PREDICTED VARIABILITY AND DIFFERENCES BETWEEN TESTS OF A SINGLE STRUCTURE

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1 Abstract

Variable models are used to describe structures that change in time, whose properties are not well known, or that are improperly modeled. When the objective of the model is to predict Frequency Response Functions (FRF), variability descriptions typically need to be considered in the physical, modal and FRF parameter spaces. The present study uses simulations of a fairly complex variable finite element model to seek a better understanding of limitations that can be expected in forward and reverse propagation of variability descriptions between those parameter spaces. The finite element model represents the GARTEUR SM-AG-19 testbed so that predicted variabilities can be compared with differences between the test results of participants of this Round-Robin exercise.

2 Introduction

When performing different tests of a single structure, one always finds a certain amount of variability between the results. When the tests are performed using different hardware set-ups (because different teams test the structure or because different configurations of the test equipment are considered) the variations are even more visible but are now also expected since actual modifications were made. In general, such modifications are not fully characterized, so that the test configurations are often compared as if they did not affect the structure. The test results should thus be considered as samples of a stochastic ensemble.

In a more general setting, stochastic models can be used to represent a number of situations.

- The structure can be assumed to change in time because of aging, temperature effects, loading conditions, etc.
- The same model is often used to represent a number of structures that should be identical but typically are not. Thus manufacturing tolerances, residual stresses, changes in welding point positions, etc. are known to significantly modify the properties of cars coming from a single assembly line.
- Material or geometrical properties may not be measurable at all points and so that random distributions of these properties must be assumed. Soils, for example, are typically only characterized through statistical properties.
- Cost considerations often lead to the representation of complex mechanical parts by simple assemblies of beams, plate/shells and solids which need to have *equivalent* properties leading to a similar global behavior of the model rather than being readily related to local material/geometry properties. In many cases the best values of these *equivalent* properties depends, in an unknown fashion, on the objective of the model. These parameters should thus be considered as uncertain. Similar variability should be considered when using mock-ups for complex pieces that may not yet be fully designed.

All those sources of variability can be expected to have similar effects on the response of the structure. The present study thus focuses on a particular application where modeled test conditions are variable assuming that the results found will remain representative of variability found in other cases.

The 12 members of the GARTEUR Structures and Materials Action Group 19 performed a Round-Robin exercise where each participant tested a single representative structure using his own test equipment [1, 2]. A conclusion of the exercise was that test equipment variability was a major factor in explaining the differences between the results found by different participants. The present study thus uses a well refined finite element model of the GARTEUR SM-AG-19 structure with a variable description of loading effects linked to test equipment. Comparisons with the results from tests on the real structure give a good indication of the realism of this variable model.

Predicting the Frequency Response Functions (FRF) in the low frequency range is considered to be the objective of the models. These responses are characterized by properties of low frequency modes which allow the prediction of FRF. After a discussion of variability descriptions in different forms, the study propagates a variability description in physical parameter space into modal and FRF parameter samples using Monte-Carlo simulations. The samples are then used to characterize variability in the FRF and modal parameter spaces. This analysis leads to a better understanding of limitations that can be expected in forward and reverse propagation of variability descriptions in physical, modal and FRF parameter spaces.

3 Describing variability

3.1 Physical parameter space

Structures are typically described by a set of geometrical and material properties which can be translated into a finite element model (mass, stiffness and possibly damping matrices). Variability descriptions in physical parameter space are characterizations of changes of the parameter vector

$$p = p_0 + \Delta p \tag{1}$$

In practice realistic manipulations of models require reparametrization of the model. Usually, one will write the stiffness matrix as a linear combination of elementary constant matrices

$$K(p) = \sum_{j=1}^{NB} \beta_j(p) K_j \tag{2}$$

where, for example, a plate model with variable thickness and Young's Modulus will require three parameters $\beta_1 = Et, \beta_2 = Et^3, \beta_3 = Et^2$ to account for membrane, bending and possibly coupling effects.

It is important to note that software implementations of variable models will much more easily consider the β coefficients as variable than the actual physical parameters since this approach does not require recomputation of the element matrices for each value of the parameters p. Linearized representations of the relation between p and β are of course possible but may have very limited ranges of validity. Note also that the disassembly method [3] that describes $K(p)$ as a product of the form $[C]\left[\backslash \beta(p)_\backslash\right][C]^T$ seems to be another promising approach for the parametrization of variable finite element models.

3.2 Modal parameter space

Models are used to make predictions of the response to applied forces, in other words, to find solutions of the form

$$q(\omega, p) = \left[K(p) - \omega^2 M(p)\right]^{-1} [b]\{u(\omega)\} \tag{3}$$

In most practical applications, the size of the model matrices is such that the inverse of the dynamic stiffness matrix $K(p) - \omega^2 M(p)$ cannot be computed directly and one uses a truncated modal series to approximate the response. Normal modes are solutions of the eigenvalue problem

$$-[M(p)]\{\phi_j(p)\}\omega_j^2(p) + [K(p)]\{\phi_j(p)\} = \{0\} \tag{4}$$

and are traditionally used to project the model. The general form of a projected model is

$$q(\omega, p) = [T(p)]\left[T(p)^T Z(\omega, p) T(p)\right]^{-1}\left[T^T b\right]\{u(\omega)\} \tag{5}$$

but when the vectors of T are chosen to be an incomplete basis of low frequency normal modes $T = [\phi_1(p) \ldots \phi_{NR}(p)]$, the projected dynamic stiffness is diagonal

$$\begin{aligned} q(\omega, p) = \sum_{j=1}^{NR} \frac{\{\phi_j\}\{\phi_j\}^T [b]}{-\omega^2 + \omega_j^2} = \\ [\phi_1 \ldots]\left[-\omega^2 \left[\backslash I_\backslash\right] + \left[\backslash \omega_j^2{}_\backslash\right]\right]^{-1} [\phi_1 \ldots]^T [b] \end{aligned} \tag{6}$$

The traditional approach to describe variability in modal space is to characterize changes on the frequencies and mode shapes

$$\omega_j = (\omega_j)_0 + \Delta(\omega_j) \text{ and } \{\phi_j\} = \{\phi_j\}_0 + \Delta\{\phi_j\} \tag{7}$$

An alternative that will be considered here is to project the model on a fixed basis T. Thus if this basis is given by $T = [\phi_1(p_0) \ldots \phi_{NR}(p_0)]$, the variability will be described using

$$[M_T] = \left[\backslash I_\backslash\right] + \Delta M_T \text{ and } [K_T] = \left[\backslash \omega_j^2{}_\backslash\right] + \Delta K_T \tag{8}$$

One advantage of using a fixed basis description is that variability descriptions in physical parameter space are easily propagated into the fixed basis modal space (which could be called reduced model parameter space) since the full order parametrization (2) can be projected on the basis T leading to

$$T^T K(p) T = \left[\backslash \omega_j^2 \backslash\right] + \Delta K_T = \sum_{j=1}^{NB} \beta_j T^T K_j T \quad (9)$$

In this description, the modes associated to a set of p (or β) parameters are typically estimated using the reduced eigenvalue problem

$$-\left[T^T M(p) T\right]\{\phi_{jR}(p)\}\omega_{jR}^2(p) + \left[T^T K(p) T\right]\{\phi_{jT}(p)\} = \{0\} \quad (10)$$

where the shape defined on the initial degrees of freedom is given by $\{\phi_j\} = [T]\{\phi_{jR}\}$. The use of this reduced eigenvalue problem drastically lowers the computational cost associated with the variability description while retaining a description of the correlation between the variations of the different modal properties (see section 5.2). While the simple projection on the basis of low frequency modes can be considered [4], its accuracy can be questioned and one may want to consider more complex bases adapted to a particular variability description (see Refs. [5, 6] and section 4).

3.3 Frequency response parameter space

For most test cases, the data available for analysis are estimates of the frequency responses at a finite number of points. The traditional way of describing variability on FRFs is to define for each frequency the variability of the amplitude/phase

$$H(\omega) = (|H_0| + \Delta|H|)\, e^{i(<(H_0)+\Delta<(H))} \quad (11)$$

or the real/imaginary parts

$$H(\omega) = H_0(\omega) + \Delta H(\omega) \quad (12)$$

The later description is often considered in robust control applications (in particular for so called μ-synthesis methods [7]).

4 The GARTEUR SM-AG-19 testbed

4.1 About the testbed

During 1995 and 1996, 12 members of the GARTEUR Structures and Materials Action Group 19 tested a representative structure shown in figure 1. The results of this Round-Robin exercise have been publicized in different papers [1, 2] and the current work proposes partial explanations for the variability of those results.

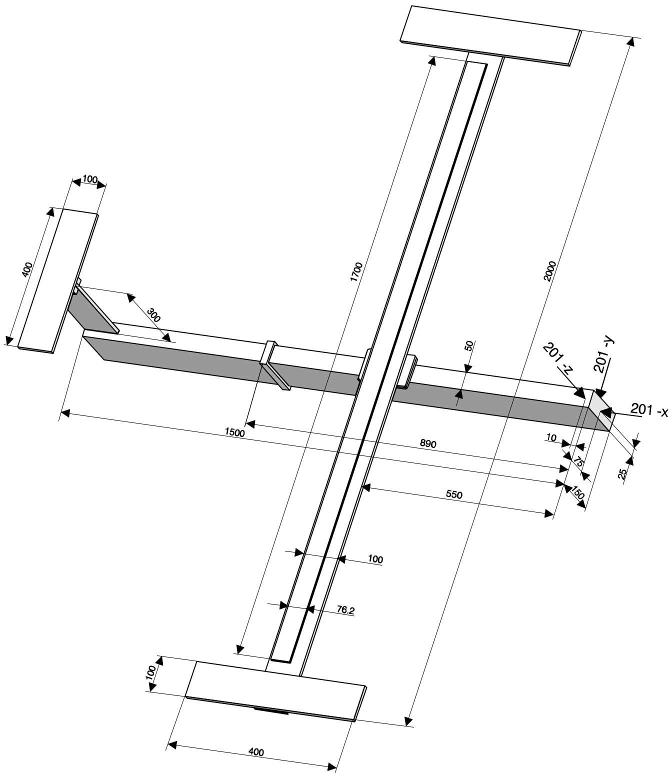

Figure 1: General view of the GARTEUR Structures and Materials -Action Group -19 testbed

4.2 The updated finite element model

The finite element model, used here and shown in figure 2, was created by DLR and refined by the author. This 485 node, 2509 DOF model contains 100 8-node plate/shell elements for the main aluminum and steel parts, 80 beam elements for the suspension and various connections between the main parts, 26 concentrated masses for the sensors and compensation masses.

Table 1 gives indications on the quality of the final test/analysis correlation. One notes good agreement of both modal frequencies and mode shapes (based on MAC at the 24 sensor locations shown in figure 2). Remaining identified problems are the following

- The true wing is not symmetric (its width actually varies between 99 and 100 mm). This significantly affects the wing torsion modes. This difference between the model and reality accounts in good part for the Modal Assurance Criterion values of 81 and 88 found for modes 3 and 4.
- The constraining layer used in the testbed to augment damping levels is difficult to model properly. The 1700x76.2x1mm constraining layer has the 3M ISD-112 viscoelastic work in shear. The actual levels of

shear energy depend on the deformation pattern and the equivalent stiffness for torsion modes should be decreased while not modifying the properties linked to wing bending. The effect of such a modification could not at the time of writing be tested using the Structural Dynamics Toolbox [8] which was used for all computations.

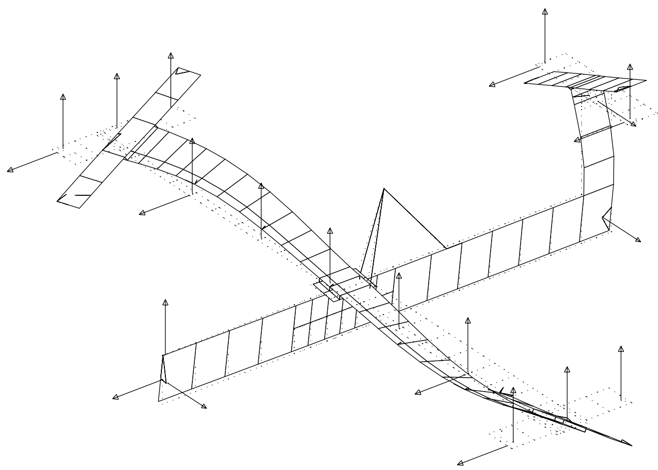

Figure 2: Mode 3 of the finite element model of the GARTEUR SM-AG-19 structure with locations of the 24 required accelerometers

Table 1: Test analysis correlation with experimental results of participant C.

Mode name	ω_{TestC} (Hz)	ω_{FE}	MAC
Two node bending	6.4	6.3	100
Fuselage rotation	16.1	16.2	99
Antisym. wing torsion	33.1	35.7	81
Symmetric wing torsion	33.5	36.1	88
3N wing bending	35.6	37.2	94
4N wing bending	48.4	49.3	100
Inplane wing vs. fuselage	49.4	52.1	98
Sym. inplane wing bend.	55.1	62.8	99

4.3 Description and validation of variable model

Discussions within the GARTEUR SM-AG-19 led to the identification of many sources of variability. Those seen as the most important were retained in the current study and are

- *Value of the additional masses* located at the front of the wing tip bodies. In the actual testing, these masses let participants, who used current driven shakers with no load cell, compensate for the weight of the shaker moving mass which becomes part of the "structure" in such test configurations. The instructions on how to compensate were often misinterpreted which resulted in significant variability. For the simulations, these masses are assumed to be evenly distributed between 160 and 240 grams.
- *Value of the sensor masses* added at the 24 nominal locations. Participants used their own accelerometers and were actually free to use more than the 24 required sensors. As a rough approximation of the sensor loading, the mass loading is assumed to be equal at the 24 nominal sensor locations and with the sensor mass evenly distributed between 5 and 30 grams.
- *Stiffness of the suspension.* In the actual test, a set of common bungees was used but participants were given some freedom on how to attach the bungee connector. Actual heave modes were estimated between 1.8 and 2.7 Hz. Similar variability is achieved here by letting the stiffness of beam model of the bungees vary by a factor 20. A equal distribution of the log of this factor is used for simulations.
- *Shaker position.* A misunderstanding of the test documentation led a number of participants to misplace the wingtip excitation points using the inboard side (where the compensation mass is located) rather than the outboard side (where the required sensor is placed, as shown in figure 2). For FRF predictions in section 5.1, the two positions are taken to be equally likely.

The first step in assessing the validity of this variability model is to look at modal frequencies. As shown in table 2, the variabilities found in the analysis are lower than those found experimentally. Especially for mode 1 (no explanation found) and 6 (shaker stiffness effects that are not taken into account here would significantly increase variability found for this mode). For the three closely spaced modes 3-5, the model seems very realistic.

Table 2: Test and analysis variability of modal frequencies, mean values, standard deviations (in Hz and % of frequency)

#	$\bar{\omega}_T$	$\Delta\omega_T$ Hz	$\Delta\omega_T$ %	$\bar{\omega}_A$	$\Delta\omega_A$ Hz	$\Delta\omega_A$ %
1	6.7	0.22	3.4 %	6.3	0.05	0.9 %
2	16.2	0.15	0.9 %	16.2	0.08	0.5 %
3	33.4	0.63	1.9 %	35.8	0.52	1.5 %
4	33.8	0.65	1.9 %	36.2	0.63	1.7 %
5	35.5	0.54	1.5 %	37.3	0.25	0.7 %
6	48.3	1.30	2.7 %	49.4	0.24	0.5 %
7	49.4	0.68	1.4 %	52.2	0.23	0.4 %
8	54.8	0.78	1.4 %	62.9	0.43	0.7 %

Figures 3 and 4 compare variability of 8 test results and of 20 finite element models selected randomly in the given range. The figures clearly indicate the same trends. The three modes near 35 Hz are significantly affected by test fixtures with noticeable changes in the resonance frequencies and associated shifts in the phase. The variability of driving point position can be seen by the shift in the frequency of the antiresonance near 20 Hz. This effect is even more drastic on other FRFs for which the level of response in the higher part of the frequency range depends significantly on the shaker position.

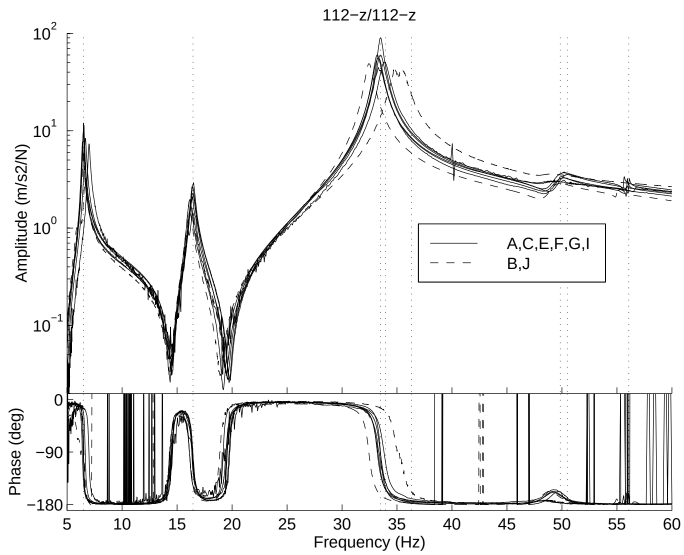

Figure 3: Measured FRFs by 8 of the GARTEUR SM-AG-19 participants

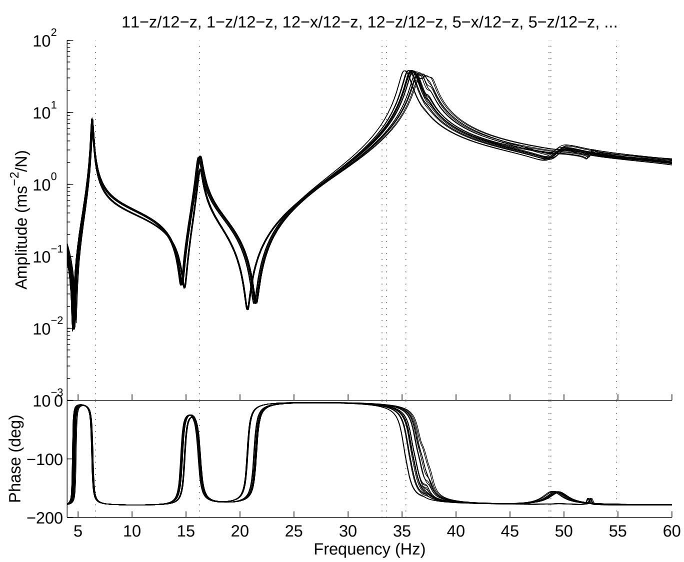

Figure 4: Predicted FRFs for a sample of 20 models

In the present study, forward propagations of variability models are obtained through Monte-Carlo simulations. For the propagation of physical parameter variability, reasonable numerical cost is obtained by introducing a fixed basis reduction before propagation. For more details on this approach see Refs. [5, 6]. The basis considered here contains 25 normal modes of the nominal model complemented by the modeshape sensitivities of the 6 rigid + first 8 flexible modes to the three considered variable parameters ($\partial\phi_j/\partial p_k, j = 1,14, k = 1,3$). The basis thus created actually only contains 59 independent vectors. It can be shown that this approach is actually more accurate than using a fixed basis projection on the first 59 modes of the nominal structure as considered in Ref. [4].

5 Properties of typical variations

5.1 Characterizing variability of FRFs

Assuming that the sample set of transfer functions is representative of actual variations between tests, it appears that traditional FRF variability descriptions are not useful near resonances. Thus, although the variability near the first bending mode seems quite small in figure 4, a zoom on the frequency range shows (see figure 5) that damping is small enough for the shifts of resonant frequency to lead to significant variations of amplitude and phase near the peak.

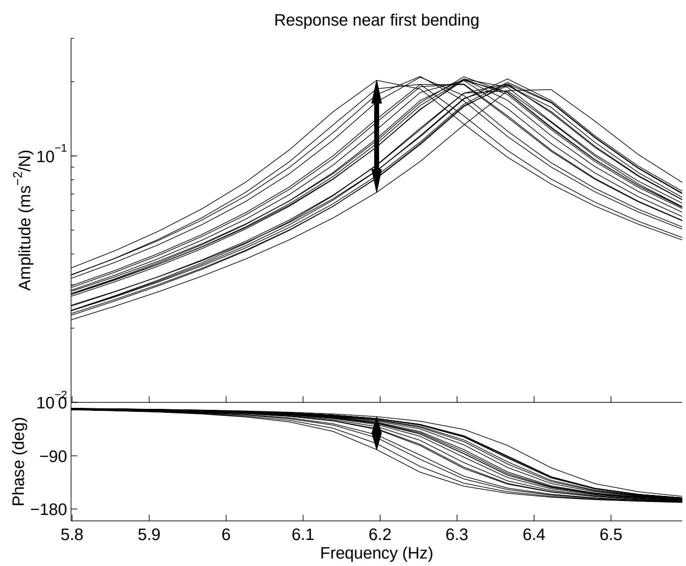

Figure 5: Predicted FRFs near the resonance of mode 1 for a sample of 20 models

Looking at the Bode plot in figure 5, the first idea is to select a variability description at a given frequency point. For the 6.2 Hz point one could describe the amplitude and phase as being in the range shown by arrows in figure 5. On the Nyquist plot shown in figure 6, this translates into a sector. Similarly the unstructured uncertainty model typically used in controls [7] would lead to a description of variability as a circle on the Nyquist plot corresponding to an error of bounded norm on the real and imaginary parts.

Figure 6 shows the variability sector linked to standard deviations in amplitude/phase, the variability circle linked to standard deviations in real/imaginary parts, as well as a dotted line linking the 20 points associated to the chosen frequency. It clearly appears that these characterizations of variability fail to account for the constraint existing between amplitude and phase shifts which leads to the area shown in gray where all the Nyquist plots are located. Note that the effect would be even more pronounced if ranges (as shown in figure 5) rather than standard deviations were used to show the variability sector or circle.

Using the terms of robust control theory, the non parametric uncertainty models (11) or (12) can only give an extremely conservative (unprecise) representation of the parametric uncertainty linked to physical parameter variations.

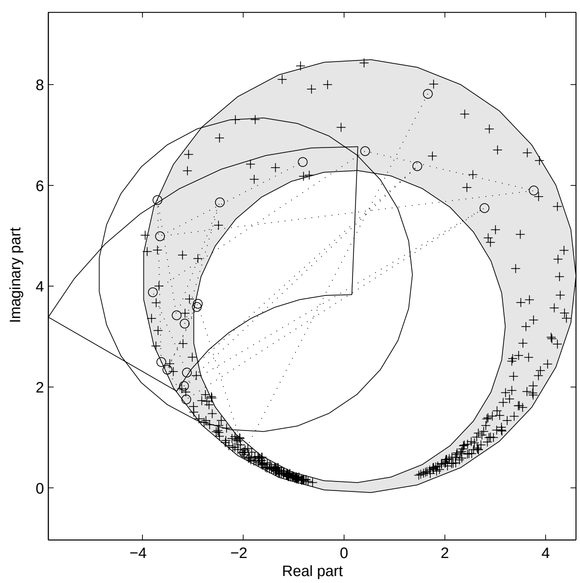

Figure 6: Nyquist plots for a set of 20 models with envelope for all points shown in gray. $(\cdots)$ links all 6.2 Hz responses. (—) envelopes of magnitude/phase or real/imaginary part variability descriptions for a pole at 6.2 Hz

5.2 Characterizing and propagating variability of modal properties

In a first phase, one could consider that the mode shapes are constant and that the variability only affects the modal frequencies. A Monte-Carlo simulation of the resulting responses was performed while focusing in the 35 Hz range. Figure 7 shows the result of a sample of 20 models. It clearly appears that the interactions between the two torsion modes are quite different from those seen in figure 4. In particular, an antiresonance sometimes appears near 36 Hz which significantly lowers the predicted level of response as well as leads to an increase of the phase in the 35.5–36.5 Hz range.

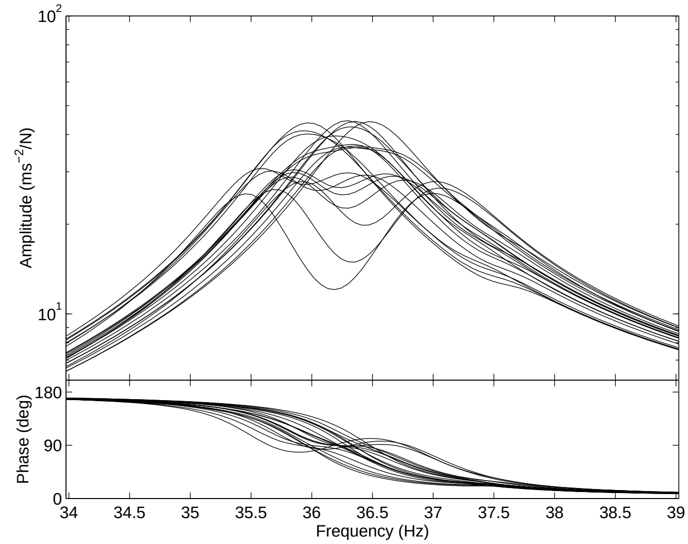

Figure 7: FRF variability associated with propagated modal frequency variability is not representative.

These poor predictions of FRFs are linked to the interactions between the two torsion modes which are close in frequency. The three considered modifications do not actually allow the frequencies of these modes to move independently, as clearly apparent in the fixed basis projection (9) of the full order variability model.

Another way to demonstrate the correlation between variations of different modes is to plot the modal masses at a particular sensor versus modal frequency. Thus figure 8 indicates that the modal mass of mode 5 (3 node bending) at the shaker position 112z (wing tip) changes by more than an order of magnitude (the node is fairly close to a node line which explains the high and sensitive modal mass). The almost constant slope of the lines linking the three points of a given model clearly indicate a strong but fairly difficult to entangle relationship between the properties of the three modes.

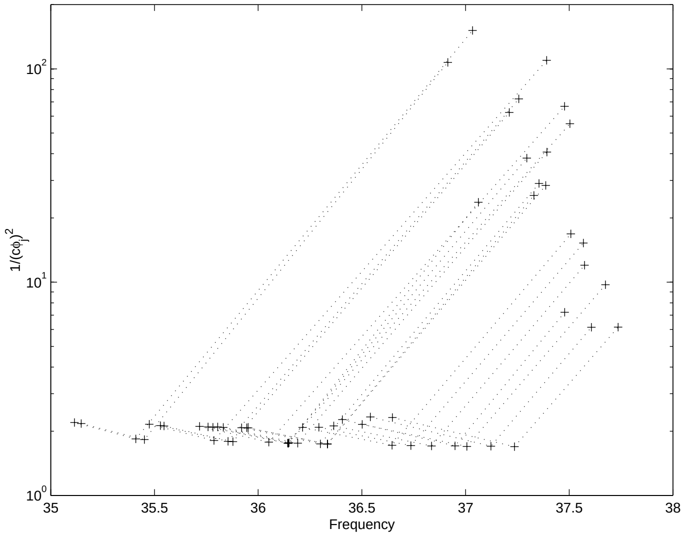

Figure 8: Plot of modal mass at shaker position 112z versus frequency. The dotted line links the two torsion and 3-node bending modes of each model.

6 Conclusion

The illustration trough realistic simulations of a variable model of the GARTEUR SM-AG-19 testbed leads to the following statements.

The parametric nature of variations is essential for a realistic representation of variability of the dynamic behavior of structures. In particular, none of the usual descriptions of FRF variability based on variations of the response at a given frequency point can account for the actual properties that are typical of structural modifications (this is particularly true near resonances and anti-resonances).

The typical variations are not well described by a characterization of frequency and modeshape variability. Given a variable full order model, a reduced representation can be obtained by projection on the fixed basis of nominal low frequency modes (eventually complemented by appropriate corrections [5, 6]). Such projected models are low order and thus useful even for very large full order finite element models, but retain key information on the links existing between variations of the different modeshape and frequencies.

These conclusions given on a forward propagation study give indications on what can be expected for reverse propagation. FRF variability descriptions cannot be expected to be useful to estimate physical parameter variability that is typical of structures. Mode shape variability is also very limited as soon as some of the modes are close to each other. A method to define a fixed basis projection for experimental results seems the missing piece to allow an experimental characterization of variability by other means than doing series of finite element model updates for samples of experimental results.

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