SENSORS, DEGREES OF FREEDOM, AND GENERALIZED MODESHAPE EXPANSION METHODS

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ABSTRACT

Modeshape expansion techniques deal with the spatial incompatibility linked to the measurement of modeshapes through a limited set of physical sensors and their analytical prediction at a (larger) number of finite element degrees of freedom. Expansion methods are formulated here with the minimal assumption that sensor measurements be linearly related to finite element DOFs. This allows the localization of experimental points independently from the FE mesh, the measurement of translations in arbitrary directions, as well as the combined use of translation and strain sensors. Various methods to interpolate sensor motion from DOF motion are discussed. Generalized static, dynamic, and minimum residual expansion methods are introduced and it is shown how reduction methods traditionally used in Component Mode Synthesis applications allow the use of the proposed methods for industrial size finite element models.

1 INTRODUCTION

Expansion methods seek to estimate the motion at all DOFs of a finite element model based on measured information (modeshapes or frequency response functions). While a significant literature exists in the area, well established method still present major shortcomings.

The theory is always presented using the assumption that measurements corresponds to DOFs which leads to major and unnecessary limitations. DOFs of finite element models are translations/rotations at finite element mesh nodes. Confusing sensors and DOFs thus imposes that the sensors be located at mesh nodes. Furthermore, using sensors that correspond translations in arbitrary possibly non-orthogonal directions or strain measurements is either impossible or very cumbersome to implement. The present paper shows that the use of an observation equation giving a linear description of the relation between measurements and DOFs gives a sound theoretical basis to solve expansion problems even for very complex test configurations.

Most expansion methods can be categorized by how they relate to, modify, or combine the static (based on Guyan reduction [1]) and modal/SEREP [2,3] expansion methods. Dynamic expansion [4] is often acknowledged as the best extension of static expansion but its use has been limited because of its high numerical cost. The limitations linked to the lack of residual terms in modal expansion have shown the need to create hybrid methods [5,6]. But such combinations imply the setting of parameters (coefficients or selection of target modes) which leads to a possibly dangerous form of user involvement.

The second part of this paper shows how observation equations, static/dynamic expansion, and finite element model reduction methods can be combined to formulate generalized static and dynamic expansion methods. The discussion of the assumptions underlying the proposed Reduced Basis Dynamic Expansion (RBDE), then leads to the introduction of a Minimum Residual Expansion (MRE) and its extension using quadratic inequality constraints on measurement errors (MRE-QI) in a fashion similar to the QI versions of standard methods discussed in [7]. The proposed methods are then briefly illustrated.

2 SENSORS AND DOFS

2.1 Motivation

To illustrate the distinction between sensors and DOFs let us consider the laser vibrometer test of a harp resonator shown in figure 1.

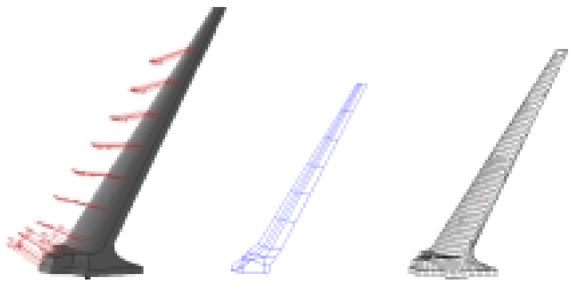

Figure 1: Sensors, test nodes, DOFs

When doing the test, one measures translations in the line of sight of the measurement head. The measurements made define a set of **sensors** and one can build an input/output model from the exciter(s) to the sensors. Sensors typically used for modal analysis are

- accelerometers which, at a given point, measure the acceleration in one or more directions
- laser vibrometers which measure velocity or displacement of a surface in the line of sight
- strain gauges which measure local deformation in one or more directions.

In the testing process, it is important to visualize test results using a wire frame representation of the structures linking physical points where a measurement is made (these will be called **test nodes**). Here the motion is only measured in a single sensor direction assumptions must thus be made to reconstruct the 3-D motion of test nodes. Setting motion orthogonal to the line of sight measurement to zero is easy enough and acceptable for intermediate verifications of test results. For a laser head positioned close to the harp, this however significantly differs from true motion and is not acceptable for test/analysis correlation.

To go beyond the wire-frame representation of the test configuration, one needs to start using a mechanically meaningful model which is generally a finite element model of the structure. Finite element models of linear structures lead to second order differential equations with a number of **Degrees of Freedom** (DOF) which generally correspond to translations and possibly rotations of finite element mesh nodes. For this example and most industrial structures, the mesh is generated by a CAD system and the number of nodes is strongly dependent on the geometric complexity of the structure.

From the analysis of this example it appears that a complete methodology for test/analysis correlation

- must allow arbitrary numbers of sensors (scalar measurements of translation, rotation, strain) at each test node
- must allow an independent selection of test and mesh nodes
- should be independent from the element formulation

The first item is needed for non triaxial measurements. Such measurements are often used to overcome limitations of the correlation methodology in cases where mono or biaxial measurements would be sufficient. Cost effectiveness dictates an optimal repartition of measurements and thus the use of non-triaxial sensors.

Non coincidence of test and FE nodes is a practical constraint. The FE mesh and test nodes are often positioned by different teams, at different times, with different objectives. While bringing the test and analysis people to cooperate better is certainly desirable, one must provide methods that do not require full coordination.

The last item comes from the fact that correlation is generally performed using software packages that differ from the general purpose finite element codes used for analysis. It is important to provide methods that will work without insider knowledge of how finite elements are formulated for two reasons. First detailed element formulation and source code, is not publicly available for major commercial finite element codes. Second, methods based on particular elements would need to be adapted to the countless elements available in most codes.

2.2 Practical methodology

The distinction of DOFs $\{q\}$ and outputs $\{y\}$ through the use of linear observation equations of the form

$$\{y(t)\} = [c]\,\{q(t)\} \tag{1}$$

is the key of the proposed methodology. One thus considers that the dynamics of a system are described by an **evolution** equation (2) and a set of **observation** equations (1). This description is common in control theory (state-space models are composed of two sets of equations) but rarely used in mechanical applications. Its usefulness will be shown here.

In the simplest case where sensors are positioned at FE nodes and measure in DOF directions, the **observation matrix** $[c]$ is just a Boolean matrix often called a localization matrix. An objective of this section is to show that in general, one should use observation matrices that are not just Boolean.

For translation measurements, which is the common case, it is useful to consider two levels of observation. First, one relates the 3-D motion of test nodes to DOFs. Then, one projects this motion along an arbitrary sensor direction to obtain the sensor measurement (this allows the use of an arbitrary number of possibly non-orthogonal sensors). The first level of observation is clearly the difficult part.

For test nodes that do not coincide with finite element nodes (second requirement), the optimal approach would probably be to get inside the formulation of the element to which the sensor is connected and to use its shape functions to determine the displacement of a particular node in physical space. We have however already mentioned that this is not practical (third requirement).

The simplest alternative is to consider the **nearest node**. For coarse meshes or movements with significant rotational components, the error made by neglecting the relative motion of the nearest and physical nodes can be significant.

The natural extension is thus to take rotations into account by imposing a **linearized rigid connection** between the two nodes. Rotations are typically not defined for solid elements and must be handled with caution in the case of plates and shells. Typical finite elements either eliminate the rotation around axes normal to the shell (this DOF is often called a drilling DOF) or use it to improve the convergence of the membrane properties of the element. Thus, the popular MSC/NASTRAN QUAD4 element uses drilling DOFs that have no physical meaning, while the QUADR uses this rotation to represent motion at the element mid-sides [8]. The vector of rotational DOFs thus often does not give an accurate indication of the local rotation of the physical vector linking the test/FE node pair.

A third method acknowledges the problems linked to rotations and thus uses a **rigid link with rotations estimated** based on the translations of two additional non–collinear nearby nodes.

Figure 2 illustrates the nodes used to build the observation matrix of an engine block cover (see details in section 4). The rigid links connect the circles, while the additional nodes used to infer rotations are shown as pluses linked to the nearest FEM node used. The plot and the observation matrix was here automatically generated using the *Structural Dynamics Toolbox* [9] with an additional effort to ensure that links only use nodes of the physical component on which the sensor is located.

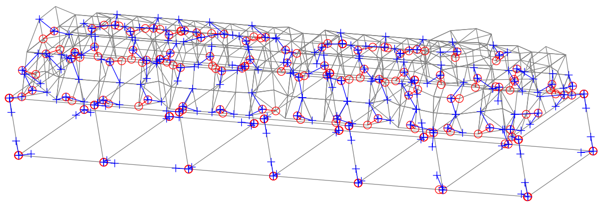

Figure 2: Nodes used to build the observation matrix of an engine block cover

The third method (rigid link with inferred rotation), is currently considered as giving a good trade-off between cost and accuracy. Improvements to this approach will obviously be introduced, but results obtained so far have always been satisfactory (which is not the case of the two other methods mentioned).

Finally, observation equations provide a theoretical basis allowing to deal with measurements of mixed nature (translations, rotations, strains, ...), this will not be emphasized in the examples but is another important reason to distinguish sensors and DOFs.

3 EXPANSION OF TEST DATA

We will now consider that the FE model leads to equations of motion of the form

$$[M_{FE}]\{\ddot{q}\} + [K_{FE}]\{q\} = [b]\{u(t)\} \tag{2}$$

but instead of the traditional partition of q into measured and unmeasured DOFs, we will use the observation equation (1).

Expansion methods estimate the motion of DOFs based on the observation of particular measurements at sensors. The basis for expansion is the fact that while a finite element model has many DOFs, the dimension of a subspace accurately representing the response to a limited sets of loads in a limited frequency range is typically small. This principle is the basis of reduction methods which are briefly summarized before introducing generalized expansion methods.

3.1 Short reminder on model reduction

The fundamental approach for model reduction is to project the FE mass and stiffness matrices on the vector space spanned by the columns of a reduction matrix $[T]$ which has less columns than rows

$$[M_R] = [T]^T[M_{FE}][T] \text{ and } [K_R] = [T]^T[K_{FE}][T] \tag{3}$$

The choice of the reduction basis has been the object of numerous publications and categories of methods known as modal analysis, condensation, component mode synthesis, sub-structuring [10, 11]. For the present paper, vectors considered for reduction will combine

- eigenvectors of the nominal model. They are used to ensure that the reduced model will be valid over a certain frequency range. Boundary conditions need not correspond to those of the test of interest.
- static responses to particular imposed displacements or loads. They ensure spatial completeness for the considered loads.

A key property for the present paper is the fact that the dynamics of reduced models only depend on the subspace spanned by $[T]$. More precisely, the dynamics of the system are characterized by the dependence of outputs $\{y(t)\}$ on the inputs $\{u(t)\}$. This relation is described by two equations (evolution and observation) given for the reduced model by

$$\begin{array}{c} [M_R]\{\ddot{q}_R\} + [K_R]\{q_R\} = [T]^T[b]\{u(t)\} \\ \{y(t)\} = [c][T]\ \{q_R(t)\} \end{array} \tag{4}$$

or in the frequency (Laplace) domain

$$\begin{array}{c} [Z_R]\{q_R\} = \left[M_R s^2 + K_R\right]\{q_R\} = \left[T^T b\right]\{u(s)\} \\ \{y(s)\} = [c][T]\ \{q_R(s)\} \end{array} \tag{5}$$

The u, y relationship is clearly identical for a model projected on $[T]$ or on $\left[\tilde{T}\right] = [T][A]$ with A non-singular

$$[cT]\left[T^T Z_{FE}(s) T\right]\left[T^T b\right] = \left[c\tilde{T}\right]\left[\tilde{T}^T Z_{FE}\tilde{T}\right]\left[\tilde{T}^T b\right] \tag{6}$$

We will refer to this property as the **equivalence** of models projected on various bases of the same subspace.

3.2 Generalized Static Reduction/Expansion

The standard Guyan or static reduction method [1] partitions the DOFs in two sets of active and complementary DOFs. The active DOFs, which correspond to interface DOFs in Component Mode Synthesis applications, are for tests assumed to correspond directly to sensor measurements

$$\{q_a\} = \{y_T\} \tag{7}$$

Assuming further that the inertia forces acting on the complementary DOFs are negligible, there exists an exact relationship between active and complementary DOFs given by

$$\left\{ \begin{array}{c} q_a \\ q_c \end{array} \right\} = \left[\begin{array}{c} I \\ -K_{cc}^{-1} K_{ca} \end{array} \right] \{q_a\} = [T]\{q_a\} \tag{8}$$

which can be used as an expansion method (given $\{q_a\}$, the relation above gives an interpolation for $\{q_c\}$) or a reduction method (application of (4) with T given by (8)).

When using a non-boolean observation equation, this method is not directly applicable. The fundamental assumption made in the classical method is that the only significant forces (inertial and external) are applied on active DOFs. One thus introduces a generalized static expansion defined as the *static response to forces collocated to the sensors such that the predicted response at sensors corresponds exactly to the measurement*.

For a set of sensors observed through (1), unit collocated forces (those associated with the sensors by the reciprocity assumption) are given by the columns of $[c]^T$. The considered reduction basis is thus defined by

$$[K_{FE}]\left[\tilde{T}\right] = [c]^T \tag{9}$$

In many cases, the finite element model will have rigid body modes so that $[K_{FE}]$ is singular. This is a standard difficulty in structural dynamics and the two standard approaches to solving this problem are to orthogonalize the response with respect to rigid body modes $[c]^T$ [12] or to use a mass-shifted stiffness matrix [11].

The second step of the generalized static expansion takes advantage of the invariance property (6). To go back to an equation of the form (8), one wants to find a new basis T of the subspace spanned by $\tilde{T}$ such that $cT = I$. Assuming that the observations of the vectors of the reduction basis are independent ($c\tilde{T}$ is non-singular, this assumption will be relaxed in the next section), a generalized static expansion will thus be given by

$$\{q\} = \left[\tilde{T}\right]\left[c\tilde{T}\right]^{-1}\{y\} \tag{10}$$

For a structure without rigid body modes and a boolean observation matrix ($[c]\{q\} = \{q_a\}$), one can verify that

$$\begin{aligned} \left[\tilde{T}\right] &= \begin{bmatrix} K_{aa} & K_{ac} \\ K_{ca} & K_{cc} \end{bmatrix}^{-1} \begin{bmatrix} I \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} I \\ -K_{cc}^{-1} K_{ca} \end{bmatrix} \left[K_{aa} - K_{ac}K_{cc}^{-1}K_{ca}\right]^{-1} \end{aligned} \tag{11}$$

so that $\left[\tilde{T}\right]\left[c\tilde{T}\right]^{-1}$ is indeed the static expansion considered in (8) and the proposed method can be called a *generalized static expansion*.

3.3 Generalized Dynamic Expansion

The assumption that inertia forces are negligible is often poor for some deformations. Dynamic expansion was thus introduced to take into account that in many applications the considered deformation is associated to a known frequency (modeshapes, measured FRFs). The *Kidder dynamic expansion* [4] thus generalizes static expansion by considering the exact steady-state response to an imposed harmonic displacement at sensors that correspond to DOFs

$$\left\{ \begin{array}{c} q_a(\omega) \\ q_c(\omega) \end{array} \right\} = \left[\begin{array}{c} I \\ -Z(\omega)_{cc}^{-1} Z(\omega)_{ca} \end{array} \right] \{q_a(\omega)\} \tag{12}$$

This is a simple generalization of the static expansion but guarantees that the exact modes will be found if $\{q_a(\omega)\}$ is a restriction of the shape and the associated frequency are exact. While this method is clearly much more accurate than the static expansion, it uses a different reduction basis for each frequency. For industrial FE models in an model updating phase, dynamics responses to unit loads at sensors have to be computed for each target frequency and each step of the updating procedure. Numerically this is often very expensive and difficult to justify for a method that typically only has an effect on few modes.

For an arbitrary reduction basis $\tilde{T}$ with more vectors than sensors, one can define a **generalized dynamic expansion** (GDE) as the search for harmonic inputs $\{u(\omega)\}$ collocated to the considered sensors and such that the predicted response corresponds to the measurement

$$\begin{array}{l} \left[\tilde{T}^T Z_{FE}(\omega)\tilde{T}\right]\{q_R(\omega)\} = \left[c\tilde{T}\right]^T\{u(\omega)\} \\ \text{and} \\ \left[c\tilde{T}\right]\{q_R(\omega)\} = \{y_{Test}(\omega)\} \end{array} \tag{13}$$

The statement of this problem is clearly invariant by any change of basis in the subspace spanned by $\tilde{T}$. Assuming that the rank of $\left[c\tilde{T}\right]$ is equal to the number of sensors, it is possible to build a transformation A such that

$$\left[c\tilde{T}A\right] = [cT] = \left[[I]_{NS\times NS}\ \ [0]_{NS\times NR-NS}\right] \tag{14}$$

The last $NR - NS$ columns of A correspond to any basis of the kernel of $\left[c\tilde{T}\right]$ (which can be found using the singular value decomposition of this matrix for example [9]). Given a basis of the kernel, the first NS column of A can be found by applying (10) to a basis of a subspace orthogonal to the kernel.

With a reduction basis T verifying (14), the generalized DOFs of the reduced model are of the form $\{q_R\}^T = \left\{y_T^T\ \ q_{Rc}^T\right\}$ and the solution of problem (13) is

$$\{q\} = [T_e]\{y_T\} = [T]\left[\begin{array}{c} I \\ -Z(\omega)_{RcRc}^{-1} Z_{RcT} \end{array} \right]\{y_T(\omega)\} \tag{15}$$

which is very similar to (12) but only requires the inversion of $Z(\omega)_{RcRc}$ which has dimensions $NR-NS$ rather than $N-NS$ which carried an unacceptable numerical cost.

Note that the invariance of problem (13) by any change of basis in the subspace spanned by $\tilde{T}$, implies that the solution found in (15) is independent of the choices made for the coordinate change leading to (14).

The proposed *generalized dynamic expansion* (GDE) has the advantages of the standard dynamic expansion (no sensitivity to mass effects) while allowing low cost computations by using a model reduction (RBDE : reduced basis dynamic expansion). A typical reduction basis would combine static responses to unit sensor loads (to ensure that the result is at least as good at static expansion), analytical target modeshapes (to guarantee exact expansion for these modes), and possibly other vectors (target modeshapes of other FE configurations, modeshape sensitivities, etc. [13,14]). It should be noted that this method can be viewed as the definition of a projector in the general class of hybrid methods discussed in Refs. [5,6]

3.4 Minimum dynamic residual expansion

The assumption that dynamic loads used for the expansion are only applied at sensor locations is not particularly realistic. In particular for finite element updating procedures where the model is known to be incorrect, the dynamic residual ($R_j = Z(\omega_j)\phi_j$ for modeshapes or $R_j = Z(\omega)q - F$ for frequency response to the harmonic load F) should be non zero at most DOFs.

One thus defines here a *minimum dynamic residual expansion* (MDRE) which seeks to minimize the strain energy associated to the dynamic residual. For the case of a modeshape and a reduced basis coordinate change such that $\{q_R\}^T = \left\{y_T^T \;\; q_{Rc}^T\right\}$, one thus seeks the solution of

$$\min_{q_{Rc}} \left|\left| \left\{ \begin{array}{c} \phi_T \\ q_{Rc} \end{array} \right\}^T [Z(\omega_j)] \left[\hat{K}\right]^{-1} [Z(\omega_j)] \left\{ \begin{array}{c} \phi_T \\ q_{Rc} \end{array} \right\} \right|\right| \tag{16}$$

where $\left[\hat{K}\right]$ is a mass shifted stiffness for cases with rigid body modes and the standard stiffness otherwise.

The numerical cost associated with this expansion method is only acceptable for cases with q_{Rc} not exceeding a few hundred DOFs. The MDRE should thus only be considered for reduced basis dynamic expansion using bases similar to those considered in the previous section.

3.5 Estimation error and smoothing

As a result of measurement and estimation errors (bias and variance), identified modeshapes are never exact. The observation of the expanded vector should thus be allowed to differ somewhat from the measurement.

For cases with a reduction basis with less vectors than sensors, the problem is easily solved through a least squares minimization which has a solution of the form

$$\{q\} = [T]\{q_R\} = [T]\left[[cT]^T[cT]\right]^{-1}[cT]^T\{y_{Test}\} \tag{17}$$

It can easily be verified that for Boolean $[c]$ selecting measured DOFs and a reduction basis containing a set of target modes, the application of (17) is known as the SEREP method [3].

Ref. [7] showed how classical expansion methods can be reformulated in terms of minimization problems and how the classical least squares problem with quadratic inequalities (LSQI) [15] allows to account for errors in test results. The same work can and should be done for the extensions proposed in this paper.

The principle of LSQI based expansions is as follows. Assuming that the measurement $\{y_T\}$ is inexact, one seeks a smoothed $\{\hat{y}_T\}$ close to the measurement (verifying the quadratic inequality $||\{\hat{y}_T - y_T\}|| \leq \alpha||\{y_T\}||$) such that the expanded vector is more *realistic*. Various definitions of *realistic* lead to different expansion methods.

Thus, the minimization of the strain energy of the dynamic residual associated to a modeshape leads to a MDRE-QI method where one solves

$$\min_{q_{Rc}} \left|\left| \left\{ \begin{array}{c} \hat{y}_T \\ q_{Rc} \end{array} \right\}^T [Z(\omega_j)] \left[\hat{K}\right]^{-1} [Z(\omega_j)] \left\{ \begin{array}{c} \hat{y}_T \\ q_{Rc} \end{array} \right\} \right|\right| \tag{18}$$

with

$$||\{\hat{y}_T\} - \{\phi_T\}|| \leq \alpha||\{\phi_T\}|| \tag{19}$$

4 ILLUSTRATIONS

4.1 Observation equations

The first part of this paper motivated the need to use observation matrices to distinguish sensors and DOFs and discussed various methods to interpolate test node displacement from finite element DOFs. Figure 3 illustrates this aspect for the case of an engine block cover (data kindly provided by *Renault-DR*). The test configuration uses 182 sensors distributed at 91 nodes with measurements made in two directions at each node. The (coarse) finite element model uses 2218 nodes, 7758 DOFs (solid elements have no rotational DOF), 444 plate/shell elements for the cover and 1386 solid elements for the base.

Test DOF motion was interpolated based on analytical modeshapes computed with MSC/NASTRAN using the nearest node, linearized rigid link and alternate rigid link (interpolated rotations) methods discussed in section 2. The plot in figure 3 illustrates the difference between the first 2 methods and the 3rd (assumed to be "exact"). The plot clearly indicates that the nearest node method shows less difference which, at first, seems surprising.

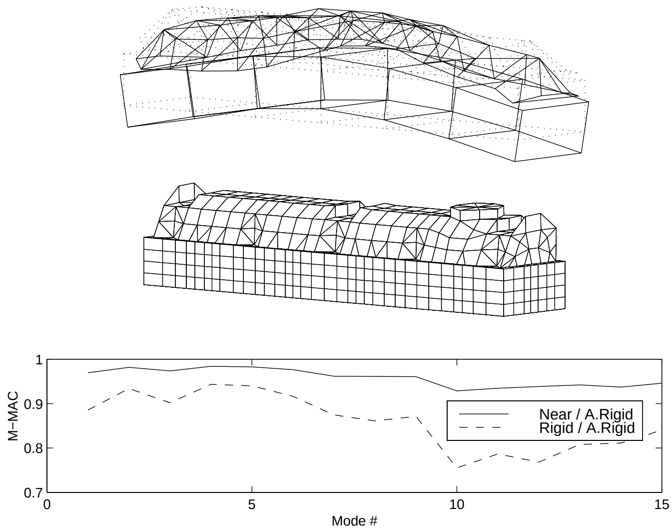

Figure 3: Wire frame representation of test configuration, finite element model, MAC comparisons of different methods for test node interpolation

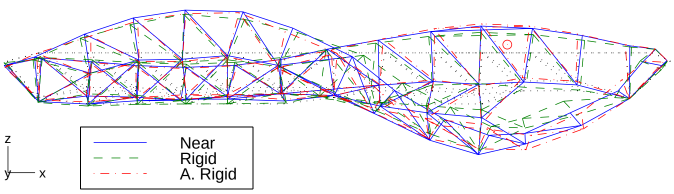

Figure 4: Interpolations of first analytical torsion mode on test mesh.

The origin of this difference is actually linked to drilling DOFs of the QUAD4 elements used in the NASTRAN model. At the joint between the cover and the support block (top in figure 4), these DOFs are mostly oriented around the y direction so that off-sets of test nodes in the xz plane will, for the standard rigid link method, lead to non-physical corrections linked to the drilling rotation. In figure 4, when comparing the response at the joint level (top of the structure in the figure), one indeed sees that the basic rigid method significantly differs from the two other results.

4.2 Computational times

Computational times are always dependent on many factors including computer, level of software optimization, ... All computations and illustrations are here performed using the *Structural Dynamics Toolbox* for use with MATLAB [9]. The only area where version 3.1 of the *SDT* is known to be fundamentally slower than fully compiled codes is in the computation of element matrices. The comparisons made in table 1 are thus quite representative of the real cost of expansion methods.

Full order dynamic expansion has a high cost (linked to the

TABLE 1: CPU times for modeshape expansion of the engine block cover test (on an SGI-R10000 processor running MATLAB 5.2.1 and SDT 3.1) .

Model assembly	72 s
Eigenvalue solution (20 modes)	21 s
Full order Dynamic Expansion (14 modes)	77 s
Reduction for RBDE (202 shapes)	60 s
Reduced Basis DE (14 modes)	4 s

block extraction and factorization). This cost is directly proportional to the number of modes to be expanded.

The cost of reduced basis version of the generalized dynamic expansion (RBDE) is composed of an up-front cost linked to the reduction and an additional cost linked to each expansion. The later part is very small for this model and only depends on the reduction basis size so that it would not grow for larger models. The reduction is here fairly expensive but it is driven by the number of shapes in the reduction basis (here 182 static responses associated to each sensor + 20 modeshapes already computed).

This case was chosen because it has many sensors and few modes to be expanded, the full and reduced order methods are thus found to have similar computational costs. The advantage of the reduced order method becomes significant when the model size or number of expanded modes is increased, the number of sensors is decreased, or if the reduction basis is already computed for other reasons (estimation of modeshape sensitivities, ...).

4.3 Selection of expansion method

The relative merits of various expansion methods discussed in this paper are difficult to establish since they all work in many cases. This section will thus seek to illustrate typical difficulties of that have motivated the development of the MDRE-QI method which is currently considered as best by the author.

The illustrations will be made using the example of the GARTEUR SM-AG-19 testbed [16, 17]. This test article is a simple structure with publicly available test results (contact the author for more information). The simple 816 DOF/90 element model of the structure shown in figure 5, the nominal 24 sensor configuration, the first 14 modeshapes measured by participant C will be used here.

Figure 5 compares, for mode 11, static and dynamic expansion to the finite element modeshape. For the mode shown, the 3 sensors available in the x direction (shown as arrows on the plot), cannot capture the bending of the fuselage. Static expansion thus gives a significant rigid body contribution for the fuselage and wings which is very unrealistic.

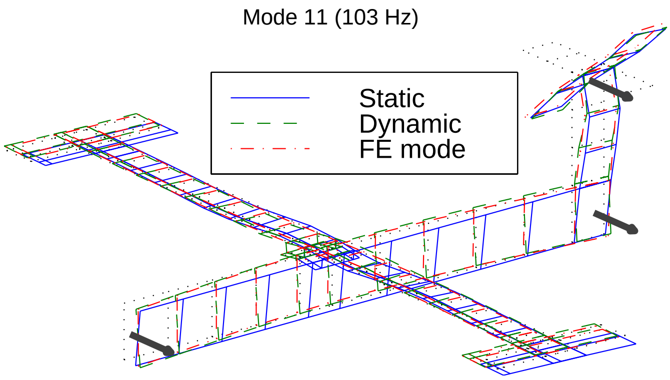

Figure 5: Modeshape expansion for the GARTEUR SM-AG-19 testbed.

Most modal test have enough sensors for static expansion to give correct results for many modes. But this example shows that even in simple tests, it can occasionally fail miserably to represent inertia effects whereas dynamic expansion gives much more robust results.

Figure 6 analyzes modal/SEREP results for various selections of modeshapes. Keeping the 6 rigid body modes works well with only 8 flexible modes (1:14 case) and poorly with 14 (1:20). Keeping 14 flexible modes (7:20) works well but adding more 7:26 and 7:30 shows clear deterioration. The modal method thus shows a lack of robustness which eliminates it as a good alternative to static expansion.

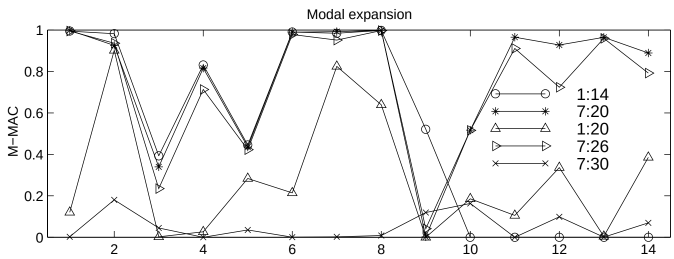

Figure 6: Mass weighted MAC of modal expansions of GARTEUR test data versus FE model

Figure 6 confirms the result mentioned previously that dynamic expansion is more accurate than static (modes 11:14). Modal results (given for the case with 8 flexible modes) is good in this range. No distinction is made between dynamic and reduced basis dynamic expansion as differences are minimal. The RBDE, MDRE and MDRE-QI methods give an increasingly good correlation which could be expected but is not necessarily a good indication of true correlation.

Finally, the difference between MDRE and MDRE-QI is illustrated in figures 8-9 where the strain energy distribution associated to the dynamic residual $\{R_j\} = \left[\hat{K}\right]^{-1}\left[K - \omega_{j\mathrm{id}}^2 M\right]\{\phi_{j\mathrm{ex}}\}$ (a particular case of the error in

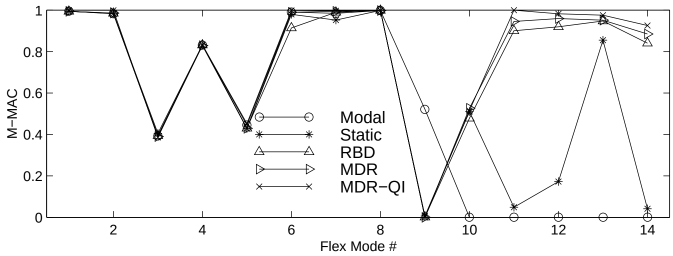

Figure 7: Mass weighted MAC of modal expansions of GARTEUR test data versus FE model

constitutive law criterion) is displayed for the first 4 modes.

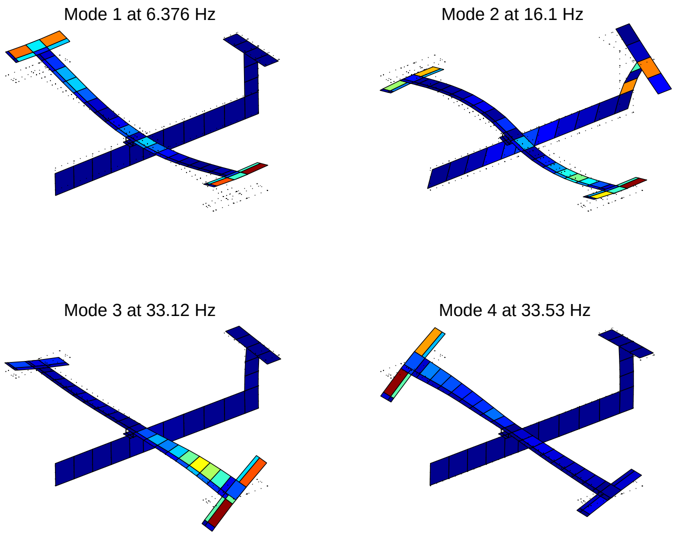

Figure 8: Strain energy distribution of dynamic residual for RBDE

The experimental modeshapes used show a slight calibration error of the sensors at the middle of the drums so that they appear to bend. The direct MDRE result thus indicates large error levels on the drums which is indication of experimental error and not of model errors which are of interest.

When gradually increasing α (how to do this is really the problem with the method), the MDRE-QI method accounts for measurement errors and gives the correct result that the first mode is mostly in error because of a poor representation of the viscoelastic constraining layer [18], mode 2 shows an error on the tail connection, ...

5 CONCLUSION

The use of observation matrices gives a sound theoretical basis extend expansion methods to non trivial test configurations (non coincident test and FEM nodes, non-orthogonal sensors, strain measurements, ...). Combining observation matrices and model reduction gives a good framework to extend traditional modeshape expansion methods and provide computa-

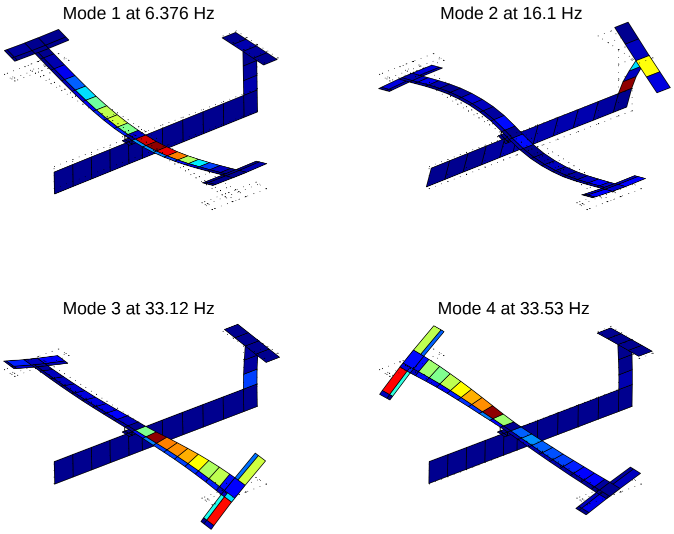

Figure 9: Strain energy distribution of dynamic residual for RBDE-QI with $\alpha = 3\%$

tionally efficient methods.

While all methods work on many cases, some are more robust than others. Modal based methods tend to be sensitive for mode selection. Static expansion gives poor results in configurations with few sensors in certain key areas. Dynamic and minimum residual expansions combine static and modal results and their reduced basis versions have relatively low numerical costs, they thus seem to be the best. One of the advantages of the modal approach was to allow some level of smoothing trough a least-squares formulation. Accepting that measurements are not exact seems a very important factor in any expansion method. The proposed formulation using quadratic inequalities seems a promising possibility but experience on how to set the error level parameter and how to choose a proper norm for the measured deformation still needs to be gained.

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REFERENCES

[1] **Guyan, R.**, *Reduction of Mass and Stiffness Matrices*, AIAA Journal, Vol. 3, pp. 380, 1965.

[2] **Kammer, D.**, *Test-Analysis Model Development Using an Exact Modal Reduction*, International Journal of Analytical and Experimental Modal Analysis, pp. 174–179, 1987.

[3] **O'Callahan, J., Avitabile, P.** and **Riemer, R.**, *System Equivalent Reduction Expansion Process (SEREP)*, IMAC VII, pp. 29–37, 1989.

[4] **Kidder, R.**, *Reduction of Structural Frequency Equations*, AIAA Journal, Vol. 11, No. 6, 1973.

[5] **Kammer, D.**, *A Hybrid Approach to Test-Analysis Model Development for Large Space Structures*, Journal of Vibration and Acoustics, Vol. 113, No. 3, pp. 325–332, 1991.

[6] **Roy, N., Girard, A.** and **Bugeat, L.-P.**, *Expansion of Experimental Modeshapes - An Improvement of the Projection Technique*, IMAC, pp. 152–158, 1993.

[7] **Levine-West, M., Kissil, A.** and **Milman, M.**, *Evaluation of Mode Shape Expansion Techniques on the Micro-Precision Interferometer Truss*, IMAC, pp. 212–218, 1994.

[8] **MacNeal, R.**, Finite Elements: their Design and Performance, Marcel Dekker, Inc., New York, 1994.

[9] **Balmès, E.**, Structural Dynamics Toolbox 3.1 (for use with MATLAB), Scientific Software Group, http://www.sdtools.com, 1998.

[10] **Craig, R. R. J.**, *A Review of Time-Domain and Frequency Domain Component Mode Synthesis Methods*, Int. J. Anal. and Exp. Modal Analysis, Vol. 2, No. 2, pp. 59–72, 1987.

[11] **Géradin, M.** and **Rixen, D.**, Mechanical Vibrations. Theory and Application to Structural Dynamics., John Wiley & Wiley and Sons, 1994, also in French, Masson, Paris, 1993.

[12] **Farhat, C.** and **Géradin, M.**, *On the General Solution by a Direct Method of a Large-Scale Singular System of Linear Equations: Application to the Analysis of Floating Structures*, International Journal for Numerical Methods in Engineering, Vol. 41, pp. 675–696, 1998.

[13] **Balmès, E.**, *Parametric families of reduced finite element models. Theory and applications*, Mechanical Systems and Signal Processing, Vol. 10, No. 4, pp. 381–394, 1996.

[14] **Balmès, E.**, *Efficient Sensitivity Analysis Based on Finite Element Model Reduction*, IMAC, pp. 1168–1174, 1998.

[15] **Golub, G.** and **Van Loan, C.**, Matrix computations, Johns Hopkins University Press, 1983.

[16] **Balmès, E.**, *GARTEUR group on Ground Vibration Testing. Results from the test of a single structure by 12 laboratories in Europe.*, IMAC, pp. 1346–1352, 1997.

[17] **Balmès, E.**, *Predicted Variability and Differences Between Tests of a Single Structure*, IMAC, pp. 558–564, 1998.

[18] **Plouin, A.** and **Balmès, E.**, *A test validated model of plates with constrained viscoelastic materials*, IMAC, pp. 1440–1446, 1999.