Superelement representation of a model with frequency dependent properties.

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ABSTRACT

The paper analyses problems linked to the prediction of the frequency response of a truss composed of tubes filled with a sand assumed to have frequency dependent properties. The full order model being too large for its solution to be affordable, it is proposed to use superelements parametrized in terms of tube length and frequency dependent complex modulus of the sand. Problems addressed in the paper include numerical representation of the parametrized model, superelement interface reduction, reduction of a parametrized model, verification of the reduced model validity, and prediction of the damped frequency response for a tube and a truss.

1. INTRODUCTION

1.1. Motivation

Isolation devices made of polymers, tubes filled with propergol, distributed vibration damping treatments and a number of other structures have vibration characteristics that have a significant dependence on the properties of viscoelastic materials. It has been shown (see [1,2] for example) that the properties of such materials can be represented over a broad frequency range by a linear elastic model with a frequency dependent complex modulus.

When using a complex modulus, the relation between applied forces u and physical displacements y is linear (there is a transfer function from u to y) and it can be computed by inverting, at each frequency of interest, the dynamic stiffness matrix. If a simple parametric model of the dependence of the modulus on frequency is available, it is possible to consider additional degrees of freedom in the model and build a model that is linear in frequency [3]. This approach, however, is limited to relatively simple parametric models of the modulus, implies the need to create specific finite elements, and leads to the use of larger models.

The frequency dependent modulus leads to simpler formulations but is not directly compatible with traditional modal methods based on frequency independent matrices. This is a problem for large models (with tens of thousands of DOFs) and/or large frequency bands (with many frequency points), for which the cost of an assembly and direct solution at each frequency point is often too high.

To resolve this difficulty, it is proposed to project the models used on constant bases of Ritz vectors. Variants of the idea of projecting a model are known as model reduction and condensation, Component Mode Synthesis, dynamic substructuring, or superelement methods, which have all been the object of numerous publications. In the considered application (see section 1.2), the reduction will be done at the component level and reduced component models (called superelements) will be assembled to achieve the system prediction.

Following the approach proposed in Ref. [4], the reduced component models will be parametrized in terms of length (which allows the creation of a generic superelement for a range of components) and modulus (which allows the use of an arbitrary law for the frequency dependence of the complex modulus). Section 2 discusses the creation of the parametrized superelements. Issues addressed are representation of the parametrized model, representation of the tube interfaces to preserve the ability to assemble a complete truss model, selection of appropriate reduction bases for the empty and filled tubes, and proper representation of damping. Computation of the truss response is discussed in section 3. Finally, numerical issues are addressed in section 4.

1.2. Considered application

The paper analyses the example of a truss made of hollow tubes filled with sand. In submarines such trusses are fixed to the hull and support all the internal machinery. The sand, which has a significant loss ratio, is used to limit the amount of vibration transmitted by the truss. A representative planar truss, whose dynamic response will be computed, is shown in Fig. 1. To account for section deformations of the thin walled tubes and 3-D motion of the sand, a 3-D model of this truss is constructed with planar motion only imposed on one face of the joints.

- Joints, shown in gray in Fig. 1, are made of solid steel ($E = 190\ GPa,\ n\ =.3,\ r\ = 7800\ kg/m^3$)
- Tubes, shown in white, are made of steel (same elastic properties as the joint) and have square sections of *220x220 mm* outer dimensions and *20 mm* wall thickness.
- Sand is packed inside the tubes (nominal elastic properties taken to be $E = 50\ MPa,\ n =0.49,\ r = 1700\ kg/m^3$).

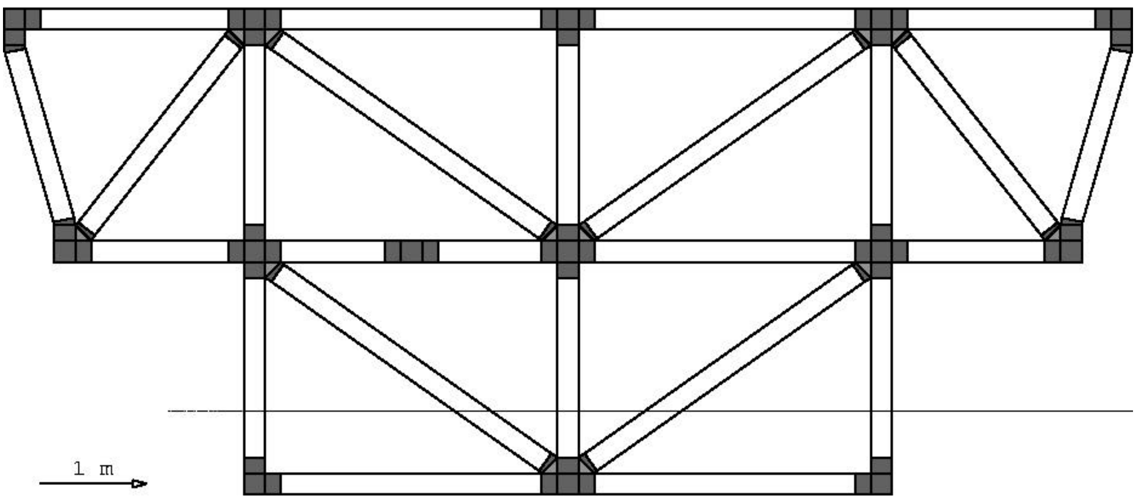

Figure 1: Sample planar truss used for the study.

Issues of linearity, cohesion, elastic coefficients, etc. are important for the creation of a good sand model. The present paper focuses on modeling of the complete truss and making predictions of the dynamic response over a relatively wide range of frequencies. It will thus be assumed, with no further discussion, that the sand can appropriately be described as a viscoelastic material with frequency dependent modulus $E(s)$ (the law for this dependence used in this study is shown in Fig. 6).

To understand why modeling this truss might be a difficulty, a rough count of degrees of freedom is needed. One wishes to model the truss up to *1kHz*. At this frequency, the nominal elastic properties of the sand give a characteristic length of $\left(E/(2\pi f)^2 \rho\right)^{1/2} = 3cm$ for axial wave propagation. A 3 by 3 division of the sand section into quadratic solid elements is thus a minimum. Using 20 node solid elements for the sand and 8 node plate elements for the tube, this leads to 468 DOFs per tube section (see node placement in Fig. 2). Elastic properties of the tubes give a characteristic length for bending motion of $\left(EI/(2\pi f)^2 \rho A\right)^{1/4} = 0.25m$ at *1 kHz*. For the considered tubes which have lengths between *1* and *3 m* a division in 10 segments is thus needed and leads to a total of 2952 DOF for a 10 segment tube (a finer mesh of the sand along the tube length would really be needed but was not considered in the present study). The sample truss being composed of 25 tubes, one would have a minimum of 75'000 DOFs. A real 3-D truss made with more than one bay similar to that shown in Fig. 1 would have at least ten time more. A directly evaluation of the frequency response is thus not affordable and design studies on the influence of damping properties even less so.

2. SUPER-ELEMENT FOR SAND-FILLED TUBES

2.1. Parametrized tube model

For the predictions of interest ones seeks to build a tube superelement with two parameters: overall length and modulus of the sand. This section discusses properties of the full order tube model and the representation of its connection to the solid steel joints.

To obtain the length parametrization, one defines an intrinsic coordinate $\bar{x}$ with the physical coordinate given by $x = l\,\bar{x}$ (*l* is the tube length). The mesh connectivity thus remains invariant for all tube lengths while physical dimensions of elements change. In this parametrization, deformations and spatial derivatives with respect to *y* and *z* (transverse to *x*) are independent of *l*, while derivatives with respect to *x* are inversely proportional to *l*.

As discussed in section 2.2, one will use constant reduction bases. To do so, it is essential that DOFs be defined independently of *l*. This is achieved here by choosing axes so that rotational DOFs (which correspond to spatial derivatives of the displacement) are either in the *x* direction or transverse to it, and replacing transverse rotational DOFs by their product by *l*. In this model, true rotations can be recovered by dividing the pseudo-rotational DOF values by *l*.

For solid elements, stiffness matrices correspond to integrals over the volume of quadratic forms of the strain shape functions which are spatial derivatives of the displacement shape functions. The parametrized stiffness matrices can thus be decomposed into a sum of three matrices that are constant, proportional to *l* and inversely proportional to *l*. It is also easily shown that the stiffness matrices are proportional to Young's modulus and the mass matrices to the length. From these considerations, the dynamic stiffness matrix of the sand elements (based here on NASTRAN CHEXA 20 node solid elements [5]) can be decomposed as

$$K_{Sand}(E,l,s) = \left[lM_{1S}s^2 + E(s)\left(lK_{1S} + K_{2S} + l^{-1}K_{3S}\right)\right] \quad (1)$$

For plate elements, bending effects lead to second order derivatives so that terms in l to l^{-3} are needed (if, as discussed above, rotations are described using definitions independent of l). Here the NASTRAN CQUAD8 elements are decomposed as follows

$$K_{Tube}(E,l,s) = \left[lM_{1T}s^2 + \left(lK_{1T} + K_{2T} + l^{-1}K_{3T} + l^{-2}K_{4T} + l^{-3}K_{5T}\right)\right] \quad (2)$$

Polynomial decompositions of element matrices of the form (1)-(2) can be found for a large variety of finite elements. The different matrices of the polynomial can be simply determined by combining element matrices for different values of the length. For bending properties, numerical conditioning is a problem and different matrix combinations leading to model (2) are not equivalent. The most important difficulty is a tendency find rigid body modes with non-zero frequencies (this is not unusual) that come close to flexible mode frequencies (this is a problem).

It is assumed here that the plates and solids can be assembled in the usual manner. This assumption would need to be mathematically validated, but its verification is clearly unrelated to the modeling process discussed here.

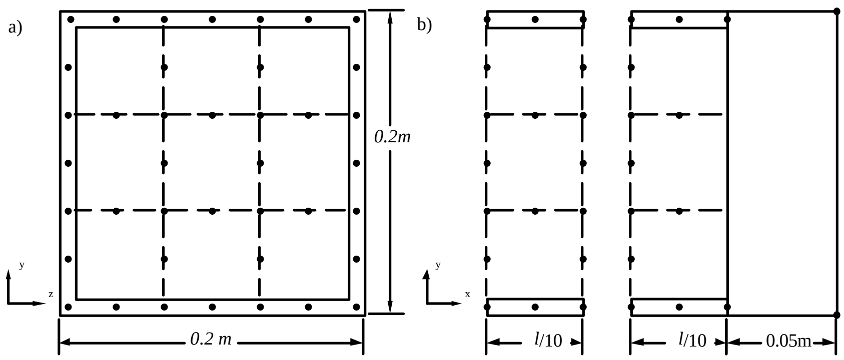

Figure 2: a) tube section, b) tube section and cap (tube section and 8 node hexahedral solid element HEXA8 linked by continuity constraints). Mesh nodes are shown as (•). Tube length l varies between *1m* and *3m*.

For the present study, the tubes are linked to solid steel joints which are not expected to deform very much. A representation of these joints using a few 8 node trilinear solid elements (see Fig. 1) is thus expected to be sufficient. As shown in Fig. 2b, an interface is used with 40 nodes on the tube side and 4 on the solid side.

As proposed in Ref. [6], interface continuity is achieved by only considering deformations linked to the 12 standard bilinear shape functions of a quadrilateral (3 translations at each corner node and bilinear interpolations). These shape functions include rigid body modes and constant strain states of the interface which are a minimum representation. The retained deformations correspond to the trace, on the interface, of shape functions used for HEXA8 elements. Thus, although meshes on the two sides of the interface are not compatible, the approach falls within framework of displacement based discretization methods and preserves all the associated convergence properties.

The interface with the plate elements (tube walls) poses the problem of rotations. Rigid body modes of elements do not always verify the conditions of geometric link between translations and rotations. Here, the rigid body rotation of the hexahedron around the x axis leads, if rotations in the x direction are defined using derivatives of shape functions, to a bending deformation of the tube walls and thus to a model with only 5 rigid body modes (modes with no strain energy). To obtain proper results, rotations are thus determined by condensation of a tube section on its edge with displacements based on the bilinear shape functions.

2.2. Principles of model reduction methods used

Model reduction methods considered here correspond to Ritz analyses, or *displacement based reduction methods*, where one seeks approximate solutions in a reduced subspace corresponding to the range (described by reduced DOFs q_R) of a rectangular matrix T

$$\{q_{\text{True}}\}_{N\times 1} \approx [T]_{N\times NR}\,\{q_R\}_{NR\times 1} \tag{3}$$

The validity of the projection is based on the assumption, that all effectively found displacements q of the full order model have a close approximation in the range of T. The projection (3) applied to loads and displacement of the full order model (T equal to the identity in (4)) lead to the reduced model (called superelement when it corresponds a component of the full structure) with NR rather than N DOFs

$$\begin{gathered}\left[T^T M T s^2 + T^T K T\right]\{q_R\} = \left[T^T b\right]\{u\} \\ \{y\} = [cT]\,\{q_R\}\end{gathered} \tag{4}$$

Note that saving the projection basis T (in practice, it can be stored on disk) permits recovery using (3) of deformations of the full order model deformations and thus through post-treatment the computation of strains, stresses or any other quantity of interest.

The choice of the projection basis T is clearly the key issue. Actual deformations result from applied loads. These loads can always be written as the product of a frequency independent input shape matrix b (which characterizes spatial properties of loads) and a frequency dependent vector of inputs u (which characterizes the frequency content of applied loads). The essential feature of this notation is that inputs u are defined independently model coordinates q. For the full order model, the response of DOFs is given by

$$\{q(s)\} = \left[Ms^2 + K\right]^{-1}[b]\{u(s)\} \tag{5}$$

The selection of the reduction basis is done by making assumptions on the spatial and frequency content of applied loads.

The spatial content is fully characterized by the input shape matrix b which is well known for most problems (here it corresponds to loads applied at the tube ends). A minimum requirement for a good reduced model is clearly to represent exactly the response to applied static loads. To do so, it is sufficient to retain static responses to the input shape matrix b (these shapes are generally called *attachment modes*) or equivalently static responses to unit deformations of the interface DOFs on which b is applied (here the 24 DOFs of tube ends). The later shapes, generally called *constraint modes*, are given by

$$[T_C]_{N\times NI} = \begin{bmatrix} [I]_{NI\times NI} \\ -\left[K_{CC}^{-1}\right][K_{CI}]_{NC\times NI} \end{bmatrix} \tag{6}$$

where I refers to interface and C to interior DOFs.

Since frequency content of true input is rarely known, one seeks to improve the low frequency representation by retaining additional vectors. Normal modes of the model with either fixed, free, or loaded boundary conditions on the interface DOFs provide a convenient way of controlling the frequency range over which the model is accurate. The Component Mode Synthesis literature (see Ref. [7] for example) has addressed many aspects of this problem.

For the parametrized models considered here, the reduction basis T could depend on the design parameters (tube length, Young's modulus). It is proposed however to create a constant basis valid for a range of parameters (this is the usual procedure of the finite element method). A general discussion of issues linked to the choice of constant bases can be found in Ref. [4]. The principle of the approach used here is to create reduction bases (constraint modes T_C and normal modes f) for different design points (values of E and l) and retain a combination of these bases

$$T = \left[T_C(l_1, E_1) \quad f(l_1, E_1) \quad f(l_2, E_2) \quad \cdots \right] \tag{7}$$

Easy assembly of superelements implies the explicit use of interface DOFs as reduced model coordinates. Traditional CMS methods choose the vectors of the reduction basis so as to achieve this objective. Here, a general procedure compatible with arbitrary bases of independent vectors (see details in Ref. [6]) was used instead. The principle of this procedure is that for a basis T, one characterizes interface DOFs by an output shape matrix $[c_I]_{NINT \times N}$ and seeks a new reduction basis $\tilde{T}$ that is equivalent ($range(T) = range(\tilde{T})$) and such that $[c_I \tilde{T}] = [I_{NI \times NI} \quad 0_{NI \times NR-NI}]$. For models reduced on the $\tilde{T}$ basis (here tube superelements), the first NI reduced coordinates then correspond to the interface DOFs.

The reduction bases (7) used here are based on the undamped model (real stiffness matrix). When reducing the model, the imaginary part of the stiffness (or the viscous damping matrix if one was considered) are projected on the same basis. The resulting projected model has complex modes but the fact that these correspond to the true complex modes of the damped full order model is not obvious. A discussion of the validity of this projection can be found in Ref. [8].

2.3. Reduced parametrized model of empty tube

As a first step, reduction of the empty tube (tube of length l and solid caps of length $0.05m$) is considered. The constant reduction basis (7) retained contains the following vectors

1) 24 constraint modes (static response to unit deformations of the interface, for $l = 1\ m$) + 2 rigid body transverse rotations for the *3m* tube.
2) 24 flexible normal modes of the free-free *3m* tube (with caps as shown in Fig. 4)
3) 24 flexible normal modes of the free-free *1m* tube

It is easily verified numerically that this basis contains no collinear vectors. The 24 constraint modes of the *1m* tube contain 6 rigid body modes. As l changes the two rotations of axis perpendicular to the tube involve some deformation (because, as shown in Fig. 2, the cap length is independent of l). The addition of these 2 rotations for another length leads to a model with 6 rigid body modes for all values of l. The 2 sets of retained flexible modes increase the bandwidth over which the model is valid. They include all modes below *1.7* and *3.6 kHz* respectively, which is expected to and in fact does guarantee a good representation below *1 kHz*.

To demonstrate the validity of the considered basis, Fig. 3 shows, for a clamped-free tube, the evolution of modal frequencies as a function of the tube length. Although boundary conditions differ from those used to create the reduction basis (clamped-free versus free-free), one sees a very good correlation between the reduced model predictions and the exact values up to above *1 kHz*. Modal multiplicities, which do not appear on the plot but exist due to the symmetry of the two bending axes, are also well predicted.

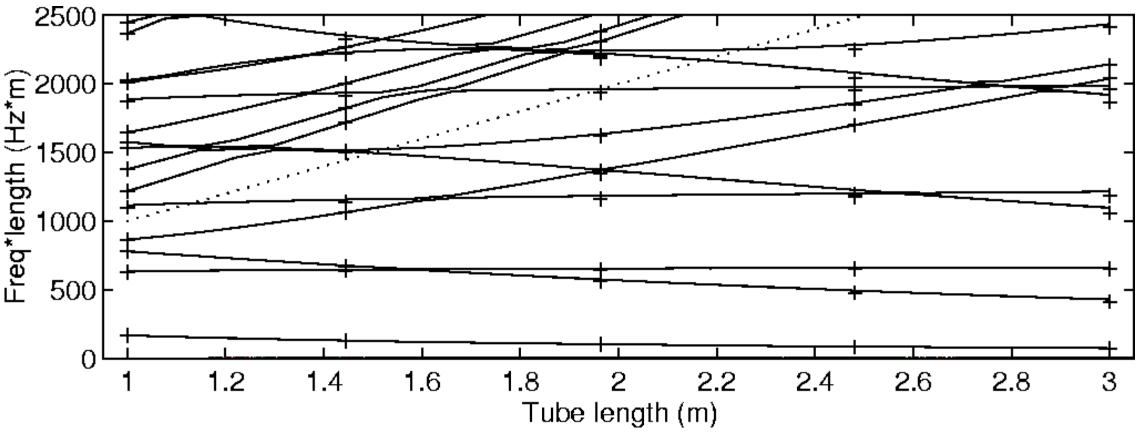

Figure 3: Frequencies (times *l*) of the clamped-free tube as a function of the tube length. (—) 72 DOF reduced model prediction, (+) exact full order model values, (····) 1 kHz.

Not all the computed modes are important for the predictions of interest where external forces will only be applied at the tube ends. This can be seen by measuring the maximum contribution of different modes to the 24 tube/cap interface DOFs. this maximum contribution (maximum norm of the interface residues for mass normalized modes) for the first 30 modes (24 flexible) of the *3m* tube is shown in Fig. 4. Two series of modes (annular modes starting a 600 Hz, and section shear modes starting at 1200 Hz) have very low contributions.

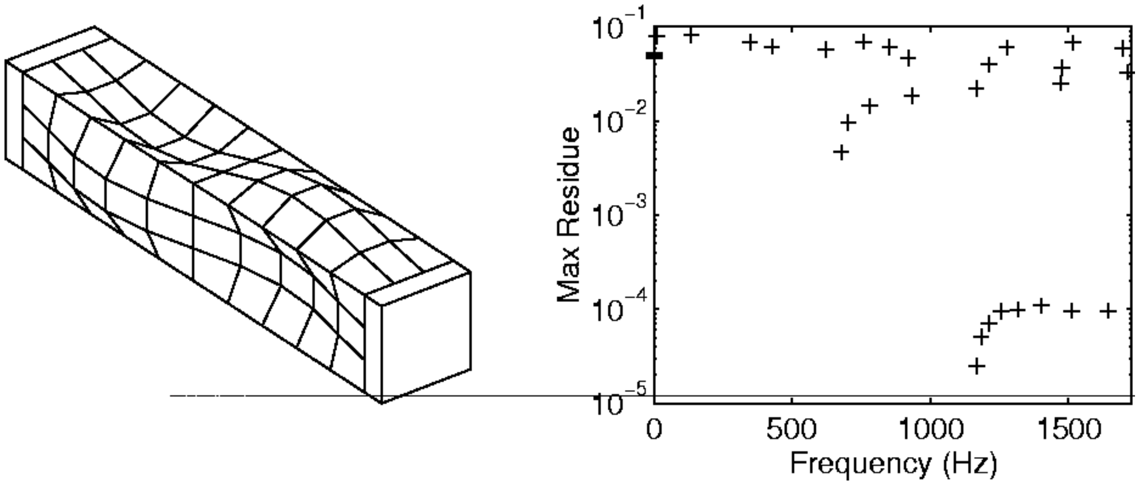

Figure 4: a) First annular mode of the *3m* tube. b) Maximum modal contribution to the interface deformation (*3m* tube).

In general, modes with very low contributions on the interface DOFs can be neglected with almost no impact the overall truss response. In particular a discussion of the fact that annular modes are not needed for a thin walled tube model can be found in Ref. [9]. In the present study however, damping is linked to interactions between tube and sand motion. The effect of eliminating certain modes on damping predictions is unclear and the approach, while thought to be very promising, was not further investigated.

2.4. Reduced parametrized model of sand-filled tube

The sand introduces much complexity to the problem of building a parametrized reduced model. After different trials, the reduced model retained is composed of the following shape functions.

1) 24 constraint modes (static response to unit deformations of the interface, for l = *1 m*, E_{sand}=*100 MPa*) + 2 rigid body transverse rotations for the *3m* tube.
2) 24 flexible normal modes of the free-free tube for l = *1 m*, E_{sand}=*50 MPa*, r_{sand}=*0 kg/m*^{3} (sand with no mass)
3) 30 sand modes with the tube fixed (l = *1 m*, E_{sand}=*50 MPa*)
4) 24 flexible normal modes of the free-free tube for l = *3 m*, E_{sand}=*50 MPa*

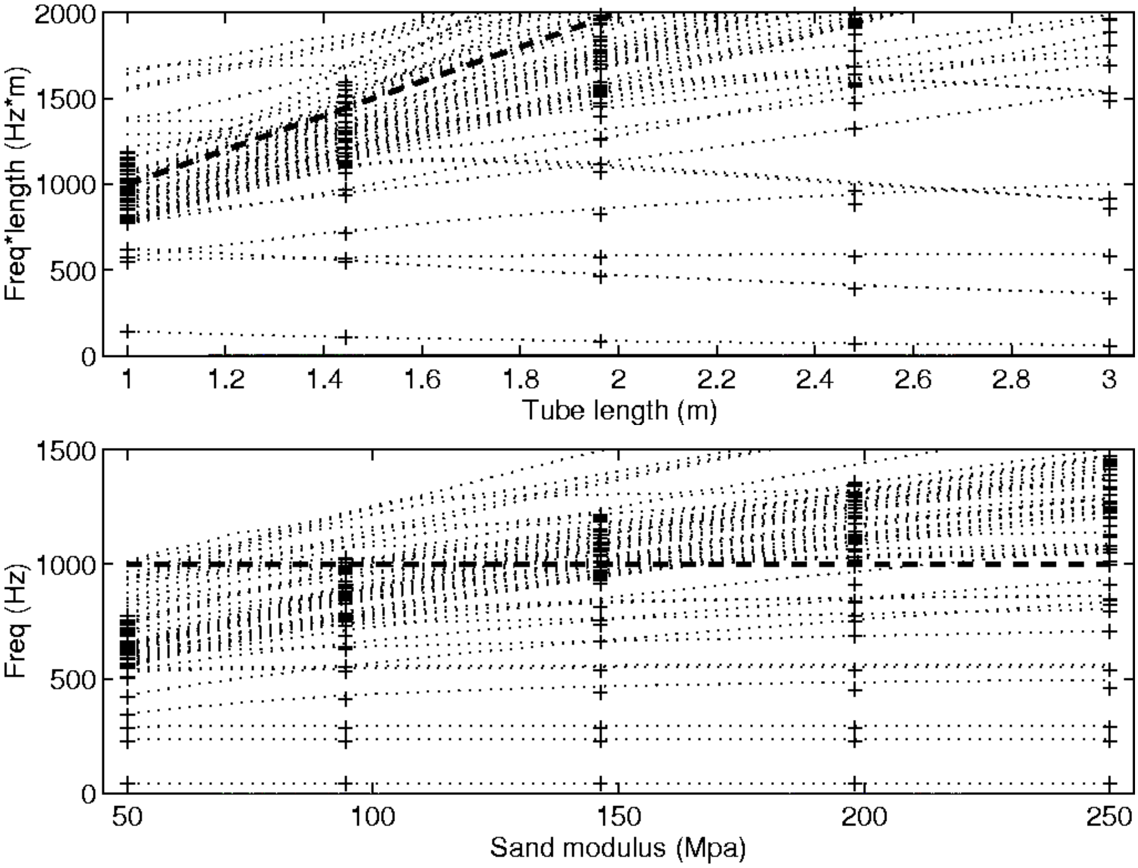

Figure 5: Modal frequencies for clamped-free tube. (····) 134 DOF reduced model, (+) exact full order model, (---) 1 kHz line.
a) Frequencies (times length) as function of length for E_{sand}=*100 MPa*
b) Frequencies as function of E_{sand} for l=*2m*

Fig. 5 shows how the validity can be verified using predictions of frequencies for a range of moduli with a mean length and for a range of lengths with a mean modulus (the selection of the reduced model was based on this plot). In the figure, one clearly notes the high modal density that limits the band of computed modes. This band of closely spaced modes is mostly linked to sand contributions and it is difficult to separate tube and sand modes. To make sure that the tube stiffness is properly represented, the considered reduced model retains for the *1m* tube, tube modes (sand with no mass) and sand modes (tube fixed). In both Fig. 5a and 5b, the last frequency of the reduced model is above the last exact frequency (mode number 48). For the frequency response predictions shown in Fig. 7, this does not significantly degrade accuracy (even though the *1m* tube is considered).

To simplify damped predictions, the tube is assumed to be undamped while the sand modulus E_{Sand} is complex and follows a law shown in Fig. 6. The validity of this law is not the object of this study and more details on existing damping models can be found in Ref. [1,2]. A significant motivation for the present study is that the parametrized superelements introduced here will allow affordable parametric studies of the influence of sand properties.

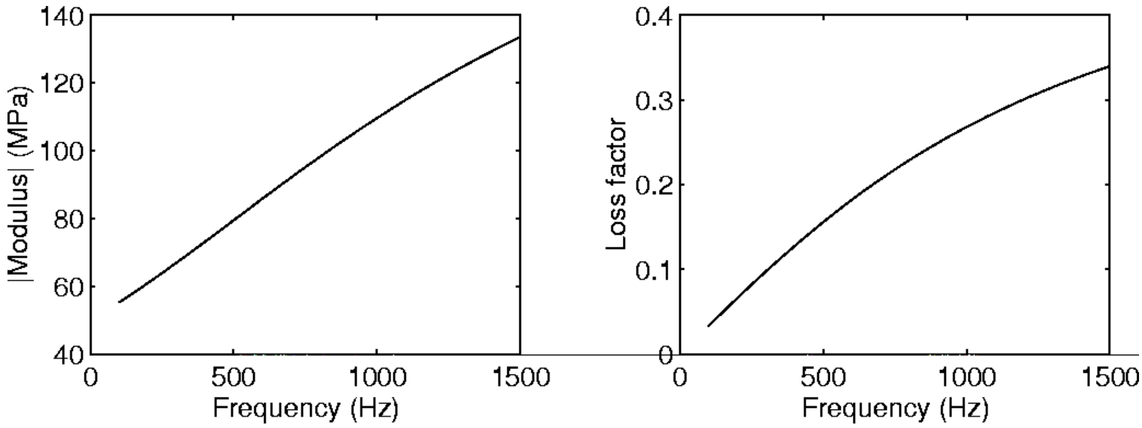

Figure 6: Frequency dependence of the sand modulus. a) Amplitude $\left|E_{Sand}\right|$, b) loss factor $\mathrm{Im}(E_{Sand})/\mathrm{Re}(E_{Sand})$

As discussed in section 2.2, the parametrization in terms of length and modulus is preserved in the reduction process. The change from a real to a complex valued modulus poses no computational difficulty, but the accuracy achieved needs to be discussed. Fig. 7 shows a comparison of exact and reduced frequency responses of the clamped free tube to a bending and an axial load applied at the tip.

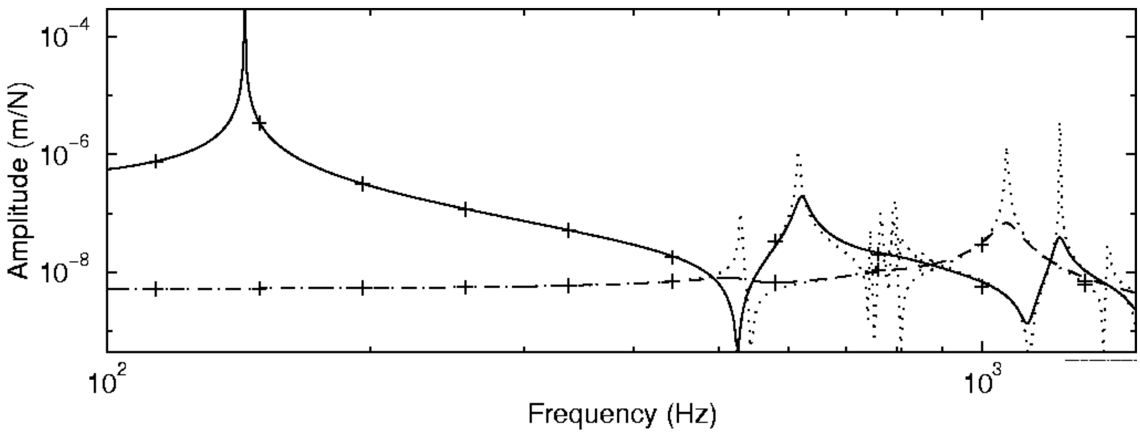

Figure 7: Reduced model FRF predictions for the clamped free *1m* tube to (—) bending and (---) axial loads applied at the tip. (+) full order model FRF. (····) undamped FRF.

From the frequencies shown in Fig. 5, one could expect relatively poor predictions above 800 Hz (the predicted frequencies for the reduced model do not directly match the exact ones). In practice however, the significant damping levels in the sand result in an almost total disappearance of some modes from the damped FRF (this is particularly noticeable for the bending loads where many of the spikes linked to the undamped model, completely disappear from the damped response).

The very significant variations in the damping of different modes is linked to the relative strain energy levels in the sand and the tube. For tube bending modes, sections tend to have small deformations so that the sand can move with relatively small strains and thus low induced damping levels. For tube axial modes, the sand is necessarily and significantly strained so that the damping level is much higher.

In the higher frequency range, many modes have high sand contributions and almost no tube contribution. Such “sand” modes are heavily damped and contribute little to the frequency response. The damping of “tube” modes also depends on strain in the sand. For a parametrized superelement where the sand modulus can change, it is not clearly possible to eliminate sand modes while preserving a correct modeling of the damping mechanisms. This was not done but is seen as a good topic for future research.

3. PREDICTION OF TRUSS RESPONSE

Fig. 8 shows three damped frequency responses for the planar truss shown in Fig. 1 and the modulus law shown in Fig. 6. In plane motion of the 3-D truss model is imposed by not allowing out of plane motion of one face of the joints. This choice eliminates non-planar modes of the truss while allowing non-planar deformation of the tubes. The tube superelements discussed in section 2 could clearly be reduced to only represent planar overall motion but doing so, while not over-stiffening the tube model, is not simple and was not done.

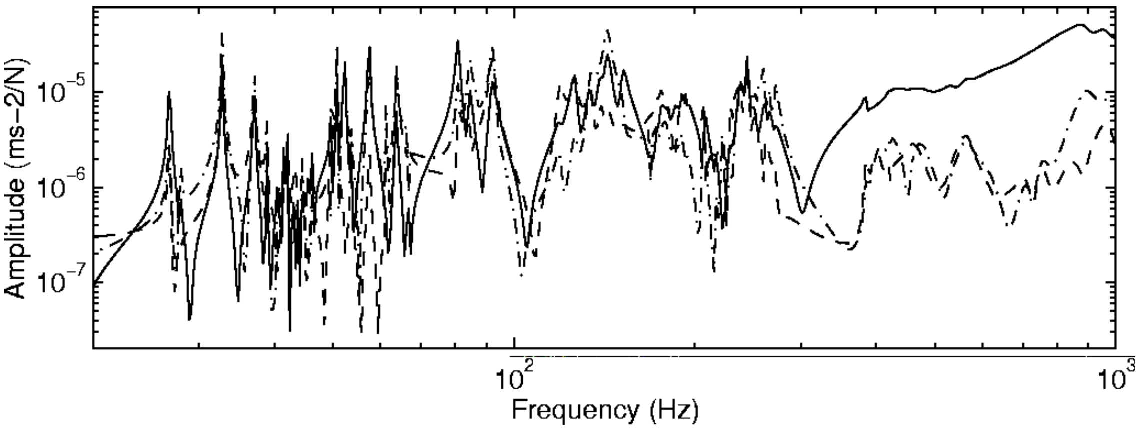

Figure 8: 3 transfer functions computed for the truss shown in Fig. 1 and the modulus law shown in Fig. 6.

In the three transfer functions shown in Fig. 8, the modal density is quite high even at low frequencies. For example, there are 40 normal modes in the 40-50 Hz band. This high modal density can easily be explained by the massive joints and the low bending frequencies of the tubes.

The demonstration of the validity of component models for the considered frequencies and a range of boundary conditions was used to validate the truss model. As discussed in Ref. [11], it is expected that individual modes of this reduced model will not always correspond to modes of the full order model while the combination of their responses will lead, for sufficiently high damping levels, to predictions of frequency response functions that are quite accurate. Here, the level of damping compared to the modal spacing only becomes high above 300 Hz (when peaks associated to different modes cannot be separated). The stability of frequency response functions with respect to small changes in the structure/model thus remains to be properly studied.

4. COMPUTATIONAL ISSUES

Element matrices for solid and plate elements where computed in NASTRAN and imported into MATLAB. All further operations were performed within MATLAB using the Structural Dynamics Toolbox [10]. The interface between the plate/solid elements of tubes and solid elements of joints is difficult to implement in NASTRAN. Verifications were thus limited to simple tube models.

The whole study is based on the use of parametrized models of the form (2)-(3). The development of routines allowing simple handling of such models constituted one of the most significant efforts for this study. The fact that MATLAB uses a sparse rather than skyline matrix storage simplified many coding operations and limited memory requirements for full order models. For reduced parametrized models, it is generally more efficient to consider the matrices as full so that storage methods are irrelevant.

In the proposed superelements, the *NR-NI* reduced coordinates that do not correspond to interface DOFs are associated to the superelement itself (and called element DOFs). In many finite element codes (and NASTRAN in particular), DOFs are necessarily linked to grid points. The implementation of the proposed approach in such codes would thus entail an automated procedure to associate interior DOFs of the superelement to particular grid points.

Many factors (the number of degrees of freedom, the number of elements, the sparsity of the matrices, the sparsity of the factored matrices, the use of real or complex numbers, the memory available, the data rate for disk transfer, the optimization of the code used, etc.) significantly impact the computational cost of a given prediction. Since the separation of all these factors is extremely difficult, an evaluation of computational costs can only be partial and fairly case dependent. Despite this difficulty the following points should be made for the present study.

The full order tube model (used for the comparison of Fig. 7) contains 2344 DOFs with a density of non-zero elements of 4.20%. On a Sun-Sparc 1000 server with 64MB memory, 5e9 flops (~50mn) are needed to compute a single frequency point, versus 2e6 (~0.75s) for the reduced model (note the difference, linked to memory requirements, in the factor ~2500 for flops and ~4000 for time). The construction of the reduced order model takes less time than the computation of a single frequency point of the full order model. Affordable computations are thus achieved with a fairly good confidence in accuracy.

The high cost of the full order model is mostly linked to the fact that solid elements have a significant interelement connectivity (which leads to well populated factors in the decomposition of the complex valued full order matrix). The truss model considered in section 4, has 2820 DOFs with a similar density of 3.33% but only requires 1e8 flops (~30s) for solving. The lower cost of this prediction is linked to the fact that tube superelements are only connected by 24 DOFs (tube/solid interface discussed in section 2.1). The influence of this simplified interface representation on the accuracy of the overall predictions still needs to be fully addressed.

5. CONCLUSION

The finite element method, with its unique ability to take geometrical detail into account, is well suited for the modeling of complex mechanical systems. For applications in dynamics and particularly in cases with materials having frequency dependent properties, full order models are often too large to be affordable. The present study showed, for a fairly complex example, that traditional model reduction methods can be extended to this type of non-linearity and that they provide computationally affordable solutions.

Key issues were the construction of reduced model parametrized in terms of length and complex modulus, reduction of interface representations, projection of damping models on a basis of real Ritz vectors. Many questions about the proposed example are left unanswered and will be the object of future research. It is hoped however that the example gave enough of an illustration to convince readers that the outlined multi-scale modeling approach (where reduced models are constructed by determining which predictions are needed and verifying that these predictions are done correctly) is applicable to a broad range of analysis and optimization problems.

6. ACKNOWLEDGMENTS

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