

Modes and regular shapes.
A perspective on CMS theory.

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Sample problems beyond coupling subsystems

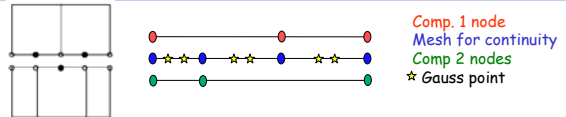
1. Structural Dynamics Modification SDM with added damping
2. Fluid structure interaction (in particular with heavy fluids)
3. Reduce a brake model while keeping
 - all elements of NL contact area
 - exact modes of linear model
4. Design of damping treatment for structure borne transfer
5. Predict response of full shaft as a function of rotation speed, mistuning, temperature



Outline

- Coupling components (substructures)
- Reducing component models
 - Assumptions on spatial and frequency content that underlie a valid reduction
 - Discuss extensions on these assumptions
- Fixed reduction for parametric problems
- Error control on reduced models

Traditional : displacement continuity



Solve with zero relative interface motion

$$\{y_{1\text{int}} - y_{2\text{int}}\} = [c_1 \quad -c_2] \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = 0$$

- Classical : Eliminate constraint
- Lagrange multiplier solution
- Penalize

$$\begin{bmatrix} Z(s) & c_{\text{int}}^T \\ c_{\text{int}} & 0 \end{bmatrix} \begin{Bmatrix} q \\ \lambda \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix}$$

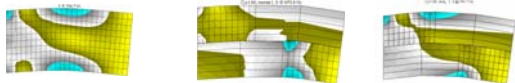
$$\left(\begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix} + \begin{bmatrix} c_1^T \\ -c_2^T \end{bmatrix} \begin{bmatrix} I \\ \epsilon \end{bmatrix} [c_1 \quad -c_2] \right) \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = [b] \{u(s)\}$$

R. R. Craig : Review of time domain and frequency domain CMS methods (1987)

A measure of compatibility

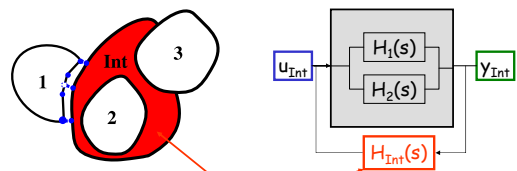
- Compatibility of a displacement $C_i(u_i) = \frac{\|\Pi_1^T u_i\|}{\|u_i\|} \in [0,1]$
- Projection characterized by scalar product $\langle q_1, q_2 \rangle_v$ given by matrices A_{11}, A_{21}, A_{22}
- $1-\epsilon$ compatibility found by solving Rayleigh Quotient

$$(1 - \epsilon(v_1))^2 = \frac{\{q_1\}^T [A_{21}]^T [A_{22}]^{-1} [A_{21}] \{q_1\}}{\{q_1\}^T [A_{11}] \{q_1\}}$$



- H. Ben Dhia, E. Balmès, Giens 2003
- Continued work volume continuity (Arlequin, partition of unity)

CMS principles



1. Reduced component model
2. Coupled by

$$\begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \{u(s)\}$$

$$\{y\} = [c_1 \quad c_2] \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}$$

- Continuity constraint
- Physical interface

Stiffness coupling

- Interface motion $\{y_j(X, s)\} = [c_{j\text{int}}(X)] \{q_j(s)\}$



- Interface stiffness

$$\begin{bmatrix} Z_{j1\text{int}} & \dots & Z_{jn\text{int}} \\ \vdots & \ddots & \vdots \\ Z_{j1\text{int}} & \dots & Z_{jn\text{int}} \end{bmatrix} \begin{Bmatrix} \{c_{j1}\} \{q_j\} \\ \vdots \\ \{c_{jn}\} \{q_j\} \end{Bmatrix} = \begin{Bmatrix} F_{j1\text{int}} \\ \vdots \\ F_{jn\text{int}} \end{Bmatrix}$$

- Coupled equations

$$\left(\begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix} + \begin{bmatrix} c_1^T & 0 \\ 0 & c_2^T \end{bmatrix} Z_{\text{int}} \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \right) \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}$$

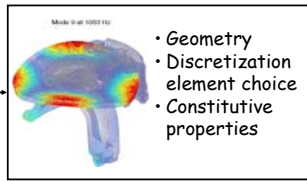


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Models of structures

$\sigma(x,t)$
 $f(x,t)$



- Geometry
- Discretization element choice
- Constitutive properties

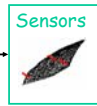
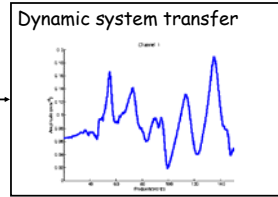
$u(x,t)$
 $\sigma(x,t)$

Reduction based on restrictions:
 •Excitation (space & freq)
 •Responses
 •Coupling ...

- CMS, Optimization
- Coupling (fluid, control), variability, non linearity

Dynamic system models

When
Where

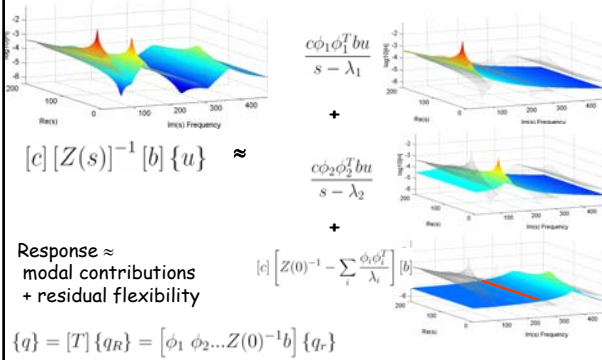


Reduction based on restrictions:
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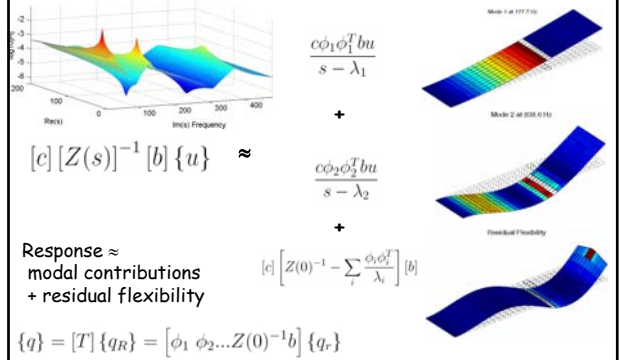
- Coupling (structure, fluid, control)
- Optimization, variability, damping, non linearity, ...

Sensors
Classical CMS
Extensions

MS 1 : Modes and residual terms



MS 2 : Modes and residual terms



MS 3 : Ritz analysis

• **Hypothesis** : Approximation of solution in subspace generated by **T** (size $N \times N_R$):

$$\{q\}_N = \begin{bmatrix} T \\ \vdots \\ \vdots \\ \vdots \end{bmatrix} \{q_R\}_{N_R}$$

• **Ritz-Galerkin approximation** :

$$[T^T M T s^2 + T^T C T s + T^T K T]_{N_R \times N_R} \{q_R(s)\} = [T^T b]_{N_R \times N_A} \{u(s)\}_{N_A \times 1}$$

$$\{y(s)\}_{N_S \times 1} = [cT]_{N_S \times N_R} \{q_R(s)\}_{N_R \times 1}$$

MS 4 : reduction bases

Reduction based on restrictions :

- **Excitation**
 - space
 - frequency
 - **Responses**
 - **Coupling ...**
- **Classical bases in CMS** :
- Free modes
 - Flexibility : static response to unit load
 - Constraint modes (Guyan) static response to unit displacement
 - Fixed interface modes

Proposed perspective :

- Any representative response to unit load/displacement will give **space validity**
- Any representative mode will give **frequency validity**
- Residual can be used to evaluate **error**
- Basis valid for **parametric range**

Redefining unit displacement

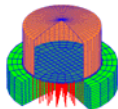
Guyan : static response to **unit displacements** on the interface

- 1 vector/DOF : often to costly
- Polynomial functions : not general
- Modes of an interface model



Interface model

- Condensation on the interface (Craig 72)
- Elements connected to the interface
- Any representative model (**mechanical SVD**)



Application : soil/structure interaction MSSMat

Redefining interface

Sensors can be considered as **interface**

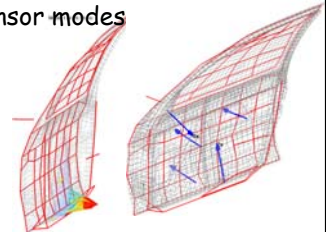
$$\{y\} = [c_{int}] \{q\}$$

Sample use fixed sensor modes

$$[K - \omega_j^2 M] \{\phi_j\} = \{0\}$$

With

$$[c_m]_{m \leq k} \{\phi_j\} = 0$$



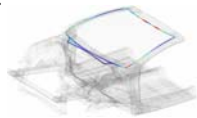
Use : place additional sensors to extend frequency band (IMAC 05)

Redefining unit load

Damped viscoelastic resp.

$$[Z(E(s), s)] \{q\} = \{F\} \text{ rewritten as}$$

$$[Z(E_o, s)] \{q\} = \{F\} - [\sum (E_o - E_n) / E_n \text{Im}(Z - Z_o)] \{q\}$$



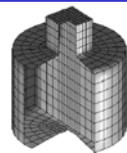
Basis contains

- **Modes** to represent nominal resonances
- **Flexibility** to viscoelastic loads associated with nominal modes

$$T = \begin{bmatrix} \phi_{1:NM} & K_o^{-1} \text{Im}(Z - Z_o) \phi_{1:NM} \end{bmatrix}$$

Modes
static response to unit load

Enhanced reduction : heavy fluid



300 Hz : 152 solid, 32 fluid modes

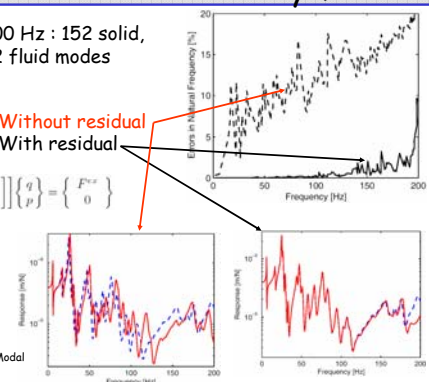
Without residual
With residual

$$\begin{bmatrix} M & 0 \\ C^T & K_p \end{bmatrix} s^2 + \begin{bmatrix} K(s) & -C \\ 0 & F \end{bmatrix} \begin{Bmatrix} q \\ p \end{Bmatrix} = \begin{Bmatrix} F^{ext} \\ 0 \end{Bmatrix}$$

$$\tilde{J}^S = [T^S [K_o]^{-1} [C] [T^F]]$$

$$\tilde{T}^F = [T^F [F_o]^{-1} [C]^T [T^S]]$$

Thanks : DGA/CTS
NASTRAN DMAP by N. Roy TopModal



Redefining modes

- Exact modes in **reduced area** & **dependent DOFs** $[\phi_j, q_0]$
- No reduction of DOFs **internal to contact area**
- Static response (initial state) : ABAQUS result exactly within contact area
- Reduced model with exact system modes

SDTools/Bosch Time domain squeal simulation
800e3 DOFs, 500e3 time steps

Redefining modes

- Nominal modes & component redesign
- Use cyclic/periodic solutions

Parametric families

Reanalysis $[[T^T K(p) T] - \omega_j^2 R(p) [T^T M(p) T]] \{\phi_{jR}(p)\} = \{0\}$

Basis can be fixed for range of parameters

- Modes $T = [\Phi(p_0)]$
- Modes + sensitivities $T = [\Phi(p_0) \partial \Phi(p_0) / \partial p]$
- Multi-model $T = [\Phi(p_1) \Phi(p_2)]$

Error control

Example : compute modes

$$[K - \omega_j^2 M] \{\phi_j\} = \{0\}$$

Approx solution $T^{(k)T} [K - \omega_j^2 M] T^{(k)} \{\phi_{j,R}\} = 0$

Load residual $R_{L,j}^{(k)} = [K - \omega_j^2 M] T^{(k)} \{\phi_{j,R}\} \neq 0$

Displacement residual $\{R_{D,j}^{(k)}\} = [K]^{-1} \{R_{L,j}^{(k)}\}$

Error evaluation $\epsilon_j^{(k)} = \frac{\|\{R_{D,j}^{(k)}\}\|_K}{\|\{T^{(k)}\} \{\phi_{j,R}\}\|_K}$

Enrichment $\epsilon_j^{(k)} > Tol \Rightarrow T^{(k+1)} = [T^{(k)}, \{R_{D,j}^{(k)}\}]$

Residue iteration (Bobillot 2002)

Error control

- Motivates existing reduction methods
- Shows strong relation with pre-conditioned conjugate gradient methods
- Provides straightforward extension mechanism

Conclusion

Coupling

- Continuity
- Stiffness

Dynamic system transfer

When

Where

Sensors

Classical CMS

Extensions

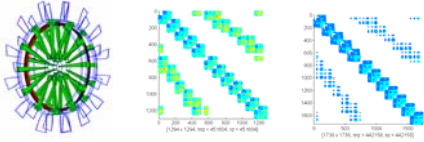
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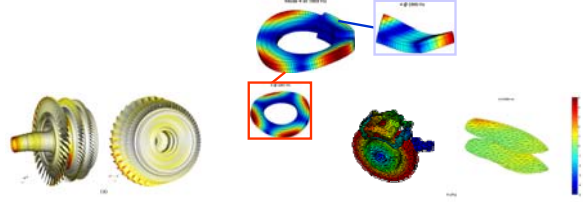
Current trends

- Non-linear contact/friction simulations
- Parametric studies associated with
 - Design and robustness (stochastic models)
 - Temp. & freq. viscoelastic dependence
 - Rotation speed
- Reduction & sparsity

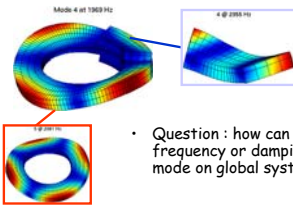


Current trends

- Coupled physics (Acoustics, MEMS, thermoelastic, ...)
- Component redesign
 - Specification on component frequencies
 - Design & restitution



Model reduction to analyze component effects



- Question : how can I see the effect of frequency or damping change of component mode on global system mode