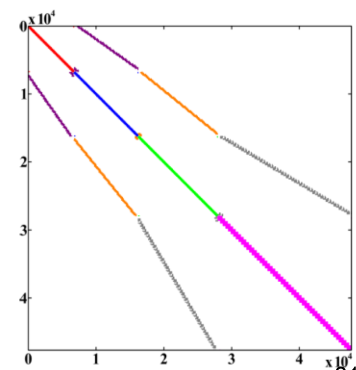








PhD Elodie Arlaud, 2016



PhD Sternshuss 2008

Outline

System models, shapes & DOF

- Subspace learning
- Subspace for dynamics = modes & residuals
- Parameter loads & subspace, error control
- Basis generation (DOF selection) & objectives (complexity)
- Sparsity

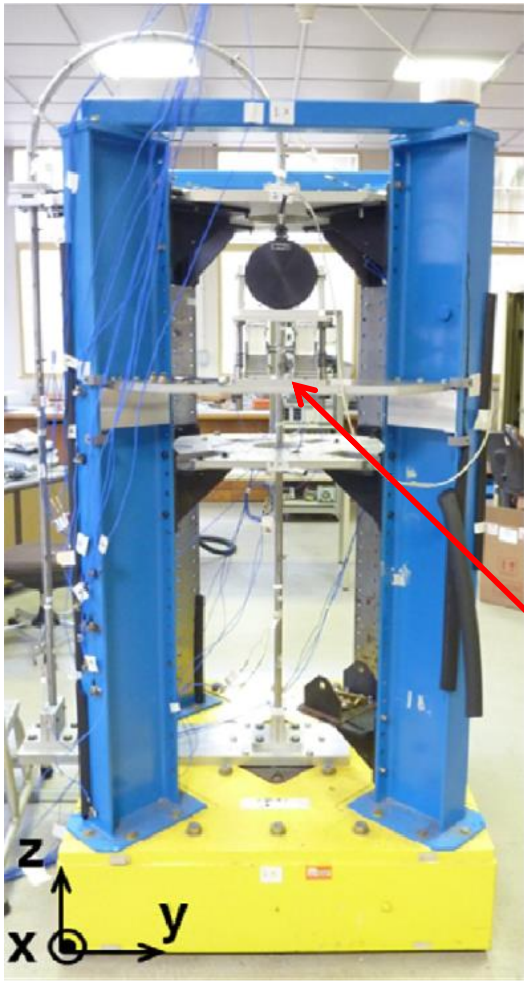
Analyzing parametric models in vibration applications

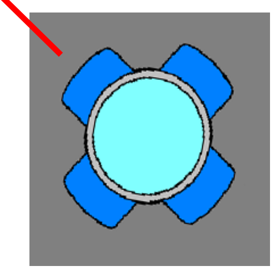
- Coupling = mechanical root locus, S shape frequency evolution
- The parametric NL equivalence
- Modal amplitudes & shape tracking
- Local modes & damping

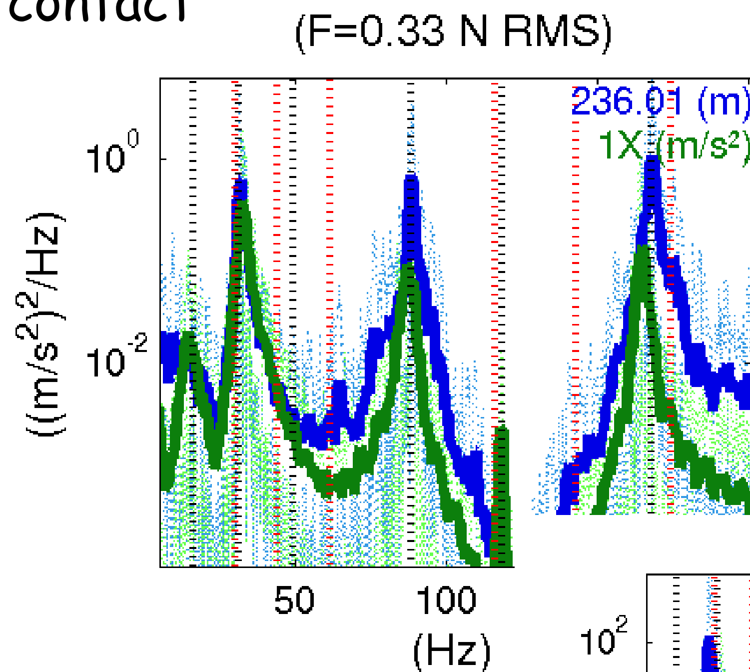
- Component modes, interfaces & design

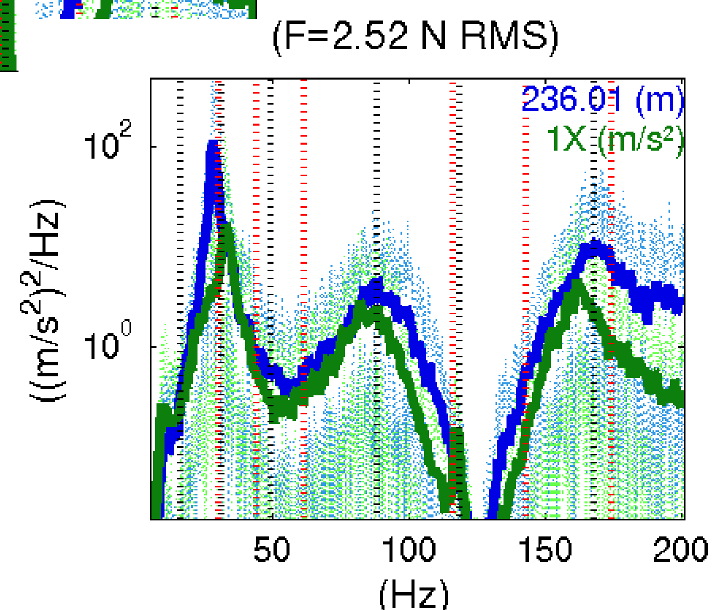
Energy coupling in & NL system

Bending beam with gap & contact
Apparent damping with pure contact



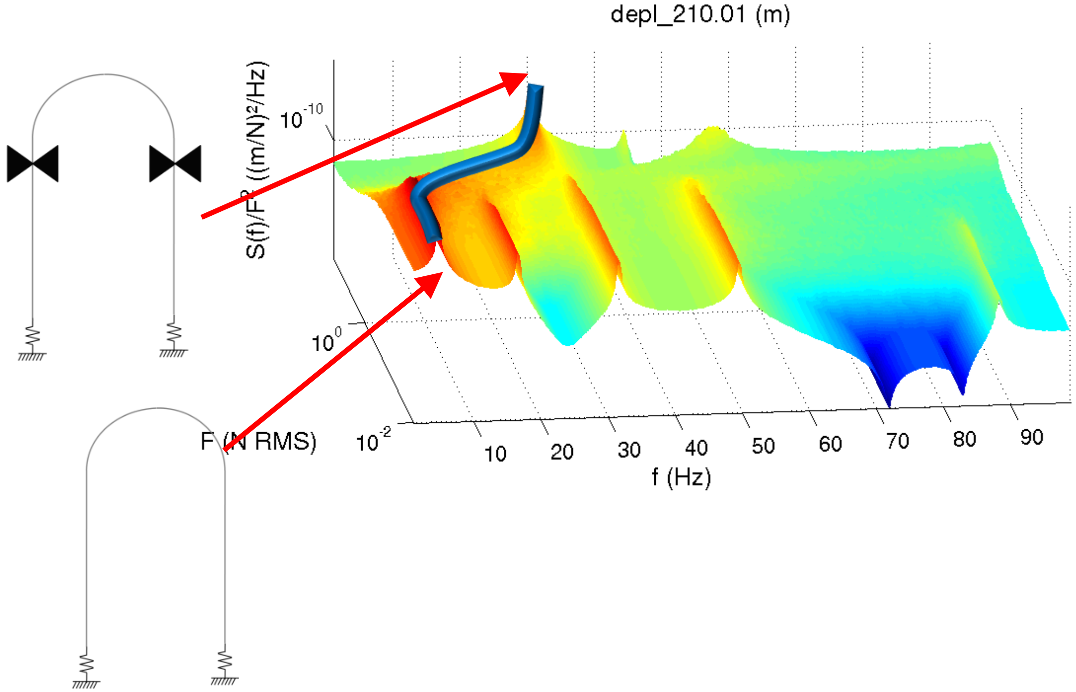


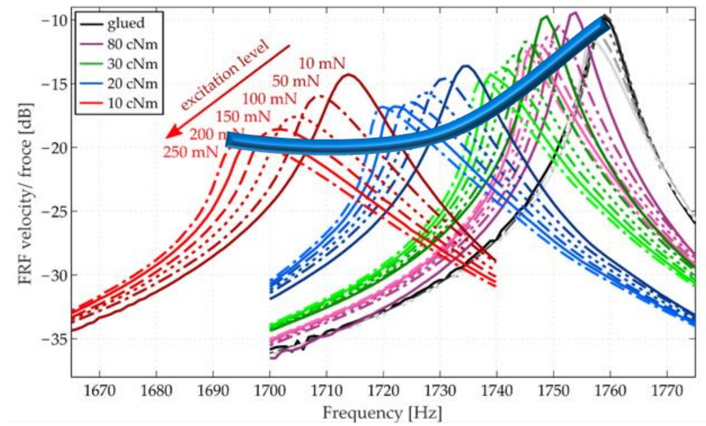




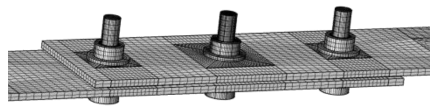
PhD. Thénint 2011

Damping occurs in the transition



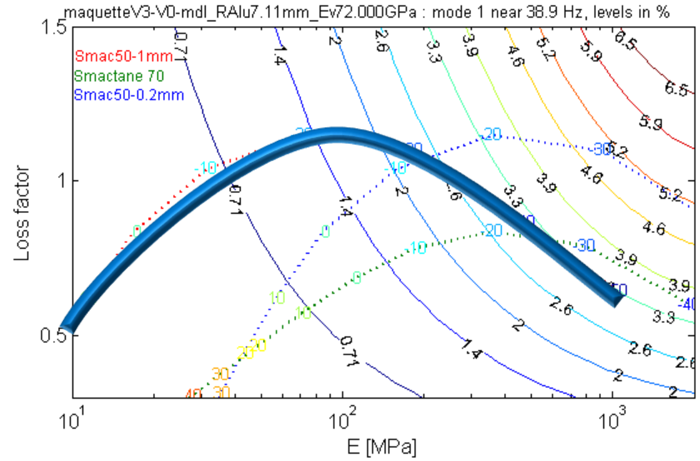


FEMTo-ST Orion Testbed



 Transition

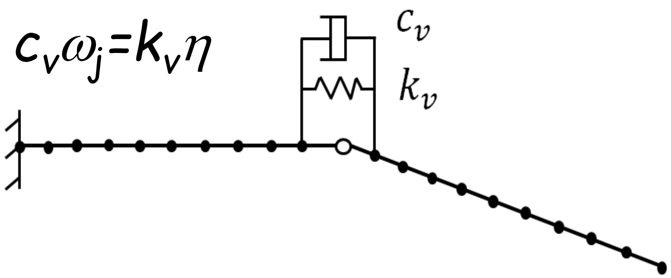
- Contact with broadband excitation level
- Friction : amplitude/normal force dependence
- Viscoelasticity : temperature/frequency dependence

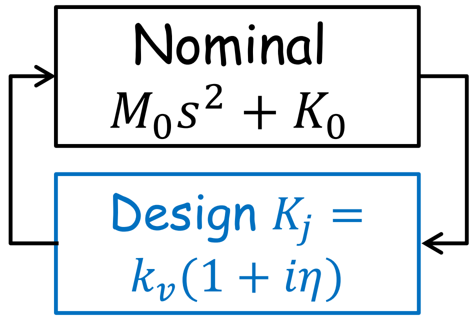


ENSAM/PhD. Hammami 2014
Hammami/Balmes/Guskov, MSSP 2015

A mechanical root locus (coupling)

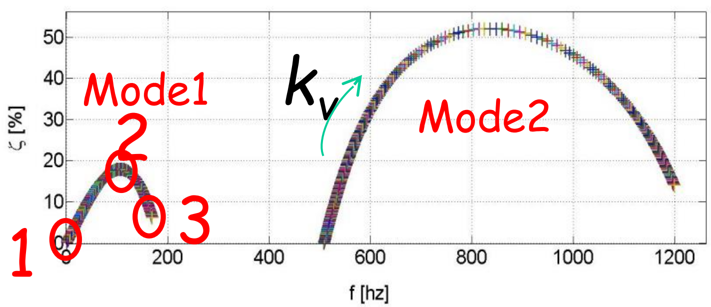
$c_v\omega_j = k_v\eta$

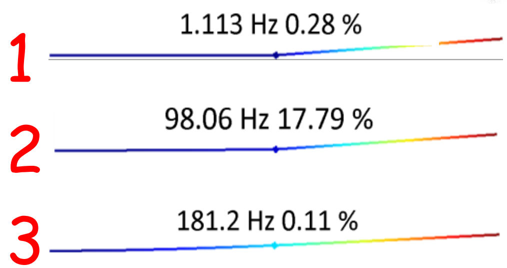




k_v variable, $\eta=1$ constant loss

Output : $f(k_v,\eta)$ $\zeta(k_v,\eta)$





Soft joint

Dissipative joint

Stiff joint

Damping can only exist if joint is « working »

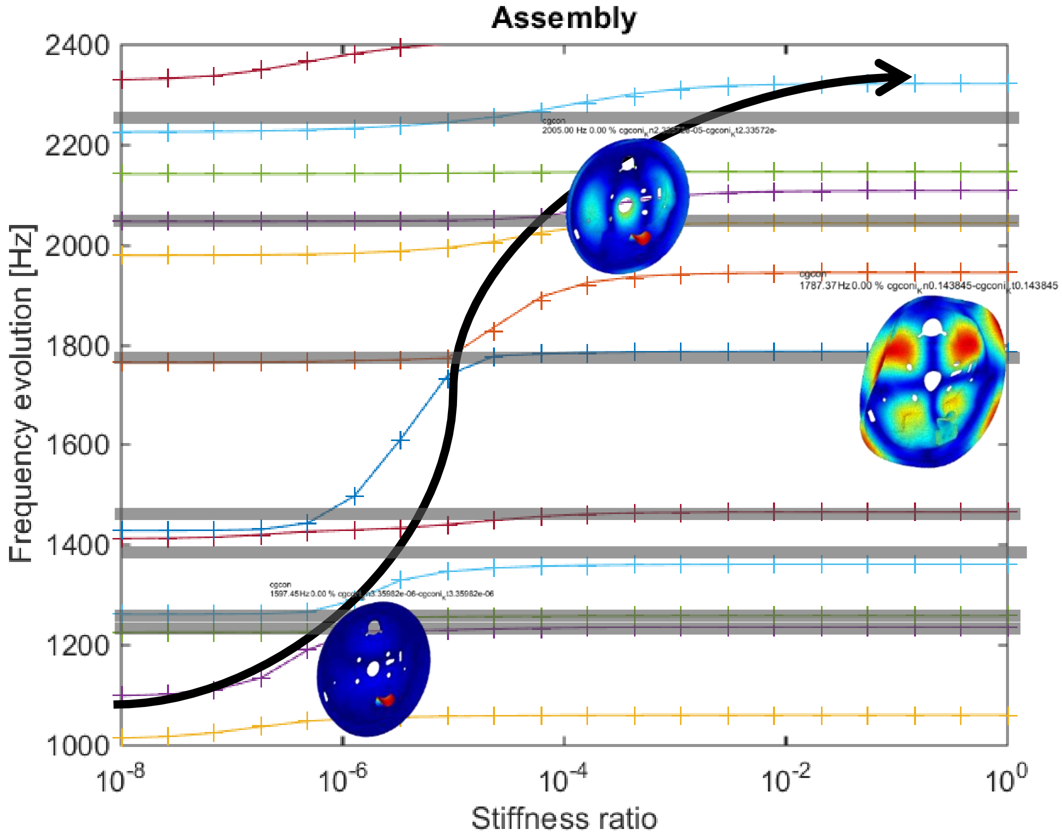
⬇

Energy fraction in joint must be « sufficient » (depends on k_v)

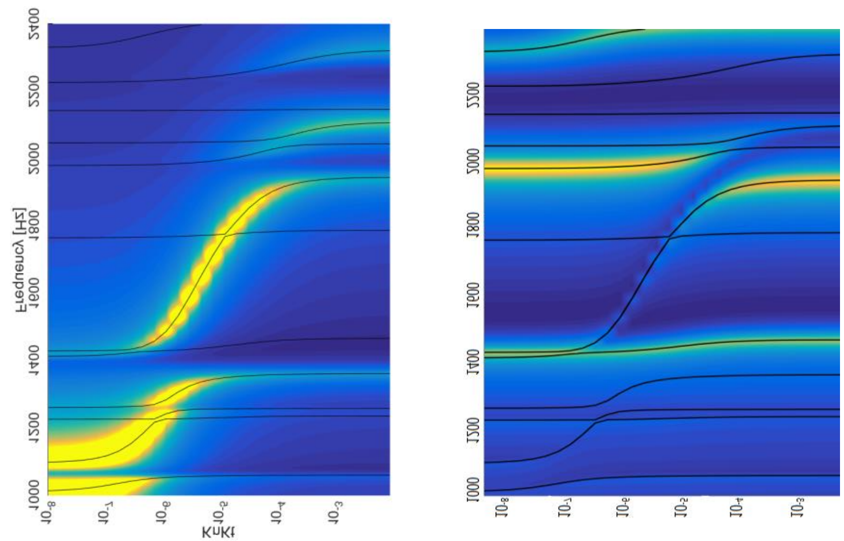
Related ideas : electro-mechanical coupling coefficient, pole/zero distance, modal strain energy

Following modes

- Horizontal lines : global modes
- S curve : local cable guide mode



Sensors can help

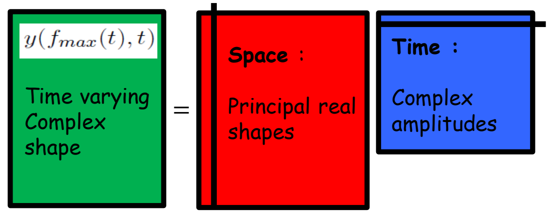


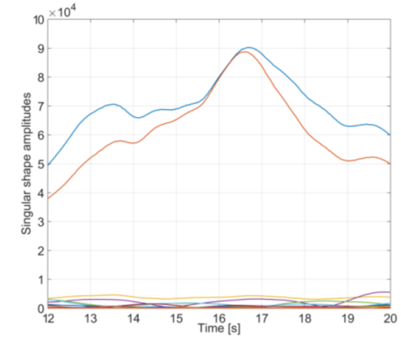
- But can we track modes better ?

Coupling & modal DOF

Squeal example

- Shapes are stable
- We looked at amplitudes





- Physical domain param. load: $[M_0]\{\ddot{q}\} + [K_0]\{q\} = \{f(q,\dot{q},t,p)\}$
- Modal domain:

- mass / stiff orthog. condition $\phi^T M \phi = I$, $\phi_j^T K \phi_j = \omega_j^2$
- Modal equation

$$[I]\{\ddot{q}_R\} + [C]\{\dot{q}_R\} + \left[\ {}^{\backslash}\omega_j^2{}_{\backslash}\ \right]\{q_R\} = \{f\}$$

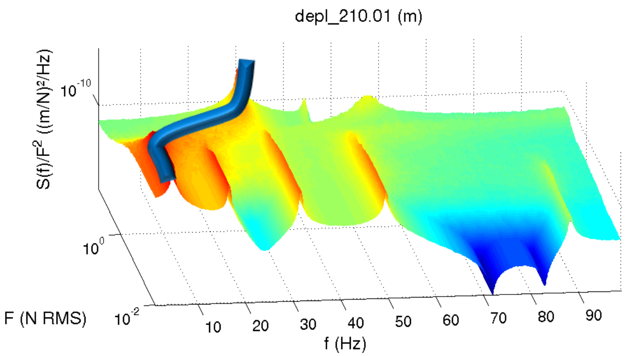
- Modal amplitudes computed for nominal model

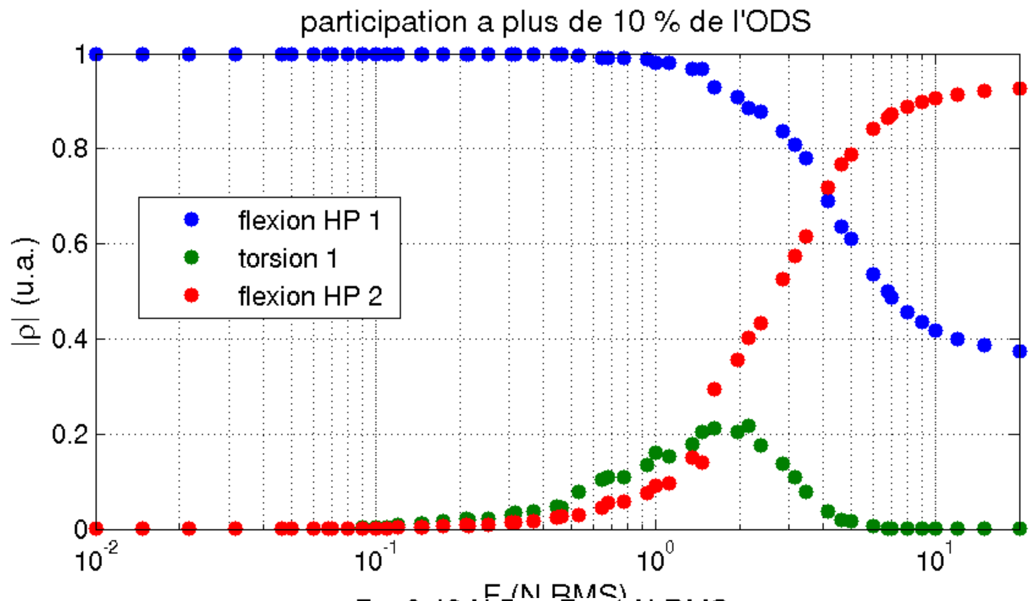
$$\{q_R\} = [\boldsymbol{\phi_0^{-1}}]\{q\} = [\boldsymbol{\phi_0^T M_0}]\{q\}$$

Associated concepts : force appropriation, modal filter

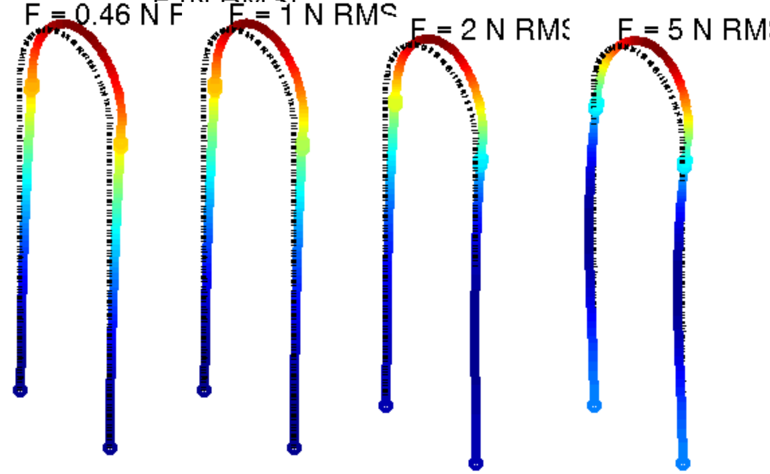
- Modal energies $e_j = \frac{1}{2}\left(\dot{q}_{jR}^2 + \omega_j^2 q_{jR}^2\right)$

Modal participations in ODS

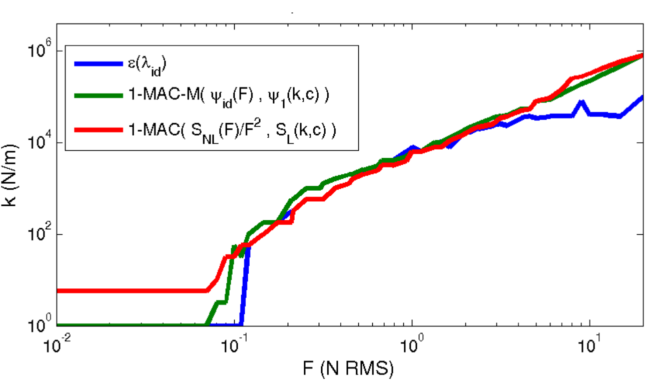


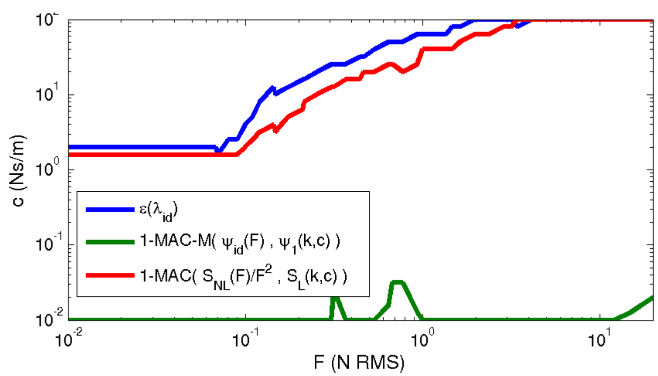


- Extract shape = SVD around « resonance »
- Obtain modal amplitudes of nominal modes $[\phi_0^T M_0]\{q\}$



- **Apparent (parametric) stiffness/damping** consistent for various methods

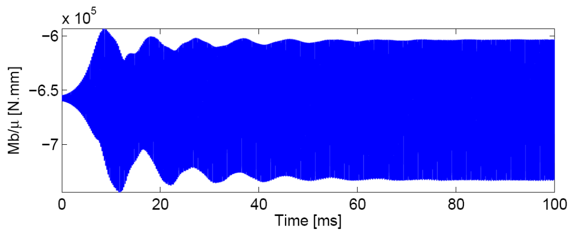


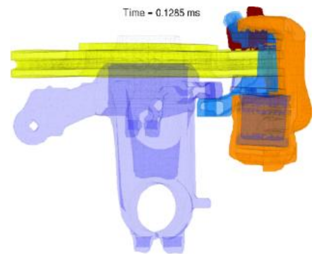


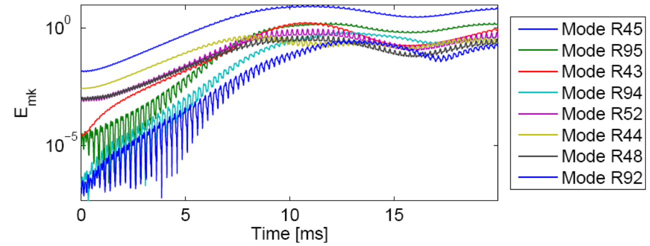
PhD. Thénint 2011

Modal energy computations

- Energy in components, energy in linear shapes

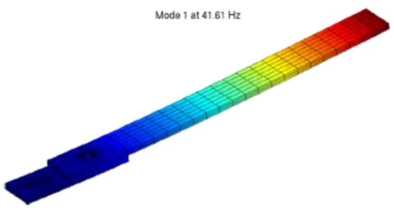


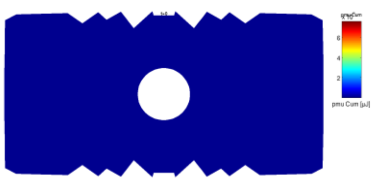


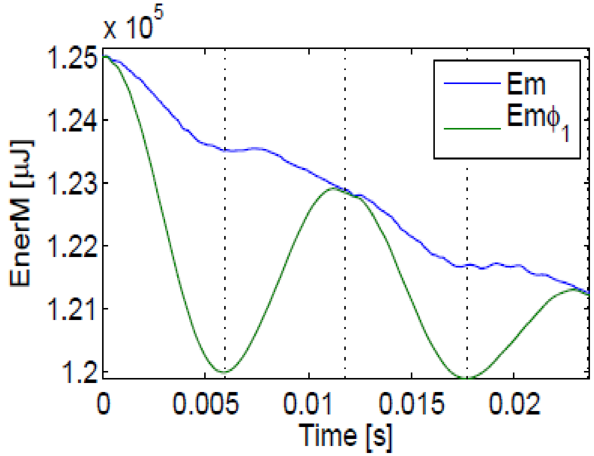


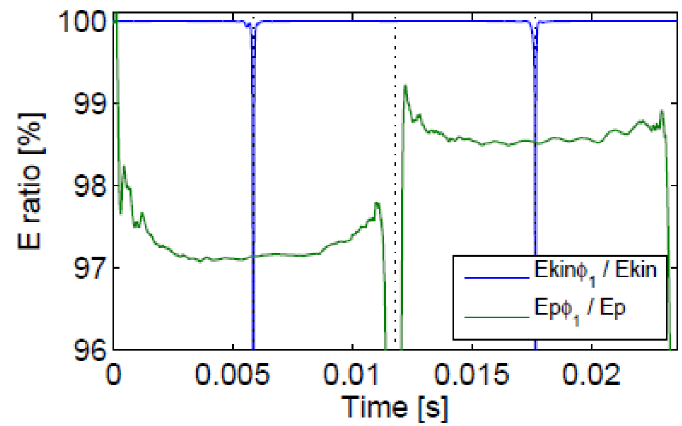
- Does the shape change in NL behavior

$$2E_{mj}(t) = \underbrace{\omega_j^2\alpha_j^2(t)}_{E_p(t)} + \underbrace{\dot{\alpha}_j^2(t)}_{E_k(t)}$$



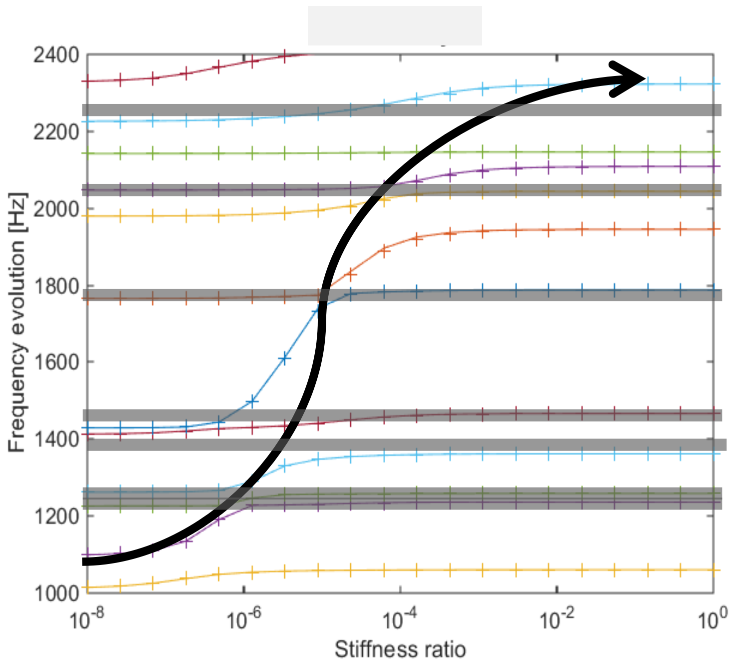




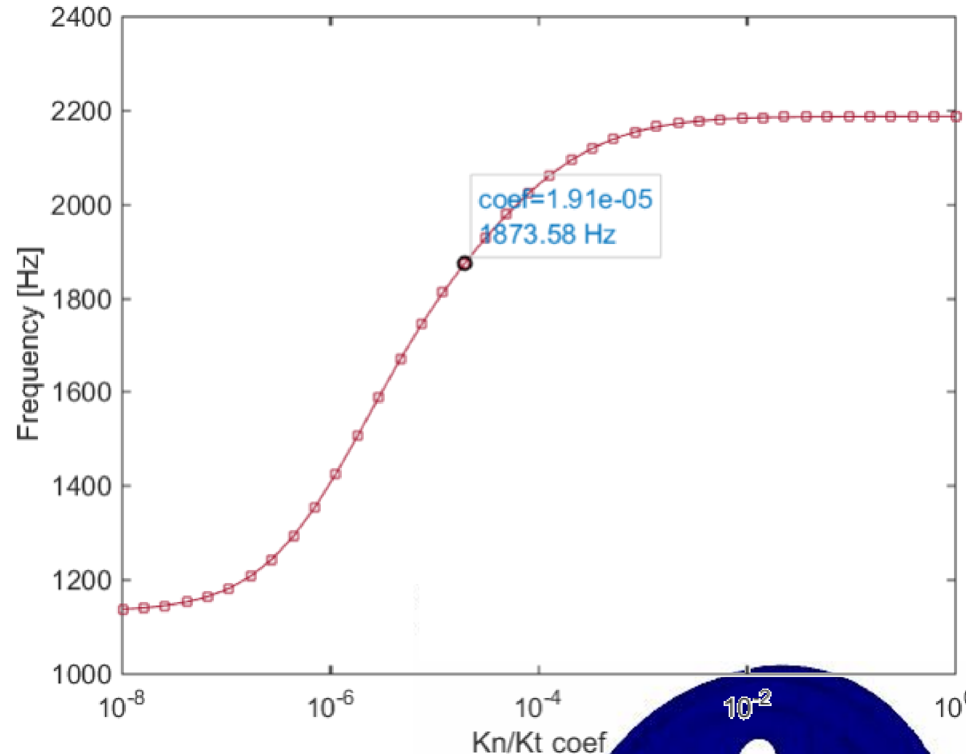


Local component mode in an assembly

Assembly



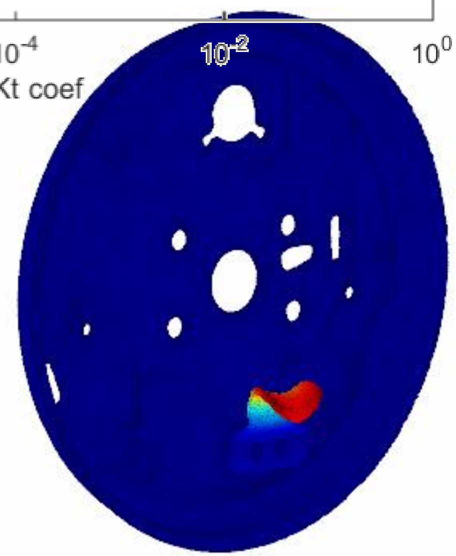
Rigid plate



$$\{q\} = \begin{bmatrix} RB_1 & & 0 & \\ & \ddots & & \\ 0 & & I & \\ & & & \ddots \end{bmatrix} \{q'\}$$

Plate : rigid motion

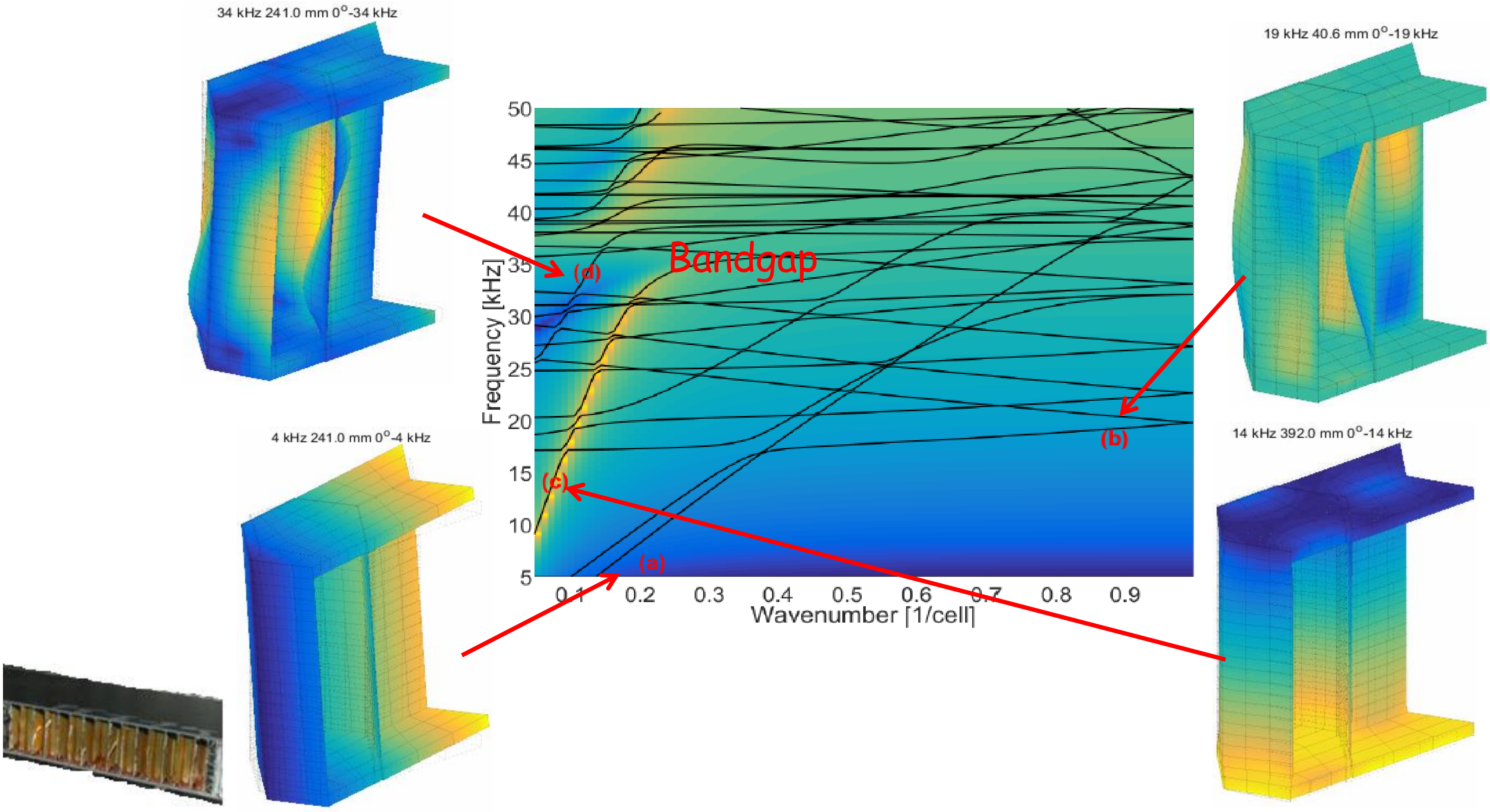
Cable guide all DOFs



PhD G. Martin 2017

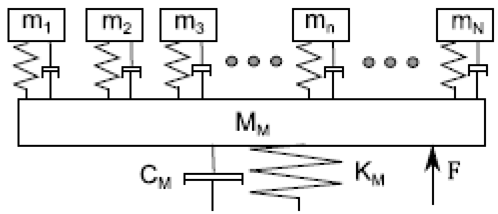
Problems with local/global modes

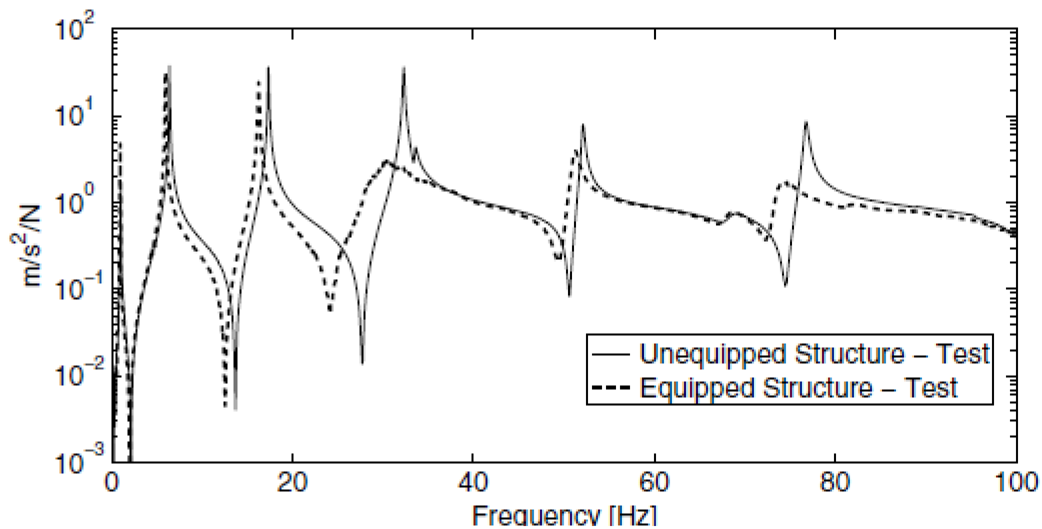
- Track horizontal wave & understand origin of bandgap (cell wall mode)

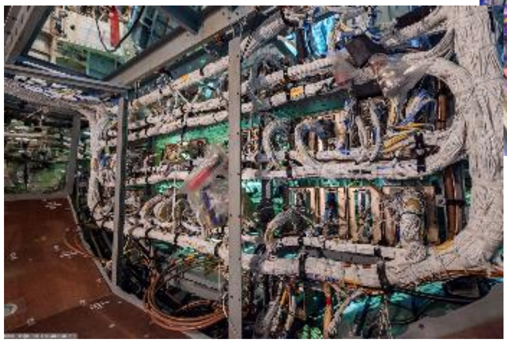


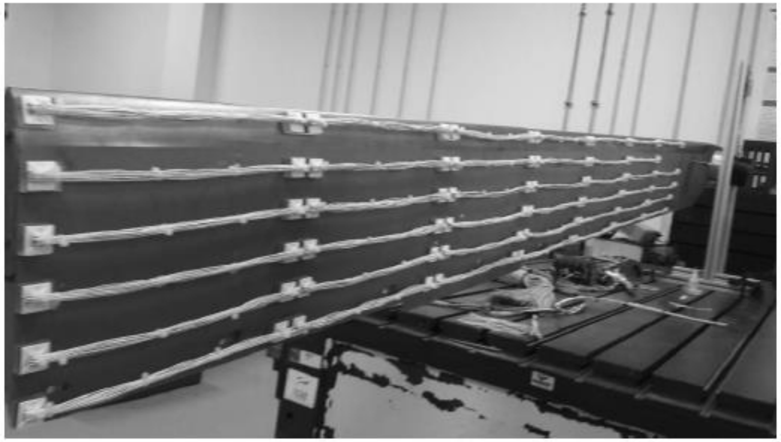
Balmes, Rébillat, Arlaud, Vishno, 2016

Damping due to local modes/non structural mass









Soize “structural fuzzy” 1987

Arnoux, A. Batou, C. Soize, L. Gagliardini. JSV 2013

Loukota, Passieux, Michon. Journal of Aircraft 2017

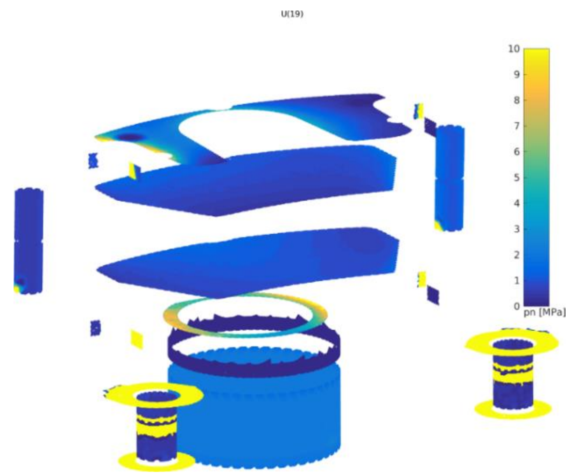
Coupling & design process

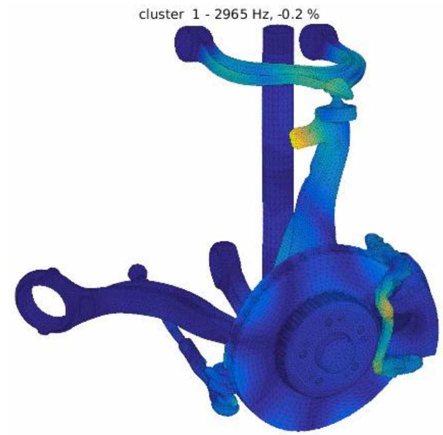
Squeal applications

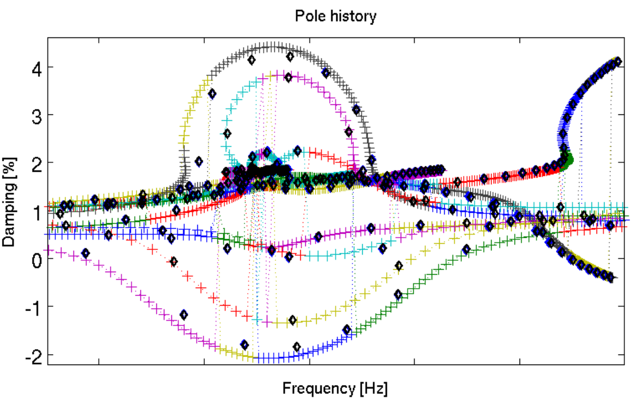
- 8-15 components
- Multiple interfaces/parameters
- 300-600 modes

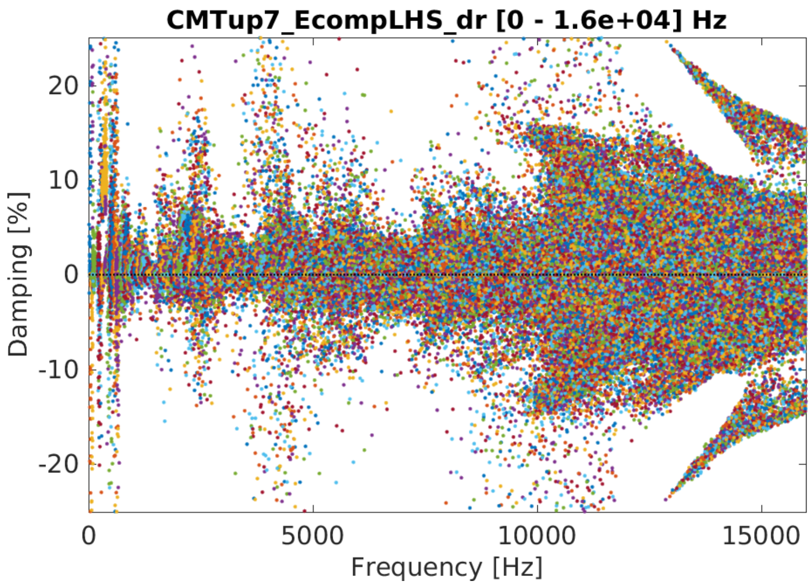
Design exploration 1000 points

- Full 80 days CPU, 22 TB
- CMT a few hours off-line learning, <1h exploration



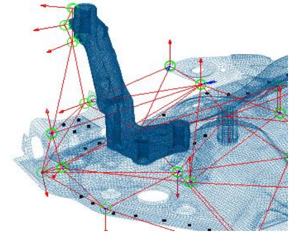




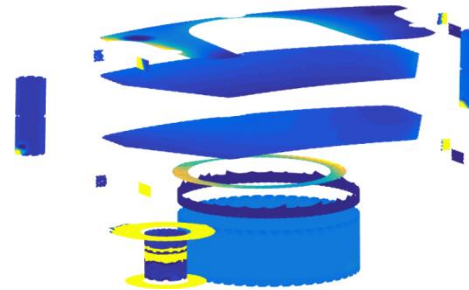


A few key challenges

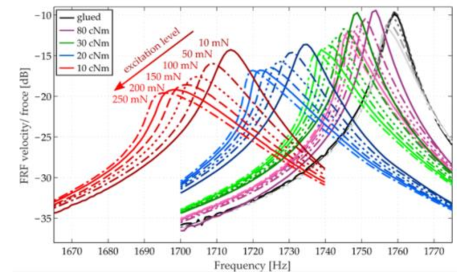
- Modes & DOFs = fully separate geometric complexity (N of FEM DOF) and system complexity (N reduced model DOF)



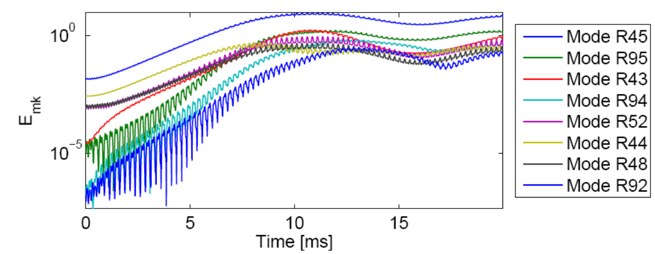
- In test & FEM
  - define complexity of junctions with distributed load
  - exploit NL/parameter equivalence
  - track using generalized coordinates (DOF with meaning)

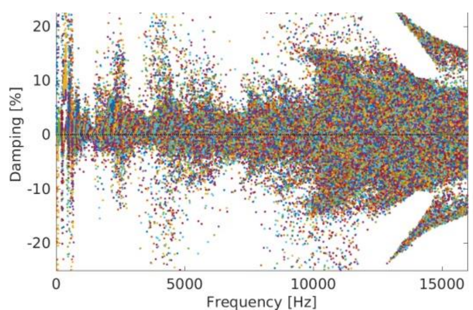


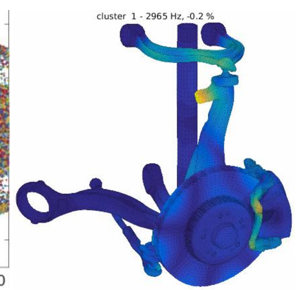
- Navigate multi-parameter design space (offline/online, memory/CPU trade-off)



- Software trade-off
  - GUI (☺ train/navigate) / Script (☺ extend/repeat)
  - Optimized/generic, extensive/easy to manipulate







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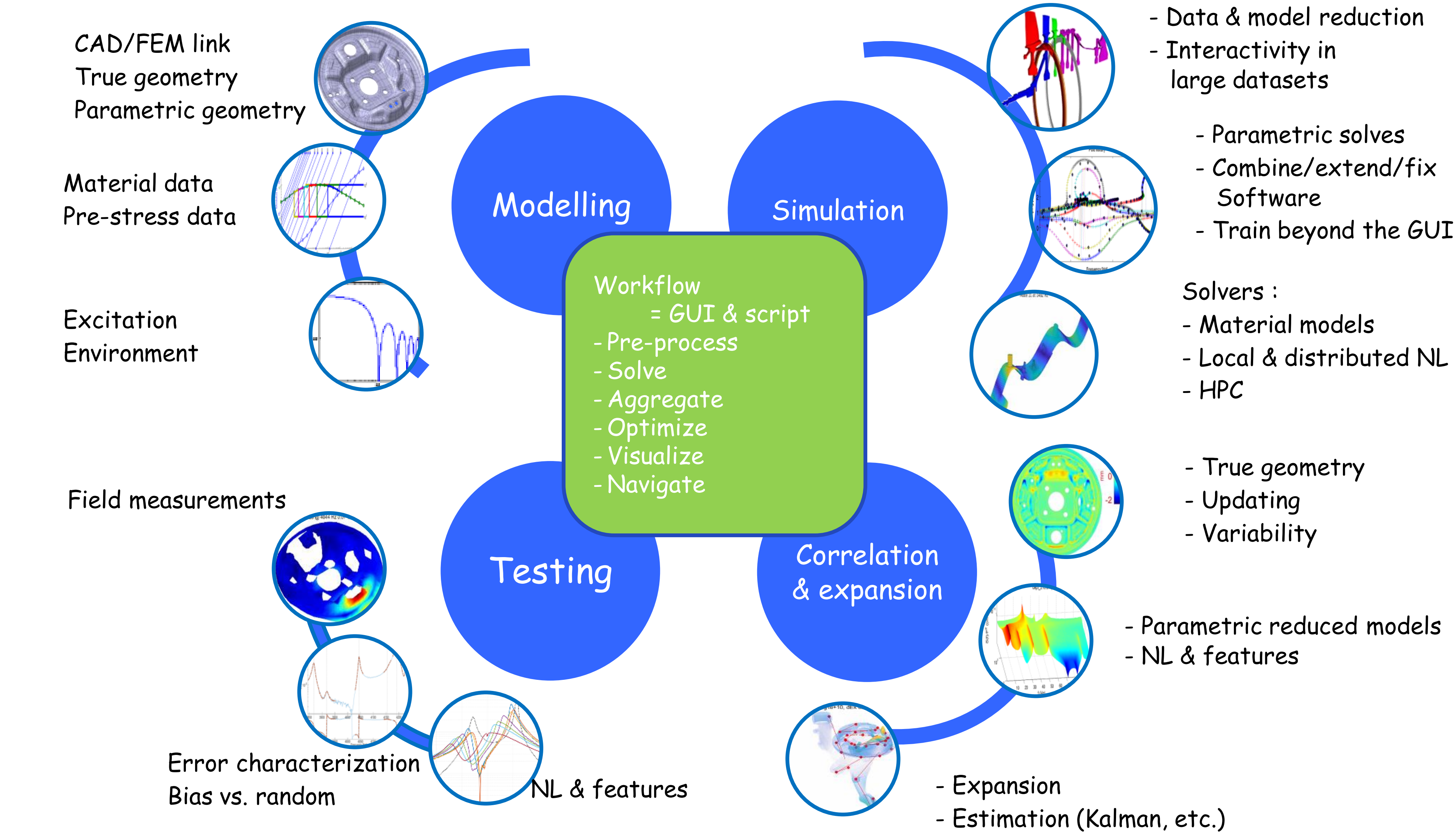
Outlook



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