IMPROVEMENT OF A STRUCTURAL MODIFICATION METHOD USING DATA EXPANSION AND MODEL REDUCTION TECHNIQUES

Mathieu Corus†,Etienne Balmès‡

† EDF DR&D, 1 Avenue du Général De Gaule, 92141 Clamart Cedex, France

‡ ECP, MSSMat, Grande Voie des vignes, 92295 Châtenay-Malabry, France

mathieu.corus@edf.fr - balmes@sdtools.com

ABSTRACT

A method designed to predict the effects of distributed modifications of structures is proposed here. This method is an evolution of the classical formulation but uses distinct measurement and coupling points; it includes a smoothed expansion procedure and two indicators to estimate the quality of the result. An academic testbed and an industrial application are use to illustrate the approach and highlight its main advantages.

NOMENCLATURE

For the sake of convenience, no distinction will be made in the following between a DOF and the displacement associated to this DOF.

N_g	Number of eigenmodes in $[\Phi_{Lg}]$
N_t	Number of sensors
$\Delta_{E_K}, \Delta_{E_M}$	Strain & Kinetic energy criterions
$\{y_t\}$	Displacements at the measurement points - test DOF
$\{q_I\}$ / $\{q_L\}$	interface / local model DOF
$\left\{q_I^{\Phi}\right\}$ / $\left\{q_I^{T}\right\}$	Coupled displacements of the interface defined using $[\Phi_{Lg}]$ / $\left[T_G^i\right]$
$\{\eta_g\}$	Generalized DOF of the local model
$[C_{tL}]$	Matrix linking the test points and the DOF of the local model
$[C_{IL}]$	Matrix selecting the interface DOF from the DOF of the local model
$[T_{It}]$	Linear operator linking the displacements at the measurement points and at the interface DOF
$[T_{gt}]$	Linear operator linking the displacements at the measurement points and generalized DOF of the interface
$[T_G^c]$ / $\left[T_G^i\right]$	Projection basis assoctiated to the static condensation of the local model *with* (^{c}) / *without the modification* (^{i}) over interface
$[\Phi_{Lg}]$	Eigenmodes of the interface
$[\Phi_{test}]$	Identified modeshapes
$[\Phi_{test}^{coup}], [\Omega_{test}^{coup}]$	Modeshapes and natural frequencies of the modified structure defined at test points
$[M^c], [K^c]$	Mass and Stiffness matrices of the local model *with the modification*
$\left[M^i\right], \left[K^i\right]$	Mass and Stiffness matrices of the local model *without the modification*
$[M^m], [K^m]$	Mass and Stiffness matrices of F.E. model of the modification

1 INTRODUCTION

Companies in charge of production processes (such as electricity, water, etc.) are not the manufacturers of their installations. They may have no access to drawings, dimensionement studies or F.E. models. When a vibration crisis occurs on a part of the installation (motor, pump, etc.), a solution must be quickly proposed. To maintain the production capability, time allowed to maintenance operations is limited. Due to these constraints on time, money and installation knowledge, the construction of a tuned F.E. model may not be allowed (or feasible) to estimate the effects of proposed modifications.

Structural modification methods permit an estimation of the dynamic behaviour of a structure after a modification when a behaviour model of the unmodified structure and a numerical model of the proposed modification are available. These methods, as presented in [1] for example, are particularly useful when reactivity is needed, since the unmodified structure can be characterized rapidly using an experimental modal test.

Few authors dealt with the problem of distributed structural modifications (W. D'Ambrogio and A. Sestieri in [2] and [3], K. Elliot and L. Mitchell in [4] or B. Schwarz and M. Richardson in [5]). Nevertheless, they all impose some of the measurement points to be on the interface, in order to estimate the behaviour of the coupled substructures, using either impedance or modal coupling (see [1] for further details). In all cases, these methods are only applicable when modal tests have been designed for this purpose, the modification being already defined.

The improvements, based on the work of E. Balmès first presented in [6], aim to take advantage of a non-specific modal test. Therefore, the measured data do not need to be obtained from the substructures interface. The difficulties generated by the lack of displacement continuity at the interface as well as the various locations of measurement and coupling points have been overtaken by means of data expansion and model reduction techniques. The expansion process uses a reduced displacement basis for describing the interface. It is derived from a linear combination of the measurement point motions. To select the appropriate basis, two indicators based upon two displacement estimators on the interface are introduced. Two case studies are presented and discussed, showing the ability of this approximate approach to accurately foresee structural modification effects.

2 FORMULATION

This section details the steps of the proposed method. The expansion of experimental data on the interface is first exposed. The finite element model retained for the computing of the displacements fields used in the expansion process, called "local model" is presented. Two indicators are introduced to select the appropriate basis for the expansion procedure.

2.1 Hypotheses and principles

Figure 1 summarises the underlying concept and shows the main difficulties :

- Measurements restricted to a limited subdomain of the whole structure,
- Distributed modification with a continuous interface,
- Non-coincidence between the interface and the measurement points.

To define the coupling between the behaviour model of the tested structure and the numerical model of the modification, displacement fields on the interface have to be known for the tested structure. Due to the experimental nature of this model, there is not enough information to describe the behaviour of the interface. Displacements have to be reconstructed

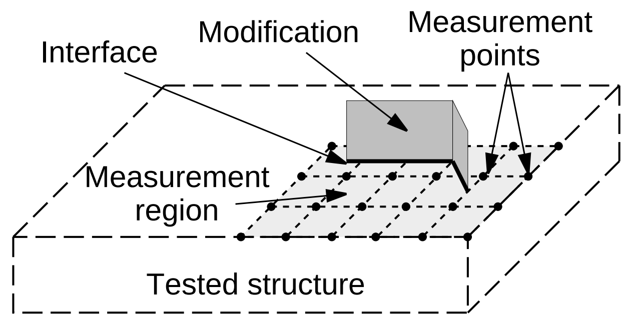


Figure 1: Difficulties overtaken by the proposed method

at points to insure the coincidence with the DOF (Degrees Of Freedom) of the F.E. model of the modification, thus containing the information on the dynamic behaviour of the tested structure.

A strong assumption is made, assuming that interface DOF derive from the DOF at the measurement points for this structure. An linear operator $[T_{It}]$ has to be built, linking DOF $\{q_I\}$ on the interface to the test DOF $\{y_t\}$. This hypothesis is represented in the figure 2.

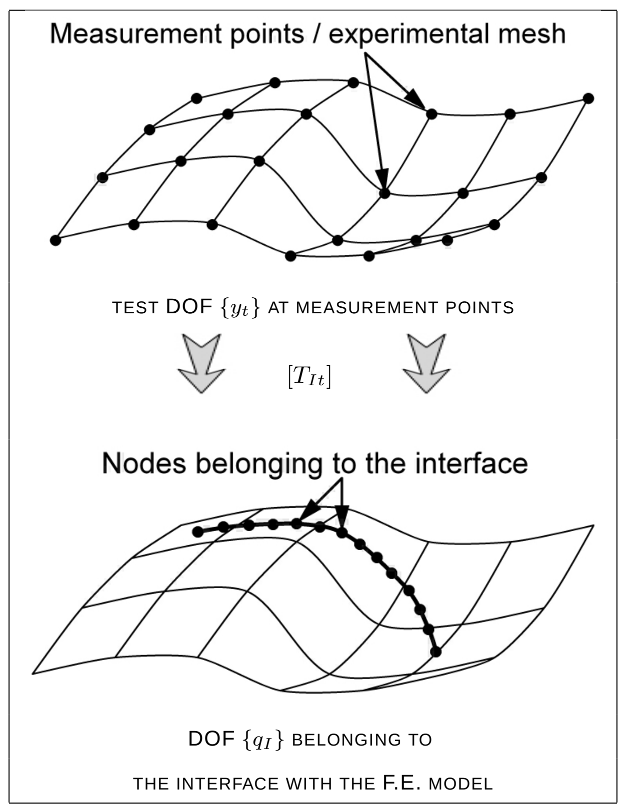


Figure 2: Reconstruction of the interface DOF from the test DOF at measurement points

2.2 Formulation of the expansion

To realise the coupling between the substructures, an operator $[T_{It}]$ must be defined to create a linear link between the test DOF and the interface DOF

$$\{q_I\} \approx [T_{It}]\{y_t\} \tag{1}$$

To build $[T_{It}]$, a basis of DOF defined both at the measurement points and at the interface is required necessary. Various methods exist to reconstruct a continuous field from discrete data set (see [7] for example). In order to add some knowledge from the mechanics, the reconstruction is done using a local F.E. model of the measurement region on the tested structure. Once this F.E. model defined, one can compute particular eigenmodes $[\Phi_{Lg}]$, called interface eigenmodes, defined both at the measurement points and on the interface. The construction of the local model and the eigenmodes $[\Phi_{Lg}]$ is detailed in section 2.3.

Considering the non-coincidence between the DOF of the local model and the test points, a matrix $[C_{tL}]$ is built to relate the test DOF $\{y_t\}$ to the DOF $\{q_L\}$ of the local model. Some ways of constructing of $[C_{tL}]$ are presented in [8]. The test DOF and the local model DOF then satisfy

$$\{y_t\} \quad = \quad [C_{tL}]\{q_L\} \tag{2}$$

The decomposition of $\{q_L\}$ on a truncated basis of the interface eigenmodes $[\Phi_{Lg}]$ yields to

$$\{y_t\} \quad \approx \quad [C_{tL}][\Phi_{Lg}]\{\eta_g\} \tag{3}$$

Where $\{\eta_g\}$ are the amplitudes associated with the generalized DOF of the local model.

Let $[C_{IL}]$ be the matrix operating the selection between interface DOF and local model DOF

$$\{q_I\} = [C_{IL}]\{q_L\} \tag{4}$$

Thus, the decomposition on $[\Phi_{Lg}]$ leads to

$$\{q_I\} = [C_{IL}][\Phi_{Lg}]\{\eta_g\} \tag{5}$$

The expression of $[T_{It}]$ defined by relationship (1) can be written

$$[T_{It}] = [C_{IL}][\Phi_{Lg}][T_{gt}] \tag{6}$$

Where $[T_{gt}]$ is given by the solution of the least square problem related to equation (3)

$$\{\eta_g\} = \underset{\{\eta\}}{\text{ArgMin}}\left(\|[C_{tL}][\Phi_{Lg}]\{\eta_g\} - \{y_t\}\|^2\right) \tag{7}$$

The operator $[T_{It}]$ then allows the reconstruction of motion of the interface deriving from the identified modeshapes $[\Phi_{test}]$. When $[T_{It}]$ is known, the classical problem of structural modification can be solved[1]. Having the displacements fields defined for both the tested structure and the F.E. model of the modification, the equations imposing the continuity of the displacements and the equilibrium balance of the forces at the interface could be expressed, leading to an eigenvalue problem. The eigenmodes and eigenvalues $\{[\Phi_{test}^{coup}], [\Omega_{test}^{coup}]\}$ provide an estimation of the behaviour of the modified structure *defined at the measurement points*.

Assuming that the truncated basis $[\Phi_{Lg}]$ contains N_g eigenmodes, N_g must be less than or equal to the number of sensors N_t to ensure the uniqueness of the solution of (7). Since the result depends on the size and eigenmodes in $[\Phi_{Lg}]$, indicators are necessary to verify the ability of this truncated basis to represent the displacements on the interface. Two criterion are proposed in section 2.4.

2.3 Local model - Construction of the interface eigenmodes

The quality of the behaviour approximation for the modified structure is directly related to the reconstruction of the displacements field on the interface. Equations (3) and (5) highlight the need of an appropriate construction of $[\Phi_{Lg}]$, and then of the local model. These two points are developed in this section.

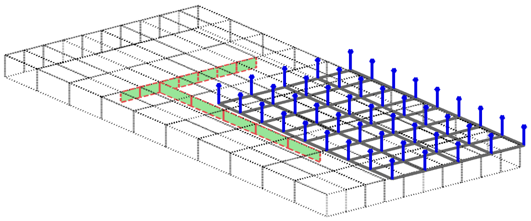
2.3.1 Local model

The purpose of the local model is to ensure a mechanical relationship between the test DOF and the interface DOF. Building a local F.E. model of the instrumented subdomain of the structure thus brings out significant advantages :

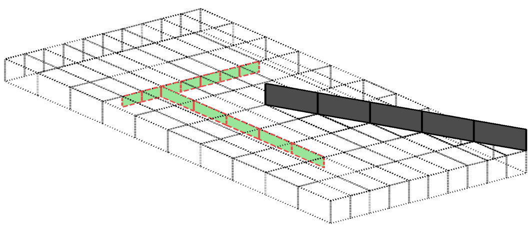
- to obtain quick design and set up of a model depicting the geometry of both the structure and the modification,
- to ease the construction of displacements fields defined at the measurement points and on the interface,
- to ensure the continuity of the displacements fields generated by the F.E. model
- to use some a priori mechanical information,
- to get a regularity of $[\Phi_{Lg}]$ with respect to the equation of the motion.

This model is based on the geometry of the tested structure, with a few mechanical properties. **The purpose is not to build a tuned F.E. model of the whole structure, but to interpolate the measured displacements**. Nevertheless, to add some a priori information on the behaviour of the interface for the coupled problem, the F.E. model of the modification is included in the local model. Thus, the coincidence of the interface for the local model and the modification is ensured. These points are illustrated by a numerical example presented in the figure 3 that emphasizes the differences between the local model and the structure.

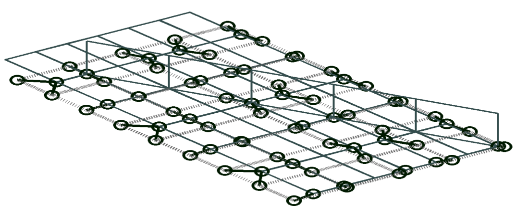
[1] Using modal coupling. Impedance coupling is also allowed, since a behaviour model of the tested structure is known for the DOF on the interface. However, the assumption of reciprocity has to be made in order to recreate the full set of FRF linked to the interface.



Tested structure - measurement subdomain



Modified structure



Local model - Links with

test mesh ($[C_{tL}]$)

Figure 3: Structural dynamic modification example - Tested structure, modified structure, local model

2.3.2 Construction of $[\Phi_{Lg}]$

The determination of vectors $[\Phi_{Lg}]$ depends upon the local model. Once this model is built, displacement fields defined by static expansion of $[\Phi_{test}]$ can be obtained. Such a basis satisfies the hypotheses stated in section 2.2, but some local deformations due to the definition of the static expansion could occur and perturb the construction of $[T_{It}]$. In order to smooth the result of the expansion, another basis is proposed.

Let nz denote the subset of indices corresponding to non-zeros columns of $[C_{tL}]$ and interface DOF, and let z be the other. Thus, we have

$$\forall j \in nz \qquad \begin{cases} \sum_i \left\| [C_{tL}]_{ij} \right\| \neq 0 \\ \text{or} \\ \text{DOF j belongs to the interface} \end{cases} \tag{8}$$

$$\forall j \in z \qquad \begin{cases} \sum_i \left\| [C_{tL}]_{ij} \right\| = 0 \\ \text{or} \\ \text{DOF j does not belong to the interface} \end{cases} \tag{9}$$

The construction of $[\Phi_{Lg}]$ is described in the following. Vectors that are involved into the expansion procedure derive from the eigenmodes of the local model condensed over nz DOF, then statically expanded over the complete local model.

Let $[M^c]$ and $[K^c]$ be partitioned into nz and z DOF. $[T^c_G]$ denotes the Guyan condensation over nz DOF. Let $\{[\Phi^c_G], [\Omega^c_G]\}$ be the eigenmodes and eigenvalues of the condensed problem

$$\left([T^c_G]^T [K^c] [T^c_G] - \omega^2 [T^c_G]^T [M^c] [T^c_G]\right) \{q\} = 0 \tag{10}$$

Where $[T^c_G]$ is defined by

$$[T^c_G] = \begin{bmatrix} Id \\ -[K^c]^{-1}_{z.z} [K^c]_{z.nz} \end{bmatrix} \tag{11}$$

$[\Phi_{Lg}]$ is then defined by static expansion

$$[\Phi_{Lg}] = [T^c_G] [\Phi^c_G] \tag{12}$$

Vectors in $[\Phi_{Lg}]$ are then refered to as "interface eigenmodes".

2.4 Selection of the interface eigenmodes

Vectors in $[\Phi_{Lg}]$ provide a smoother expansion than static expansion when less vectors than the number of sensors are used. But the choice of the optimal size of $[\Phi_{Lg}]$ is a critical point. Indicators are then necessary in order to select the appropriate basis. Selection is based on the comparison of displacements on the interface for the coupled problem obtained by two different ways

- Displacements corresponding to the j^{th} eigenmode of the coupled problem obtained using this approach : $\left\{q^{\Phi}_I\right\}_j = [T_{It}] \{\Phi^{coup}_{test}\}_j$,
- Displacement corresponding to the j^{th} eigenmode of the coupled problem obtained using static expansion over local model *without the modification* : $\left\{q^T_I\right\}_j = [C_{IL}] \left[T^i_G\right] \{\Phi^{coup}_{test}\}_j$

The reconstruction process aims to provide a good estimation of the modification behaviour. Thus, two indicators based on strain and kinetic energy of the modification are defined :

- Strain energy criterion

$$(\Delta_{E_K})_j = \frac{\left\| \{q^\Phi_I\}_j - \{q^T_I\}_j \right\|^2_{Km}}{\left\| \{q^\Phi_I\}_j \right\|^2_{Km} + \left\| \{q^T_I\}_j \right\|^2_{Km}}$$

- Kinetic energy criterion

$$(\Delta_{E_M})_j = \frac{\left\| \{q^\Phi_I\}_j - \{q^T_I\}_j \right\|^2_{Mm}}{\left\| \{q^\Phi_I\}_j \right\|^2_{Mm} + \left\| \{q^T_I\}_j \right\|^2_{Mm}}$$

No method exists at the moment to determine whether the results of the prediction are satisfactory. The indicators focus on the expansion process. The quality in predicting of the eigenvalues and eigenmodes of the modified structure is strongly related to the reconstruction of the interface behaviour, with respect to the modification. The displacements fields obtained by the mean of the static expansion process ensure the matching of displacements at the measurement points. Nevertheless, this method is sensitive to the errors resulting from the identification of experimental modeshapes. The basis $[\Phi_{Lg}]$ is used to provide a smoother expansion. The matching of displacements is then not ensured at the measurement points, but this procedure avoids the propagation of errors in the experimental modeshapes. Enlarging $[\Phi_{Lg}]$ would produce a quite similar result to static expansion. Thus, having low values of both indicators for a size N_g of $[\Phi_{Lg}]$ smaller than the number of sensors would indicate a good expansion.

However, it must be noticed that the indications provided by Δ_{E_K} and Δ_{E_M} are related to the static expansion process. Therefore, a static expansion producing poor results could lead to a bad estimation of the modified structure behaviour. An automatic procedure to detect the appropriate number N_g of interface modes is still beyond feasibility, results provided by the indicators being dependant on the static expansion.

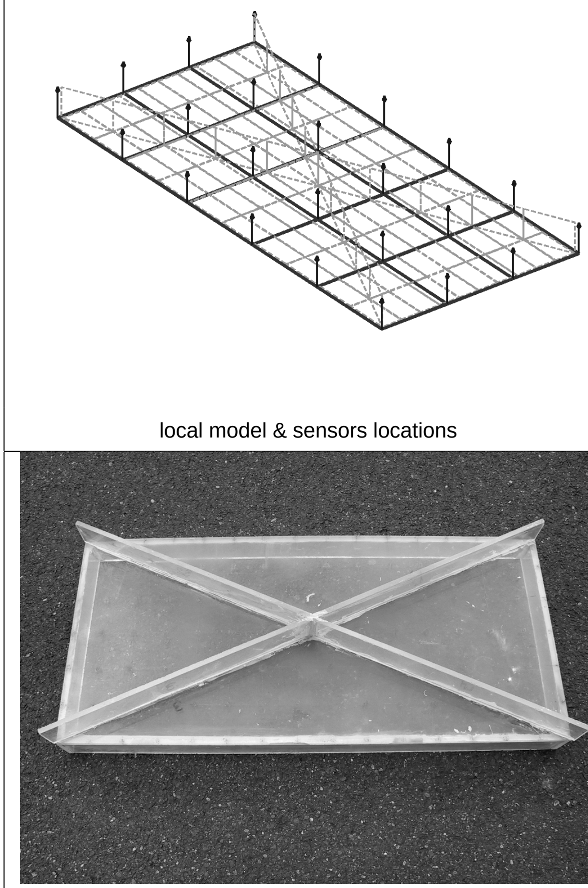
3 APPLICATIONS

To illustrate the approach developed above, two examples are presented. The first one is an academic test device consisting in a rectangular plate stiffened on its border with a rib. Modal analyses have been performed on both initial and modified structure, then compared to those obtained using this method. The second example is an electric motor driving a pump. A modal analysis has been performed on the base structure. The results are compared to those obtained with a tuned F.E. model of the motor.

3.1 Academical test device

The base structure is a rectangular plate (750x350x8mm) made of Plexiglas© stiffened on its border with a glued rib (cross section of 50x8mm). The modification is a rib glued on the diagonals of the plate. The modified structured and the measurement set-up are presented in figure 4. Experimental results are presented in table 1.

This test device is used to evaluate the accuracy of the proposed method in a case where the modification is supposed to have an important influence on the dynamic behaviour of the structure. Few sensors (located at the corners of the base structure) are located on the interface between the tested structure and the modification. This is done to demonstrate the ability of this method to provide good prediction when no other experimental structural dynamic modification method is available.



local model & sensors locations

Modified configuration

Figure 4: Academic test device

Figure 5 represent the evolution of the prediction for the first four modes (frequency and M.A.C.) with respect to the size of $[\Phi_{Lg}]$. The evolution of the indicator is also presented. Results presented in the table 1 correspond to the size of $[\Phi_{Lg}]$ indicated as optimal by both Δ_{E_M} and Δ_{E_K}. This optimum value, different for each experimental modeshape, is determined graphically.

The results presented in the table 1 are satisfactory for all considered modes. The error on the prediction of the first mode is slightly larger, due to the sensor configuration. Only out of plane displacements were measured. The first measured mode is the first bending mode of the plate, leading the ribs to act in traction. These displacements are not well estimated, because no information on the in-plane displacements is available in the experimental model. For all the other modes, the estimated frequencies and modeshapes are good. A recent study (see [9]) has shown that, in a case where some of the displacements on the interface are instrumented, the results are as good as for the classical SDM (Structural Dynamic Modification) methods.

Mode Nb	1	2	3	4
$f_{ini.}$ (Hz)	32.9	91.1	154.9	175.9
$f_{mod.}$ (Hz)	91.1	178.6	182.3	243.1
$f_{est.}$ (Hz)	75.4	167.0	183.5	240.6
Relative error (%)	17.3	6.5	-0.6	1.0
Shift error(%)	-27.1	-15.0	4.2	-3.6
M.A.C. (%)	95.4	88.5	93.0	75.8

Table 1: Frequency & M.A.C. results

- *Relative error* $\dfrac{(f_{mod.} - f_{est.})}{f_{mod.}}$
- *Shift error* $\dfrac{(f_{est.} - f_{ini.}) - (f_{mod.} - f_{ini.})}{(f_{mod.} - f_{ini.})}$

where

- $f_{ini.}$ measured frequency of the initial structure,
- $f_{mod.}$ measured frequency of the modified structure,
- $f_{est.}$ estimated frequency of the modified structure using the proposed method,
- M.A.C. M.A.C. between the identified modeshapes and the estimated modeshapes for the modified structure.

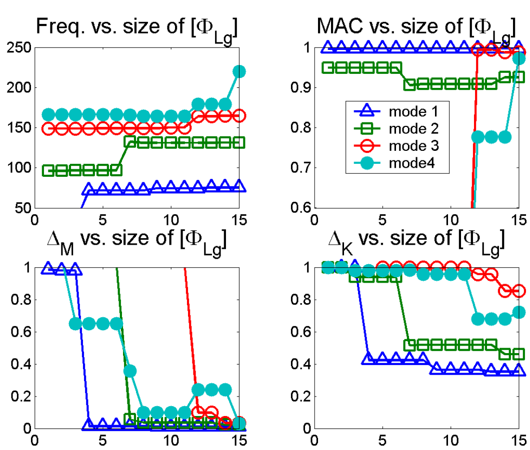


Figure 5: Frequency, MAC and criterion results vs. the size of $[\Phi_{Lg}]$

This case study demonstrate the feasibility of this approach in providing a good estimation of the effect of a modification, with no specific measurement being required. Thus, this method allows one to perform several “What if” analyzes of possible modifications with satisfactory results.

3.2 Industrial case study

This case study illustrates another aspect of this approach. The modal analysis performed in the example of section 3.1 is not really representative of an industrial structure, as the number of sensors located on the domain of interest is large. An accurate description of the local deformations of the structure is available. In industrial studies, the whole structure is generally instrumented, but not sufficiently to provide a fine description of local behaviours. The seek of information concerns the global system, while the expected modification concerns a restricted subdomain. A similar configuration has already been presented in [6].

EDF uses many electrical motors, involved in the electricity production process. On some of them, high levels of vibration have been monitored in various operating conditions. In order to explain this behaviour, a modal analysis has been performed. The sensor mesh is presented in figure 6. Displacements in three orthogonal directions have been measured at the sensors locations.

After identification, it was observed that the first mode was to blame. This mode corresponds to the bending of the motor base. Nevertheless, for security purposes, the frequencies of the second and third mode should not come close to 100Hz. Natural frequencies of the unmodified structure are presented in the table 2.

Mode Nb	1	2	3
$f_{ini.}$ (Hz)	26.1	62.0	90.0
$f_{est.}$ (Hz)	45.6	83.2	91.5
$f_{E.F.}$ (Hz)	44.0	89.1	95.6

Table 2: Natural frequencies of the motor - Unmodified / Predicted with the present approach / Predicted using a tuned F.E. model

- $f_{ini.}$ Natural frequency before modification
- $f_{est.}$ Natural frequency predicted with the present approach, after modification
- $f_{E.F.}$ Natural frequency predicted using a tuned F.E. model, after modification

In order to estimate the effects of modifications, a very fine F.E. model (around 150 000 DOF) was set up and precisely tuned. Modifications where designed and tested. The solution presented in figure 6 is one of them. This work was quite time consuming and fastidious, and a year has been spent to build a representative model. Predicted frequency results are presented in table 2.

The present approach was then applied, using the tests data used to tune the F.E. model. A local model was built. It is presented in figure 6. The total time to build the local model and to perform computations did not exceed a week. The results are presented in table 2.

It can be noticed that, for very similar results in terms of predicted natural frequencies, the proposed approach leads to considerable time savings (a factor 50), with similar confidence in the predicted results.

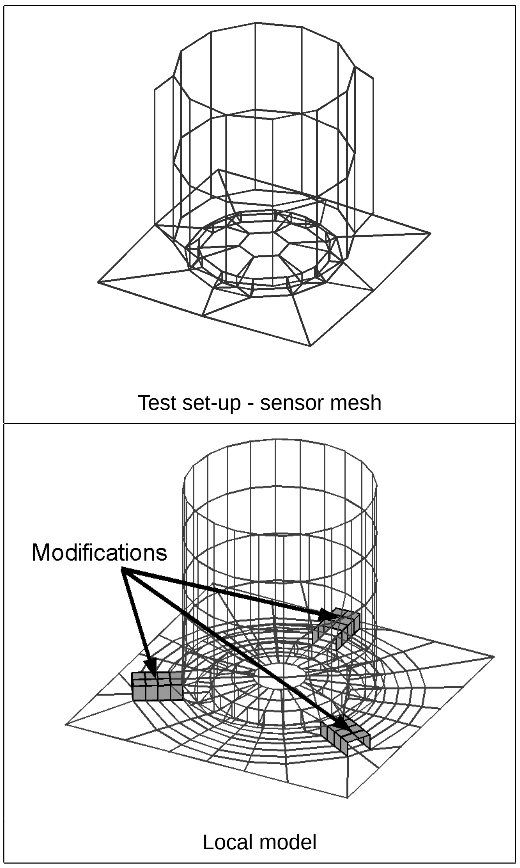


Figure 6: Models of the pump motor

4 CONCLUSION

A original approach allowing to deal with distributed structural modifications is presented. The effects of many different modifications can then be estimated using a generic test set-up. A local F.E. model of the measurement subdomain is introduced permitting the non-coincidence between measurement points and DOF at the interface of the modification and the structure. A smoothing expansion basis based on the eigenmodes of a reduced model derived from the local model is computed. Tests data are interpolated and the information on the interface between the structure and the modification is reconstructed. Two indicators are built to estimate the quality of the expansion over the interface. Based on two different evaluations of the behaviour of the interface, they allow to estimate the size of the optimal basis $[\Phi_{Lg}]$ for the expansion procedure.

Two examples are presented to illustrate some advantages of the proposed approach. An academic test device demonstrates the efficiency of the method. A modification having a great impact on the behaviour of the base structure is introduced. Using a non-specific test, with only few measurement points on the interface, the behaviour of the modified structure is accurately predicted. The ability of this method to reconstruct unknown displacements fields using few hypotheses is shown. The other example is an industrial case study. Reactivity and time saving issues are illustrated and the same results as these provided by a tuned F.E. model are obtained.

These good results are quite encouraging. Nevertheless, the influence of the many parameters involved in the formulation needs to be explored. The number of sensors, their location, the properties and geometry of the local model are subjects of interest for further research. A better understanding of the operating behaviour of the method could lead to an estimator indicating the quality of the prediction.

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