Damping and complex modes.

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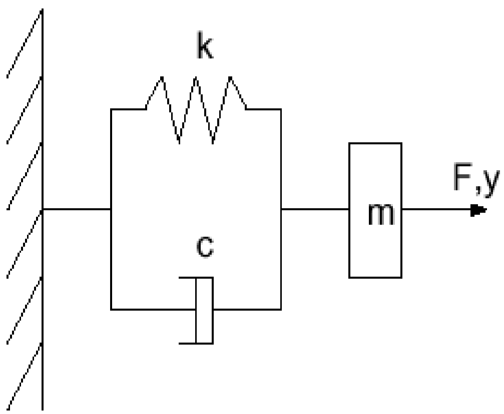
Outline

- Modes real and complex
  - Spectral decomposition
  - When are modes real ?
  - Proportional damping
  - Real from complex test modes
  - Complex non modes
- Damping modeling
  - Material, Joint, …
  - Damped FEM models
  - Local/system model

Mode ≈ 1 DOF system

$$m\ddot{q}(t) + c\dot{q}(t) + kq(t) = F(t)$$

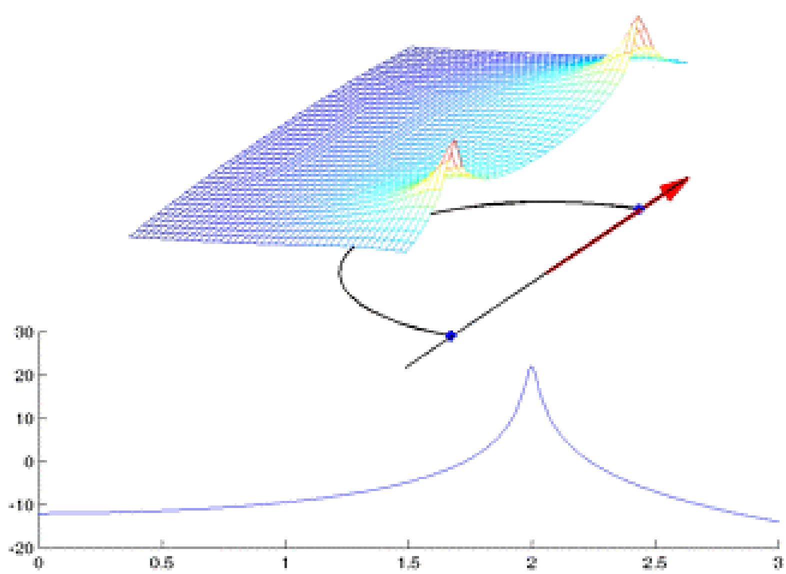
$$H(\omega) = \frac{q(\omega)}{F(\omega)} = \frac{1}{-\omega^2 m + i\omega c + k}$$



1 DOF : influence of damping

$$H(s)=\frac{1}{s^2m+cs+k}=\frac{1}{m}\left(\frac{\beta}{s-\lambda}+\frac{\bar{\beta}}{s-\bar{\lambda}}\right)$$

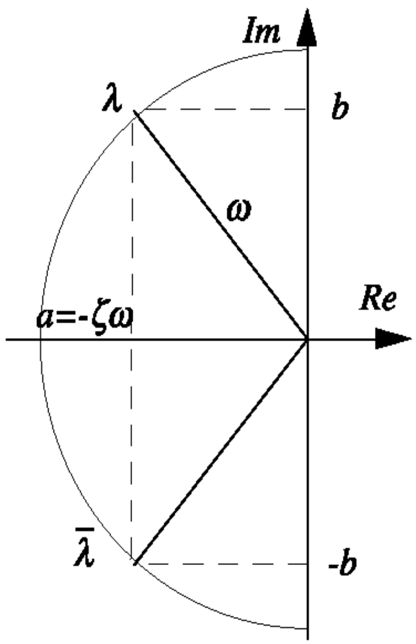
1 % damping



1 DOF : frequency domain

$$H(s) = \frac{1}{s^2 m + cs + k} = \frac{1}{m}\left(\frac{\beta}{s-\lambda} + \frac{\bar\beta}{s-\bar\lambda}\right)$$

$$\beta = \frac{1}{i\omega\sqrt{1-\zeta^2}}$$

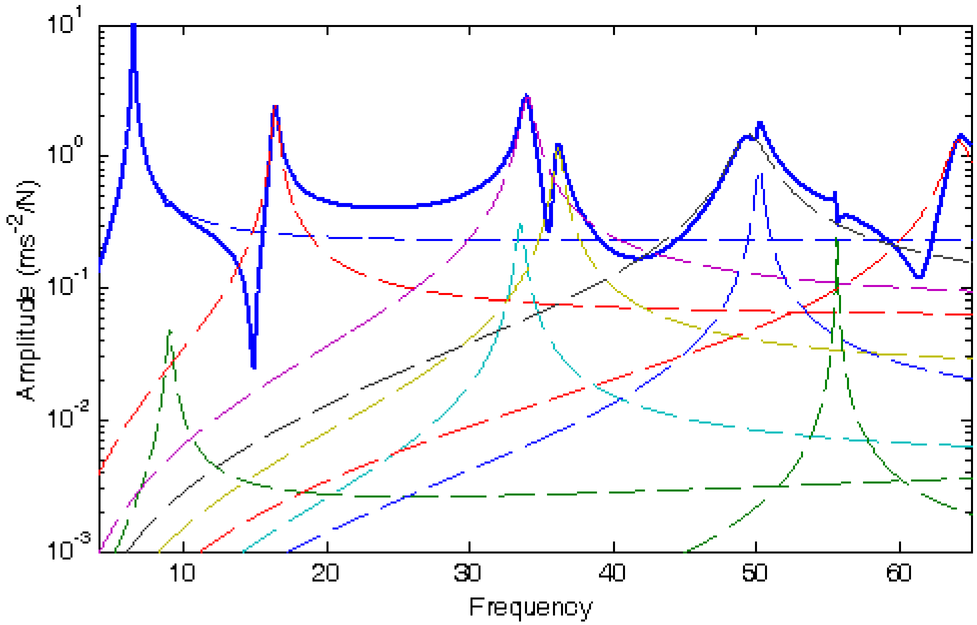


$$\lambda = -\zeta\omega_n \pm i\omega_d \quad , \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\omega_n = \sqrt{k/m} \quad , \quad \zeta = \frac{c}{2\sqrt{km}}$$

1 DOF system (single mode for mechanical system)
has
2 complex conjugate poles (linear system modes)

MDOF SISO system



MDOF multiple degree of freedom
SISO single input single output

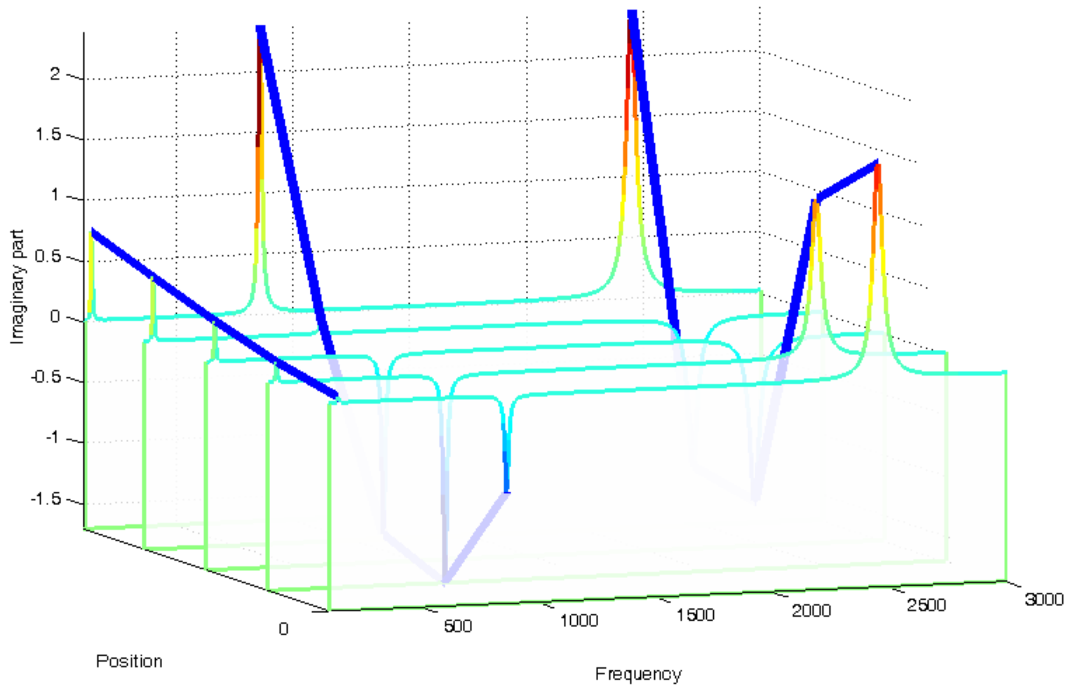
Spectral decomposition

MDOF more than 1 pole

SISO Rj is 1x1

$$\sum_{j \in \text{identified}} \left(\frac{[R_j]}{s-\lambda_j} + \frac{[\bar{R}_j]}{s-\bar{\lambda}_j} \right)$$

MDOF MIMO system



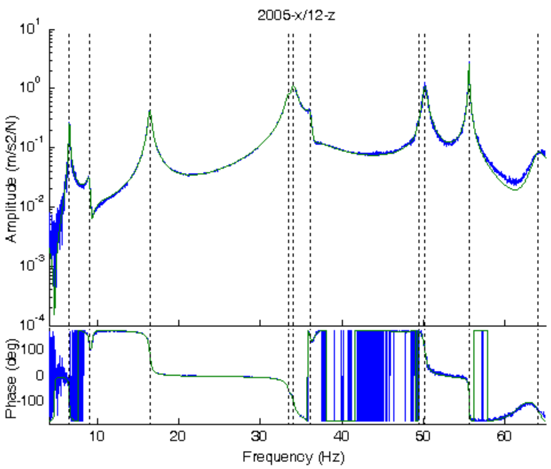
$$H(\omega) = \sum_{j=1}^{N} \left(\frac{[R_j]_{NS \times NA}}{i\omega - \lambda_j} + \frac{[\bar{R}_j]_{NS \times NA}}{i\omega - \bar{\lambda}_j} \right)$$

- Sames poles for each IO pair
- Residue matrix in spectral decomposition

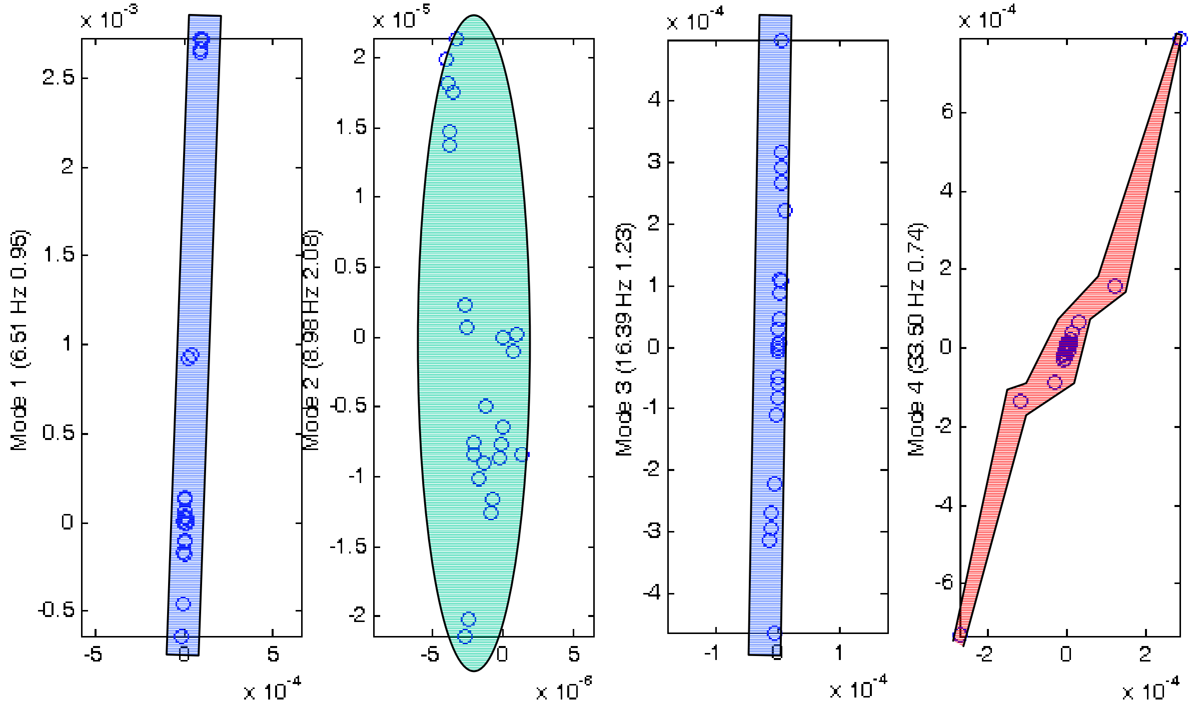
Modes
Spectral decomposition
When real ?
Proportional damping
Real from complex
Complex non modes

Damping
Material, Joint, …
Damped models
Local/system model

Experimental modes are often “real”



Garteur SM-AG19 test



“Real modes”	Residues for I/O pairs line up
“Complex modes”	Residues have a phase spread
Poor modes	Have complex residues

Modes of linear system

- Nominal state-space form

$$\{\dot{x}(t)\} = [A]\{x(t)\} + [B]\{u(t)\}$$
$$\{y(t)\} = [C]\{x(t)\} + [D]\{u(t)\}$$

- Left and right eigenvalue problems

$$[A]\{\theta_{jR}\} = \lambda_j \{\theta_{jR}\}$$

$$\{\theta_{jL}\}^T [A] = \{\theta_{jL}\}^T \lambda_j$$

Modes of linear system

- Mode shape orthogonality and scaling conditions

$$[\theta_L]^T[A][\theta_R] = [\backslash \Lambda_\backslash] \quad \text{and} \quad [\theta_L]^T[\theta_R] = [\backslash I_\backslash]$$

- Diagonal state-space model

$$\{p\}s = [\backslash \Lambda_\backslash]\{p\} + [\theta_L^T B]\{u(s)\}$$
$$\{y(s)\} = [C\theta_R]\{q(s)\} + [D]\{u(s)\}$$

Mode shape

Participation factor

Modal coordinates, state-space

- Inverting diagonal state-space leads to the spectral decomposition

$$H(\omega) = \sum_{j=1}^{N} \left(\frac{[R_j]_{NS\times NA}}{i\omega - \lambda_j} + \frac{[\bar{R}_j]_{NS\times NA}}{i\omega - \bar{\lambda}_j} \right)$$

$$[R_j]_{NS\times NA} = \{[C]\{\theta_{jR}\}\}_{NS\times 1} \{\{\theta_{jL}\}^T [B]\}_{1\times NA}$$

Residue | Mode shape | Participation factor

- Residues of linear systems have no reason to have a single phase ("be real")

Normal modes of elastic structure

- Nominal model (elastic + viscous damping)

$$\left[Ms^2 + Cs + K\right]\{q(s)\} = [b]\{u(s)\}$$
$$\{y(s)\} = [c]\{q(s)\}$$

- Conservative eigenvalue problem

$$-[M]\{\phi_j\}\omega_j^2 + [K]_{N\times N}\{\phi_j\}_{N\times 1} = \{0\}_{N\times 1}$$

- M>0 & K≥0 ⇒ ϕ real

Normal modes of elastic structure

- Orthogonality
- Scaling conditions
  - Unit mass
  - Unit amplitude
- Principal coordinates

$$[\phi]^T[M][\phi] = \left[\backslash \mu_j \backslash\right]$$

$$[\phi]^T[K][\phi] = \left[\backslash \mu_j \omega_j^2 \backslash\right]$$

$$\{\phi_j\}^T[M]\{\phi_j\} = 1$$

$$[c_s]\{\tilde{\phi}_j\} = 1 \quad \mu_j(c_s) = \left([c_i]\{\phi_j\}\right)^{-2}$$

$$\left[[I]s^2 + [\Gamma]s + \left[\backslash \omega_j^2 \backslash\right]\right]\{p(s)\} = \left[\phi^T b\right]\{u(s)\}$$

$$\{y(s)\} = [c\phi]\{p(s)\}$$

Modal damping assumption

- Assume Γ diagonal

$$[\Gamma] = \left[\phi^T C \phi\right] = \left[{}^{\backslash} 2\zeta_j\omega_j {}_{\backslash}\right]$$

- Leads to second order spectral decomposition

$$H(s) = \sum_{j=1}^{N} \frac{[c]\{\phi_j\}\{\phi_j\}^T[b]}{s^2 + 2\zeta_j\omega_j s + \omega_j^2} = \sum_{j=1}^{N} \frac{[T_j]}{s^2 + 2\zeta_j\omega_j s + \omega_j^2}$$

Mode shape — Participation factor — Residue

Modal damping assumption

This is the only widespread damping model

Why ?

- Compellingly practical
- Easy combination of test and analysis
- Sufficient mathematical conditions
  - Rayleigh $[C] = \alpha[M] + \beta[K]$
  - Caughey $[C] = \sum \alpha_{k,l} [M]^k [K]^l$

Modal also called **proportional damping**

- Often induces small modifications in behaviour

Modes
Spectral decomposition
When real ?
Proportional damping
Real from complex
Complex non modes
Damping
Material, Joint, …
Damped models
Local/system model

Spectral decompositions

- General linear system

$$\frac{[R]}{s-\lambda}+\frac{[\bar{R}]}{s-\bar{\lambda}} = 2\frac{(s\mathrm{Re}(R)) + (-\mathrm{Re}(\lambda)\mathrm{Re}(R) - \mathrm{Im}(\lambda)\mathrm{Im}(R))}{s^2-2(\lambda+\bar{\lambda})s+\lambda\lambda}$$

- Structure with modal damping

$$\frac{[T]}{s^2+2\zeta\omega s+\omega^2} = \frac{T/(i\mathrm{Im}(\lambda))}{(s-\lambda)} + \frac{T/(i\mathrm{Im}(\bar{\lambda}))}{(s-\bar{\lambda})}$$

Modal damping ⇔ R is imaginary

When are modes real / complex ?

Vibrating structures that are elastic, linear, and time invariant have real modes.

Complex modes are found for

- Damped structures — Non modal
- Non linear systems
- Time varying systems

  } Resonances rather than modes
- Periodic structures — Mathematical trick

When are complex modes nearly real ?

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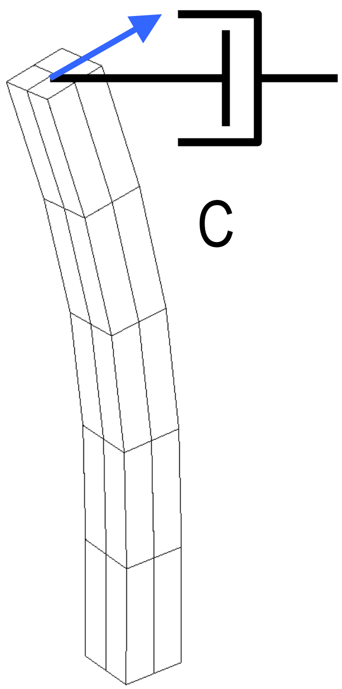
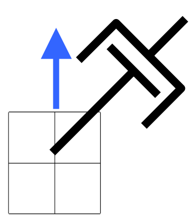
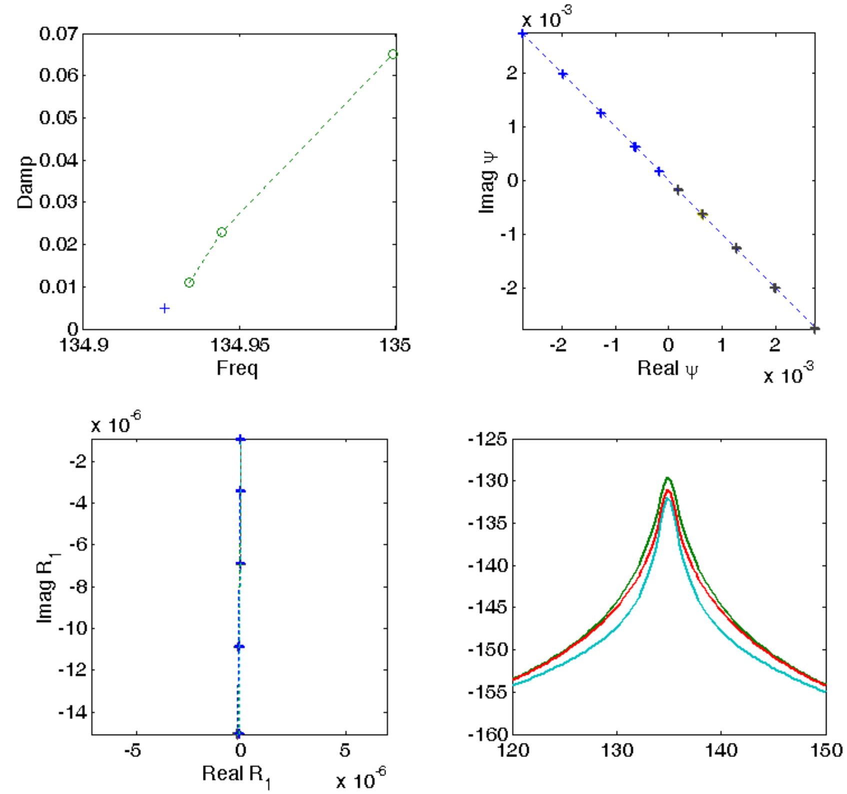
Square beam 1



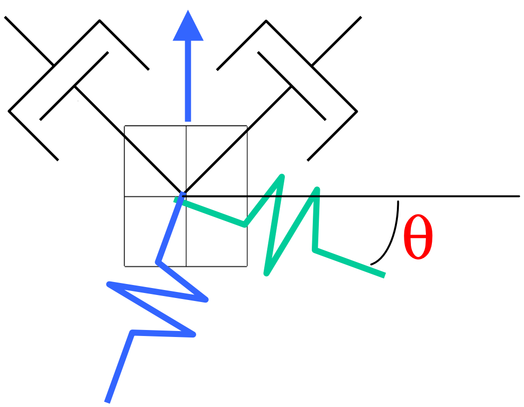


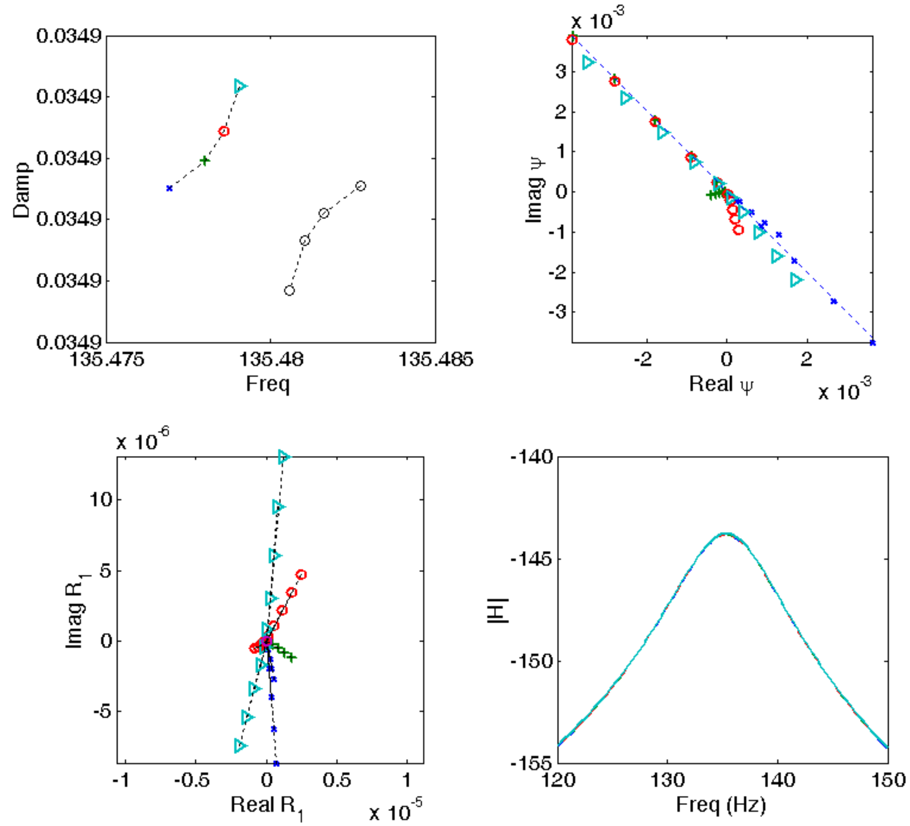


- One tip damper with variable C
- 1 pole moves
- modes remain nearly real

A case with complex modes

- C=1e3 N/m/s
- K1=1.105e5 N/m
- K2=1.095e5 N/m
- θ=0,20,30,40°

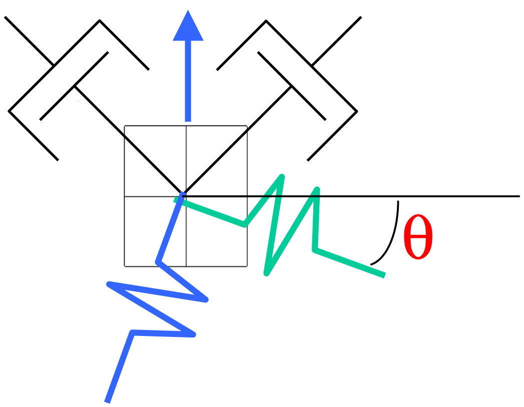


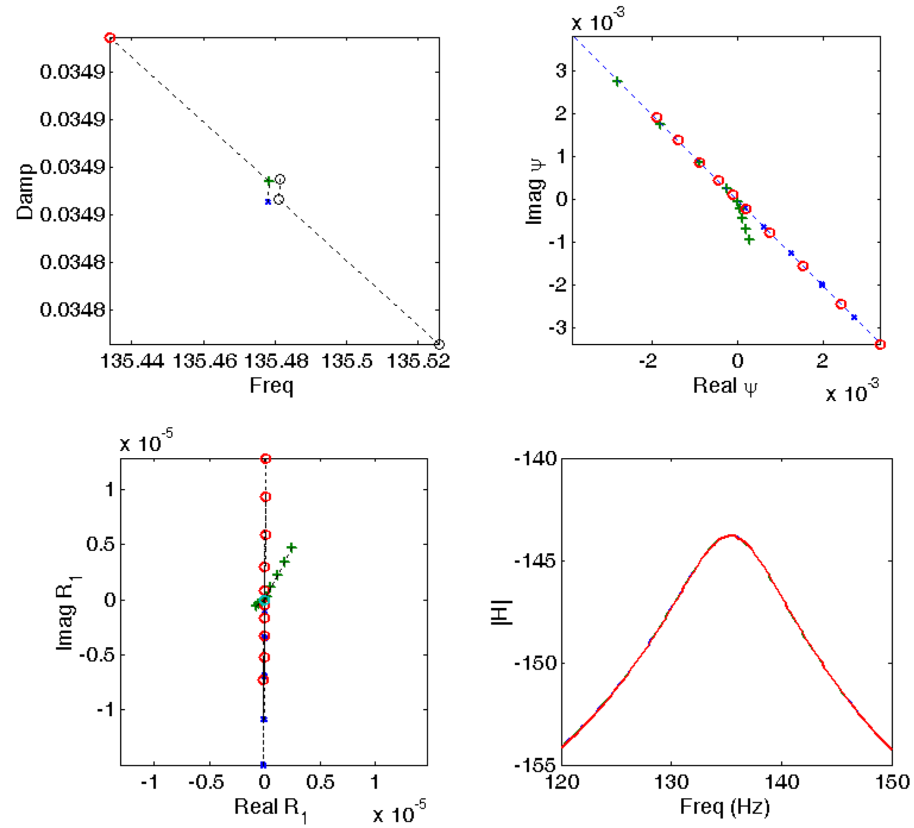


- Very close frequencies
- Damping and stiffness are not proportionnal

Losing mode complexity

- C=1e3 N/m/s
- K1,2=1.1+dk N/m
- K2=1.1-dk N/m
- θ= 22º





Different values of frequency separation ⇒ Modes nearly real

When are complex modes nearly real ?

- Coupling of two modes by viscous damping

$$\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} s^2 + \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} s + \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix}\right) \begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \{u(s)\}$$

$$p_1 = (1+e_1)^{-1} \frac{b_1 u}{(s^2 + \gamma_{11} s + \omega_1^2)} + e_2$$

Mode 1 response
Modal damping

$$e_1 = \frac{\gamma_{12}\gamma_{21} s^2}{(s^2 + \gamma_{11} s + \omega_1^2)(s^2 + \gamma_{22} s + \omega_2^2)}$$

Perturbations for
non-modal damping

$$e_2 = \frac{\gamma_{12} s b_2 u}{(s^2 + \gamma_{11} s + \omega_1^2)(s^2 + \gamma_{22} s + \omega_2^2)}$$

When are complex modes nearly real ?

- Uncoupling criterion (Hasselman) e_i<<1 ⇔

$$\min(\zeta_1\omega_1, \zeta_2\omega_2)/|\omega_1 - \omega_2| \ll 1$$

corresponds to non overlap of peaks

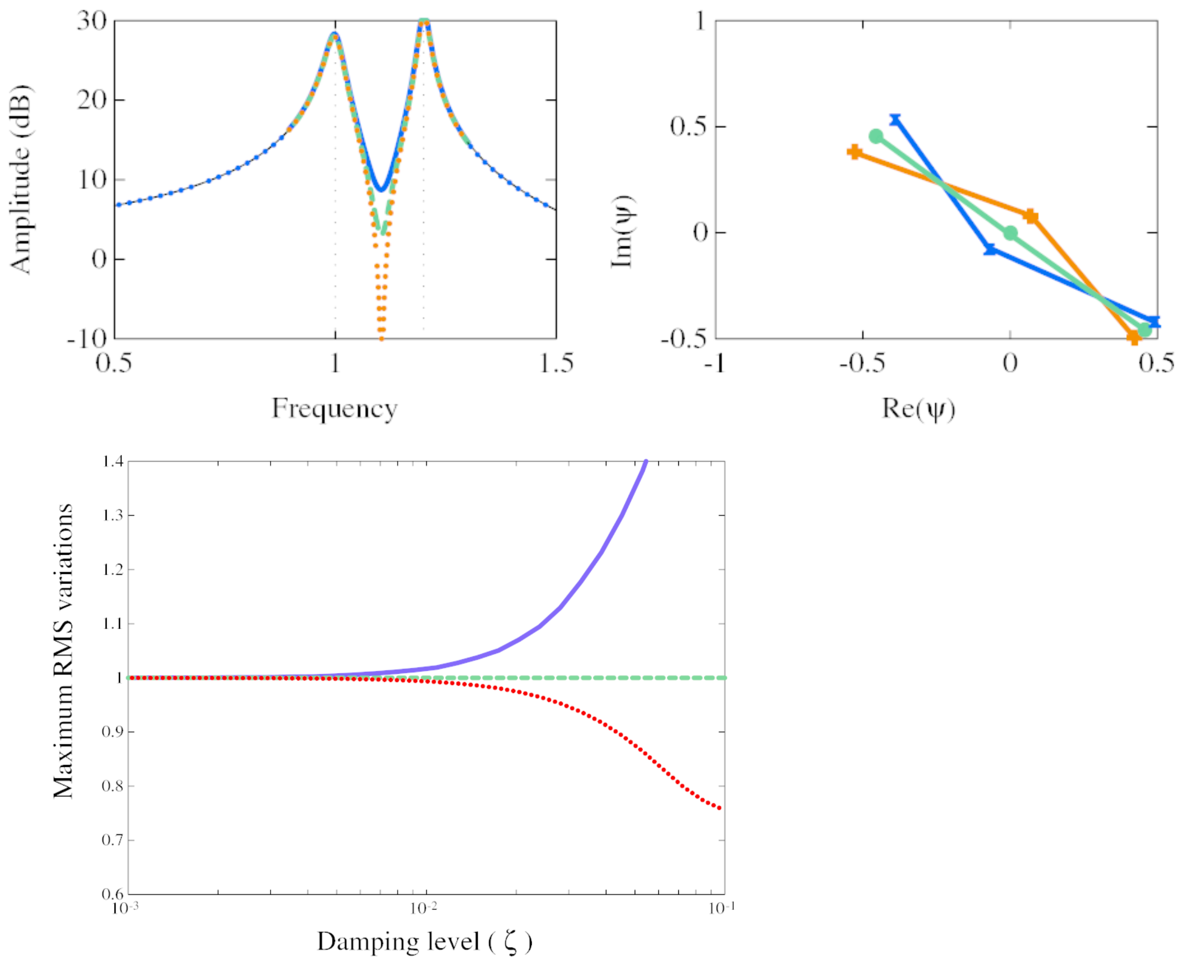
- Proof based on damping matrix positiveness

$$\gamma_{12}\gamma_{21}/(\gamma_{11}\gamma_{22}) < 1$$

- Generalization uncoupling by block (Balmes 97)

Two mode coupling

γ_{12} min, 0, max



- Influence on zero & RMS response
- Influence on modeshape complexity
- Effect only significant for high damping

First conclusion

Modal damping is a good assumption :

- Provided low modal overlap

$$\min(\zeta_1\omega_1, \zeta_2\omega_2)/|\omega_1 - \omega_2| \ll 1$$

- Errors on predicted levels are small
- Assuming real modes is then OK

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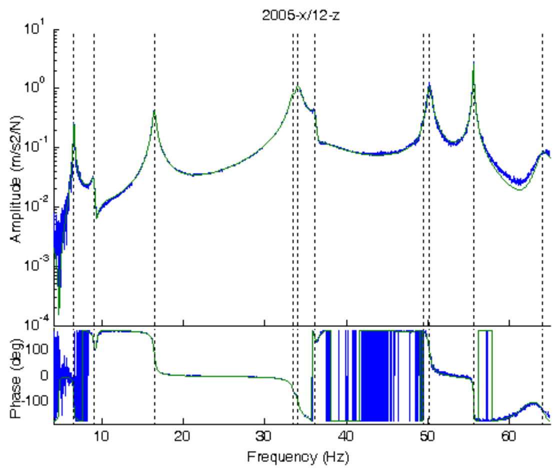
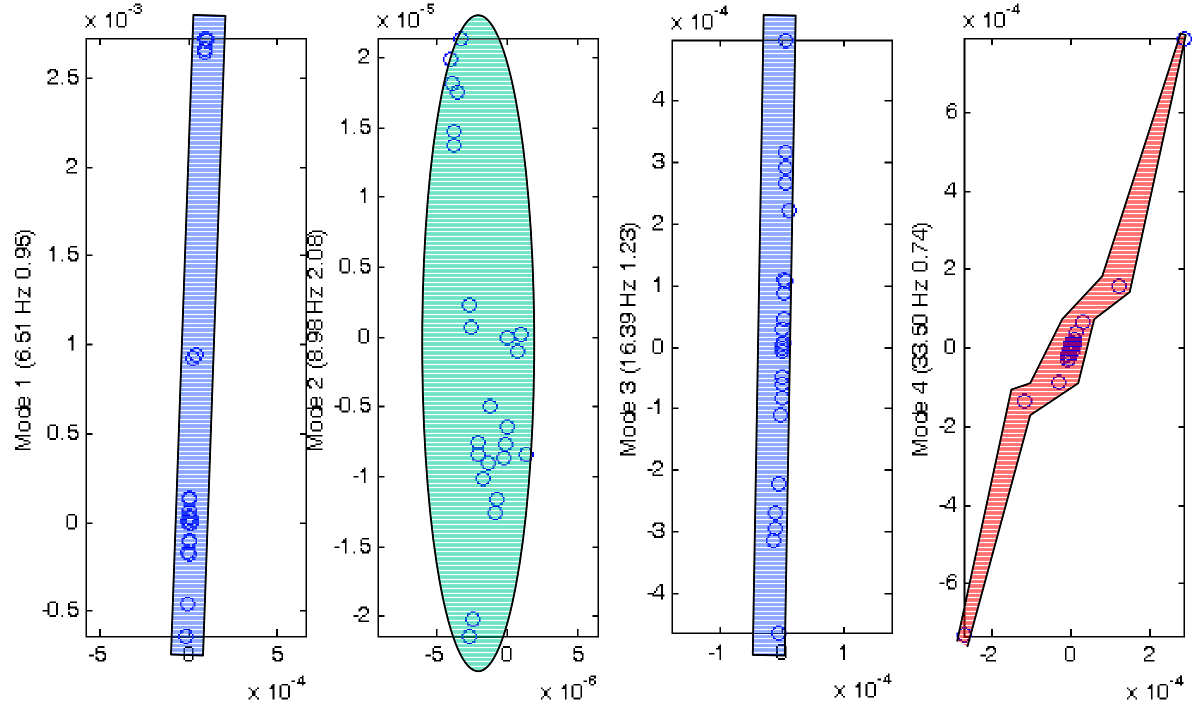
Experimental modes are often “real”



Garteur SM-AG19 test



“Real modes”	Residues for I/O pairs line up
“Complex modes”	Residues have a phase spread
Poor modes	Have complex residues

Real from complex

$$\lim_{s \to \infty} H(s) = \lim_{s \to \infty} [c]\left[Ms^2 + Cs + K\right]^{-1}[b] = O(1/s^2)$$

- Complex modes of second order system verify

$$\sum_{j=1}^{2N} \tilde{\psi}_j \tilde{\psi}_j^T = \tilde{\psi}_{N\times 2N} \tilde{\psi}_{N\times 2N}^T = [0]_{N\times N}$$

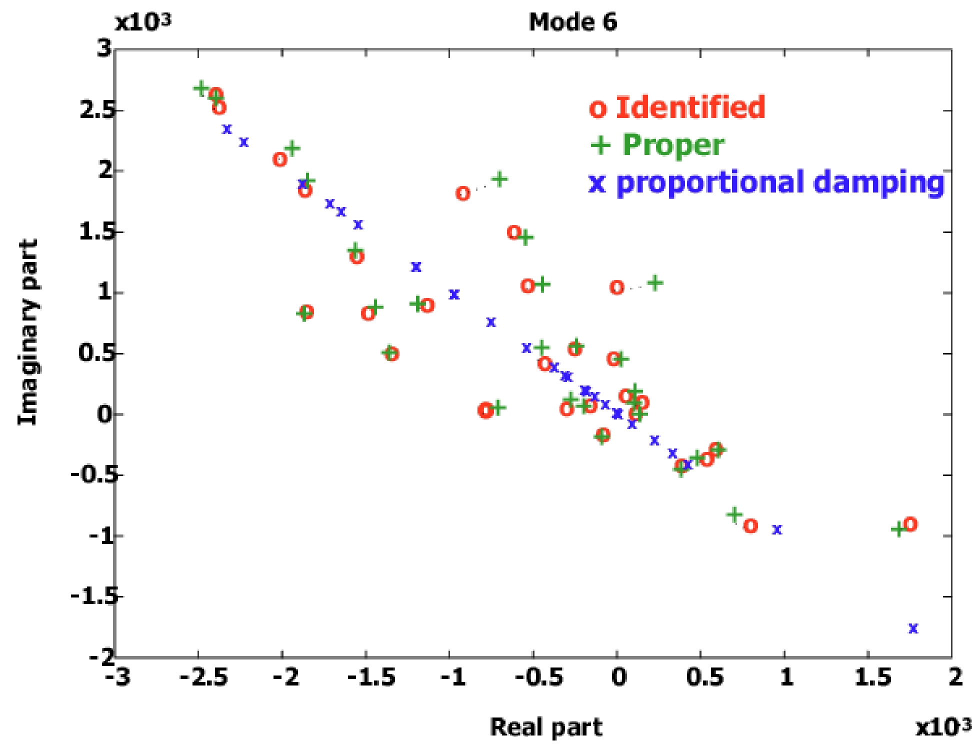
- If properness condition verified

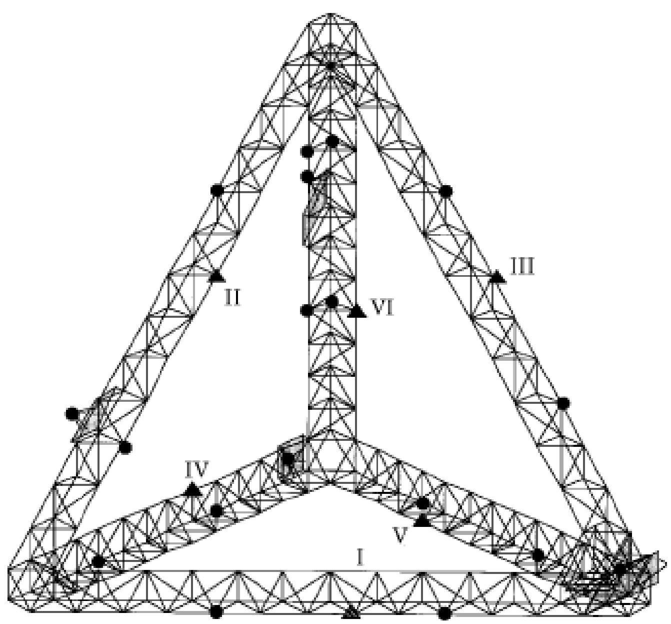
$$M = \left(\tilde{\psi}\Lambda\tilde{\psi}^T\right)^{-1}$$

$$K = \left(\tilde{\psi}\Lambda^{-1}\tilde{\psi}^T\right)^{-1}$$

$$C = -M\tilde{\psi}\Lambda^2\tilde{\psi}^T M$$

Real from complex





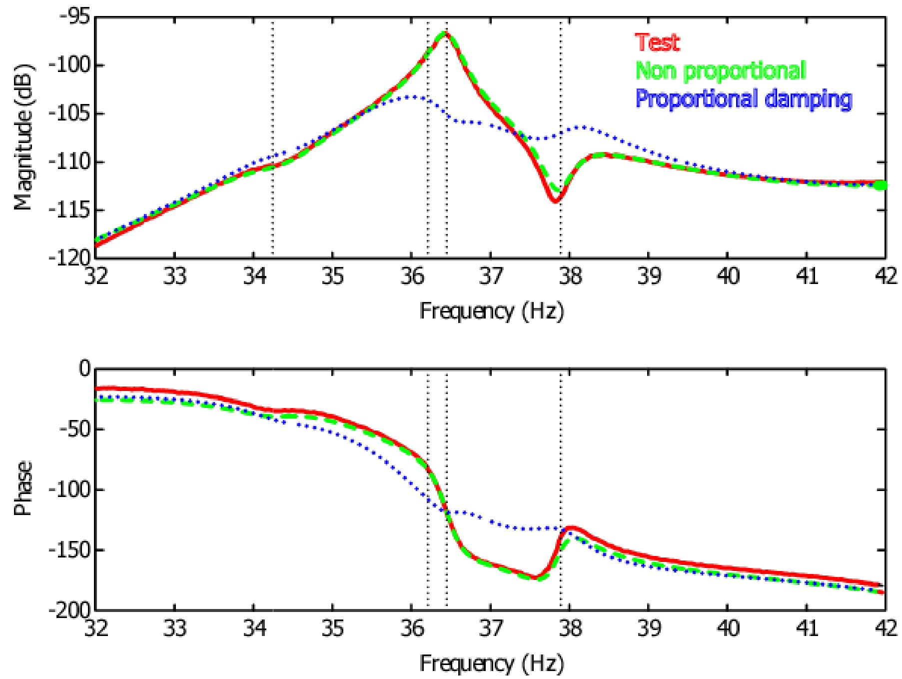
MIT SERC Active control testbed

28 sensors, 6 independent shaker locations

Balmès (PhD 93, MSSP 97)

Real from complex

- Γ consistent from test to test
- Proportional damping response significantly different



0.98	-0.02	-0.04	-0.39	0.66	-0.14	0.20	-1.00	-0.87
-0.02	**1.76**	0.13	0.66	0.06	-0.49	-0.85	-1.05	-0.00
-0.04	0.13	**1.80**	-0.06	-0.24	0.13	0.75	1.39	2.13
-0.39	0.66	-0.06	**4.68**	1.10	-1.08	-0.39	-1.78	2.46
0.66	0.06	-0.24	1.10	**6.07**	**-1.69**	0.64	-2.00	-1.41
-0.14	-0.49	0.13	-1.08	**-1.69**	**11.66**	**-3.14**	**4.11**	0.97
0.20	-0.85	0.75	-0.39	0.64	**-3.14**	**3.54**	**-0.61**	-0.12
-1.00	-1.05	1.39	-1.78	-2.00	**4.11**	**-0.61**	**4.56**	1.07
-0.87	-0.00	2.13	2.46	-1.41	0.97	-0.12	1.07	**12.72**

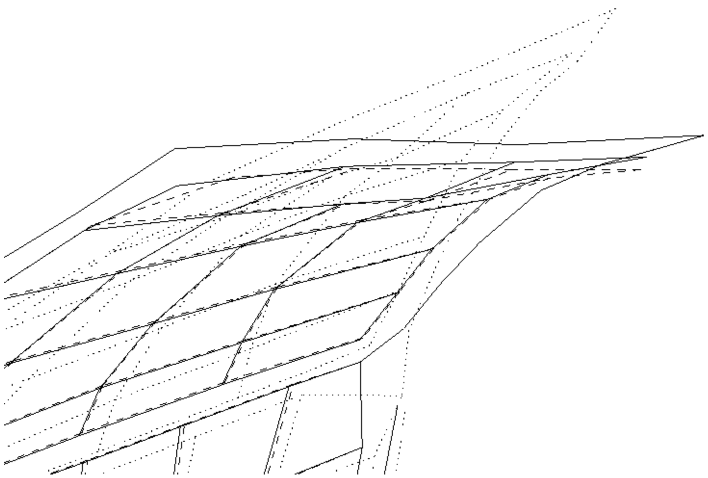
Real from complex : but …

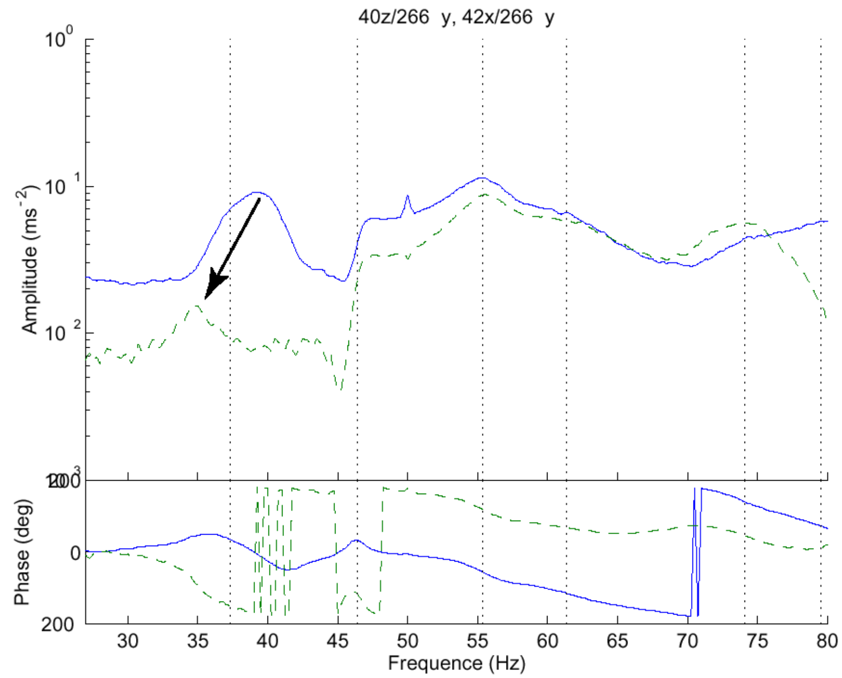
- Real data is almost never that clean
- Use of Γ_{test} difficult and does not change design

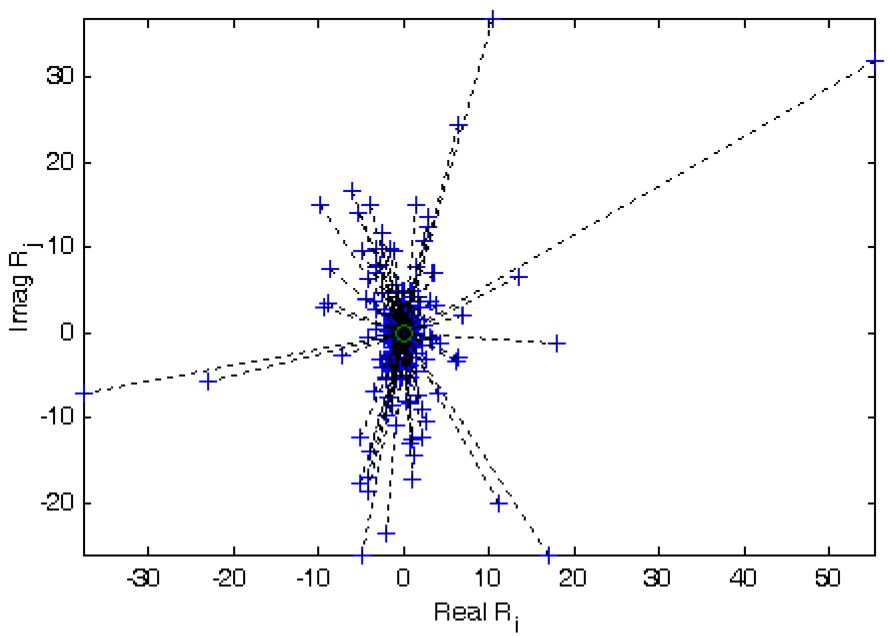
Thus for modal test derived damping

- assume modal damping
- if you really want real modes
  - Take the imaginary part of the residue
  - Use appropriation (Foltete 98)
  - Use transformations (Niedbal 84, Zhang 85, Wei 87, Imregun 93, Balmes 93 97, Ahmadian 95, …)

Complex non modes I



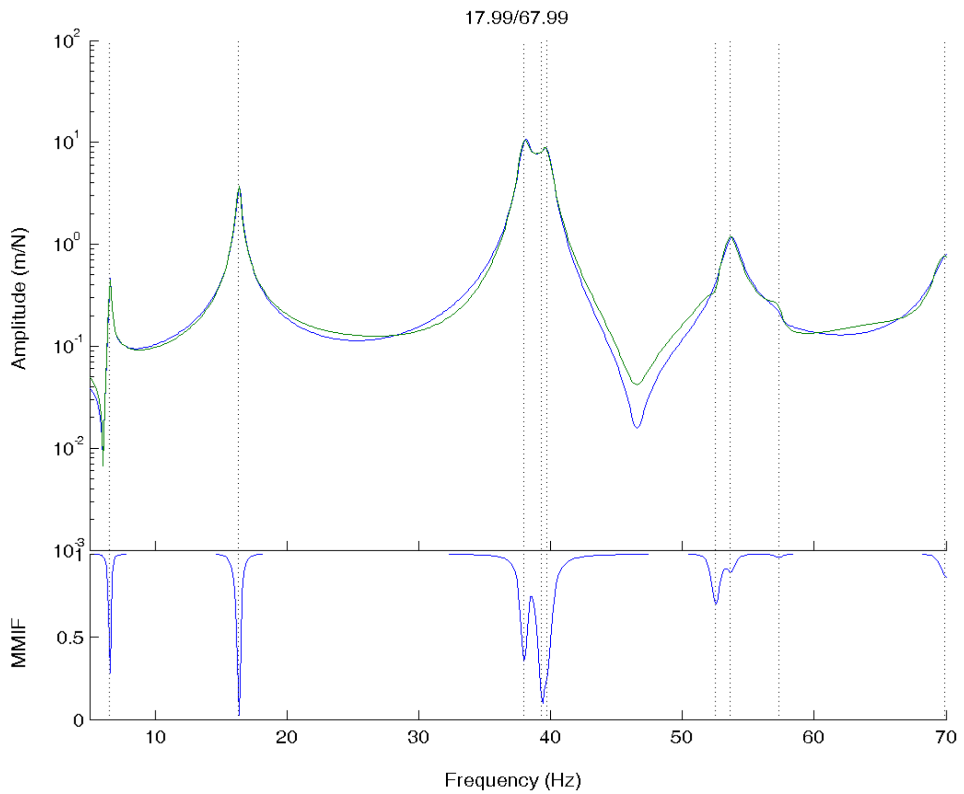


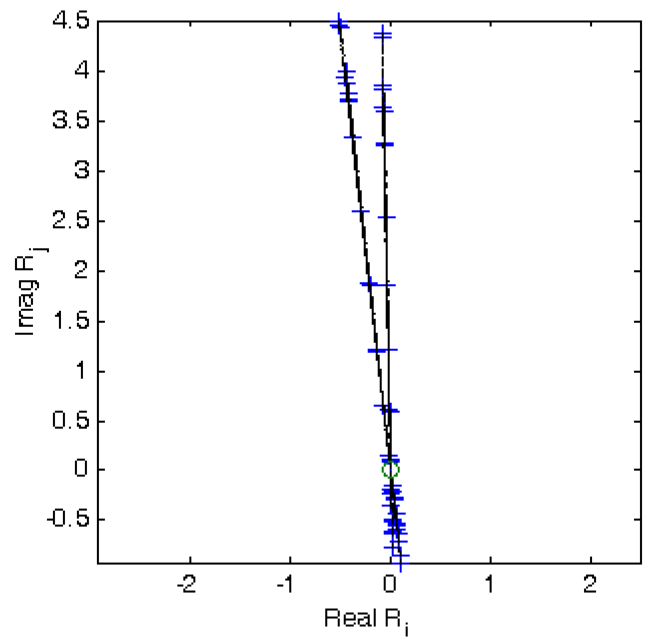


Frequency shifts in batch tests induce complexity

Complex non modes II

Simulation : frequency change of 0.2%





Frequency shifts in batch tests induce complexity

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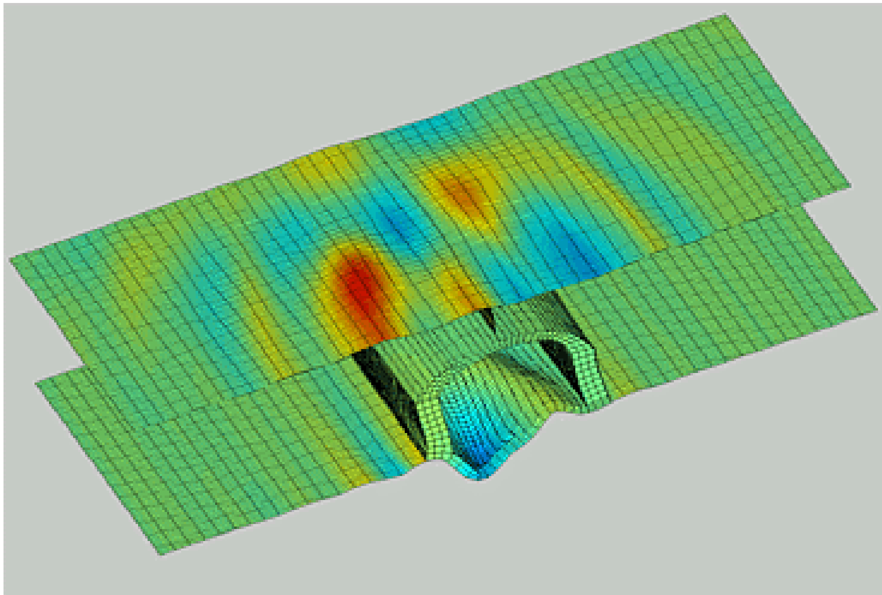
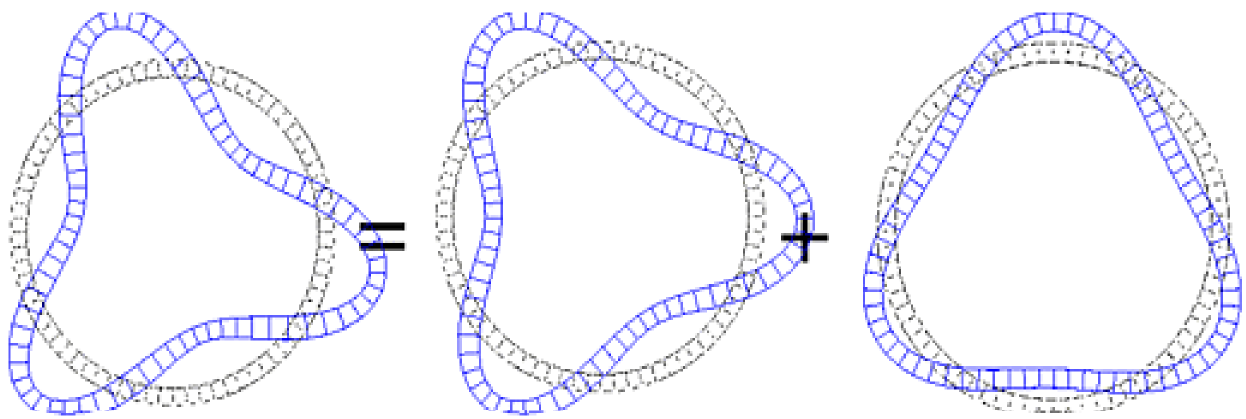
Computational complex shapes



Propagating waves
- boundary elements
- cyclic symmetry

$$\{q(t)\} = \text{Re}\left((\{\phi_1\} + i\{\phi_2\})\, e^{i\omega t}\right) = \{\phi_1\}\cos(\omega t) + \{\phi_2\}\cos(\omega t + \pi/2)$$



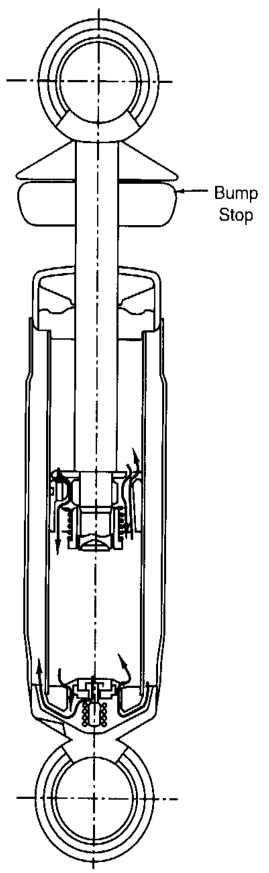
Complex shapes

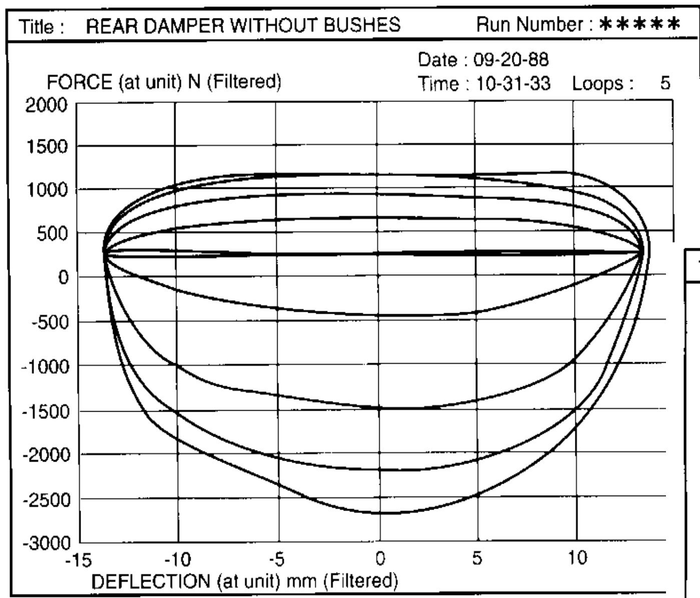
- Non modes
  - Non linear response
  - Poor identification
  - Non invariance of test article
  - Signal processing distortions
  - Operational deflection shapes, propagating waves
  - …
- Mathematical trick (cyclic symmetry)
- True complex modes (damped linear system)

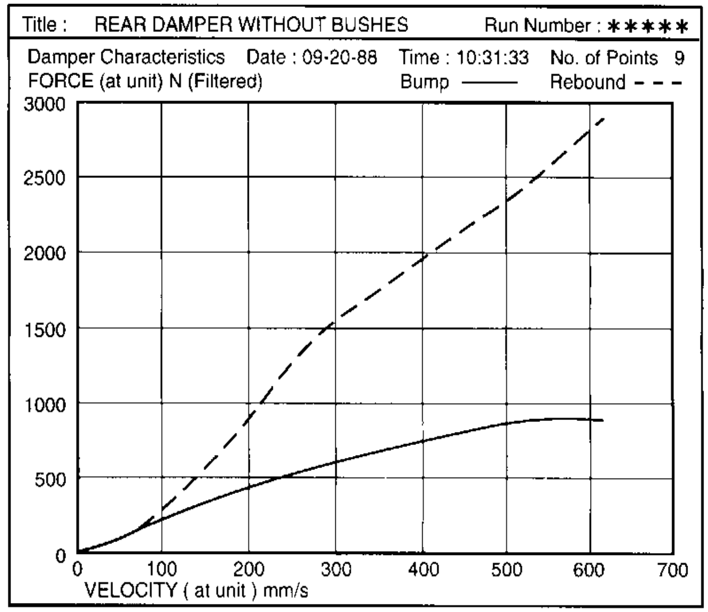
Modeling damping

- Local models (joints and materials)
- Finite element models, damping design tools
- Simplifying assumptions for system dynamics (dynamic behaviour rather than local knowledge models)

Joints





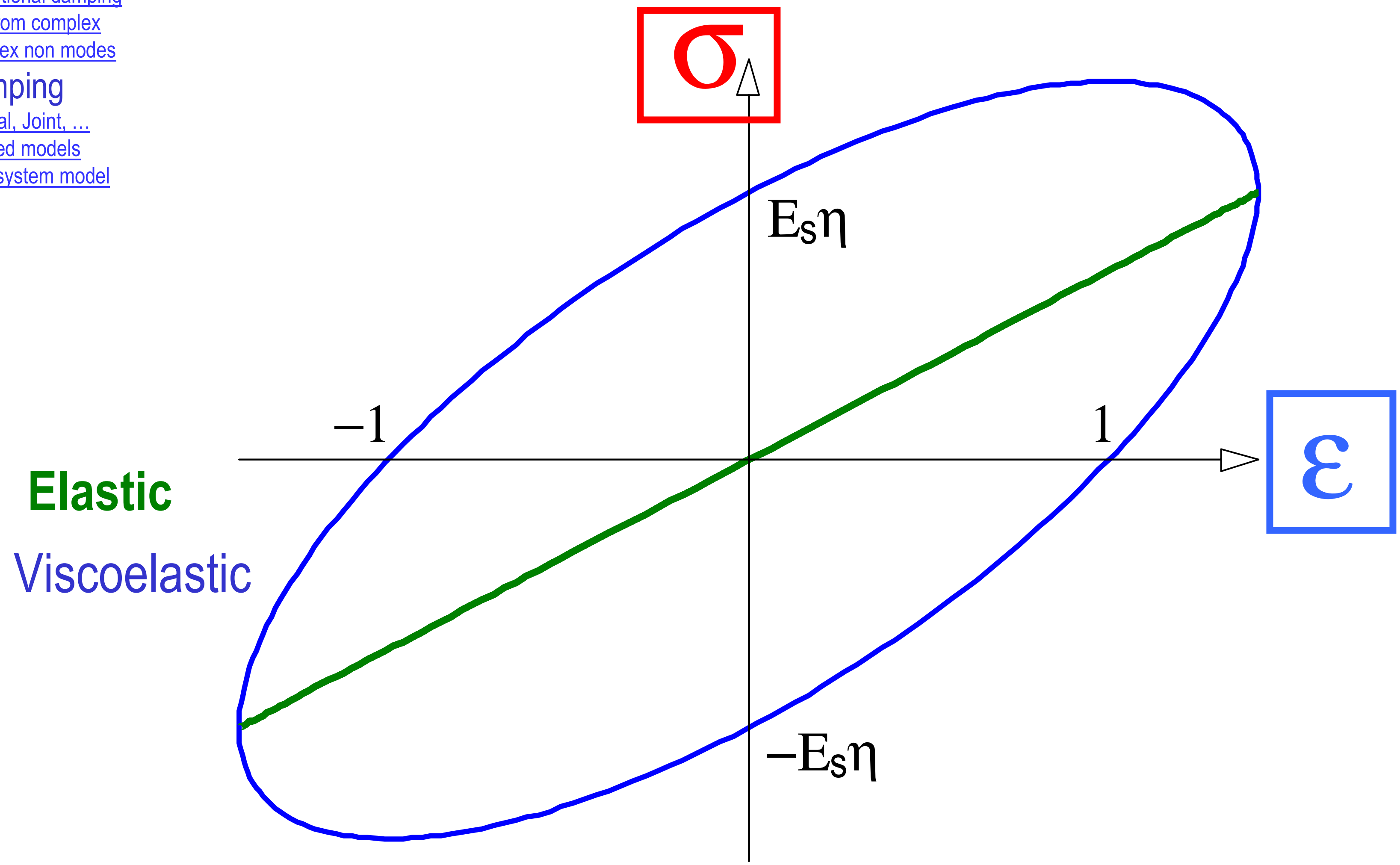


$$F = f(q,t)$$

$$F = k(s)q(s)$$

Non linear models : only practical if local

Viscoelastic materials

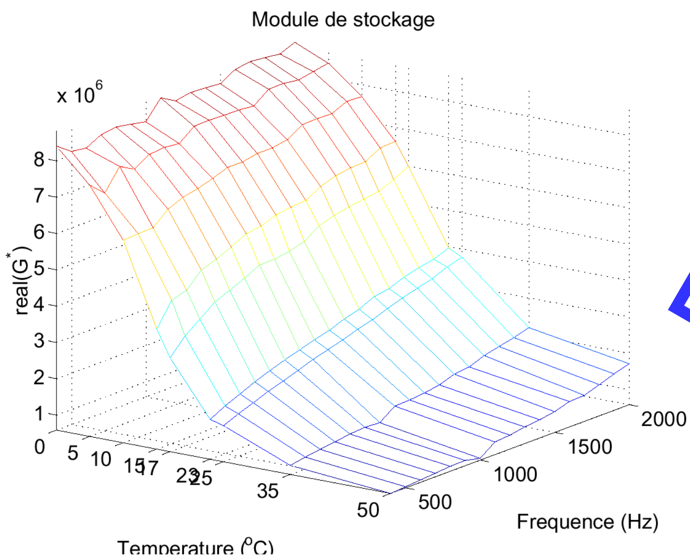


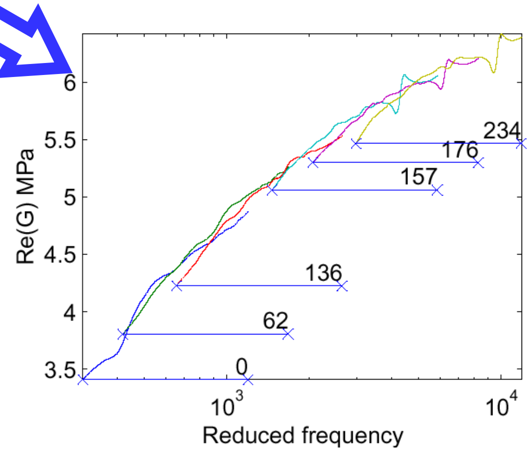
$$\sigma = \mathrm{Re}(E_s(1+i\eta)e^{i\omega t})$$

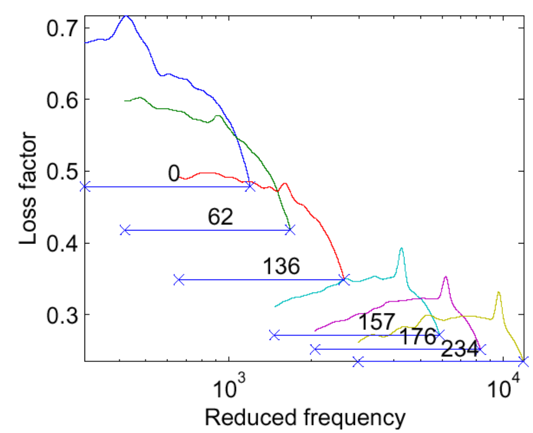
Viscoelastic constitutive relations

- Stress is a function of strain history
- Complex modulus in Laplace domain

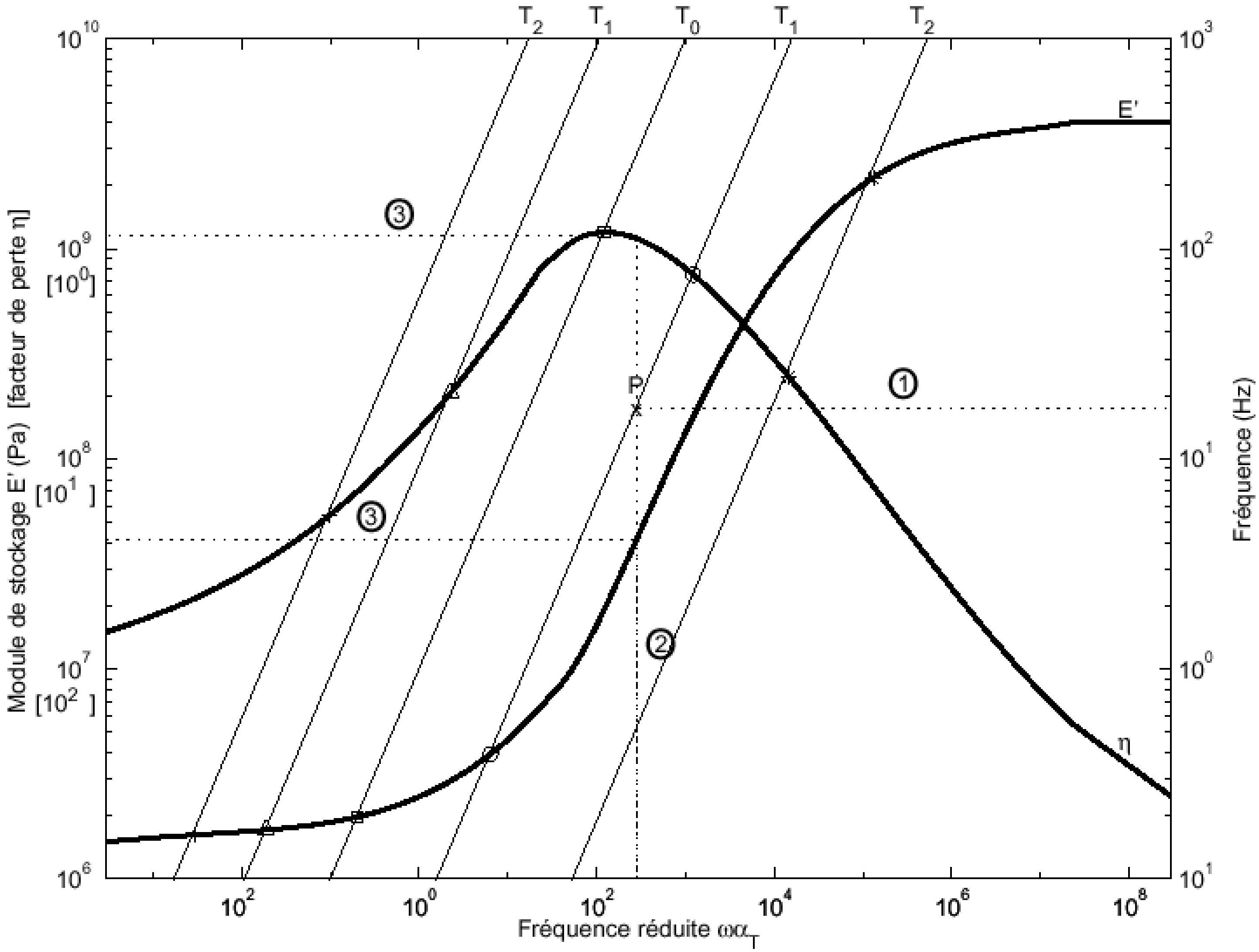
$$\sigma(s) = E(s,T,\sigma_0)\varepsilon(s) = (E' + iE'')\varepsilon(s)$$





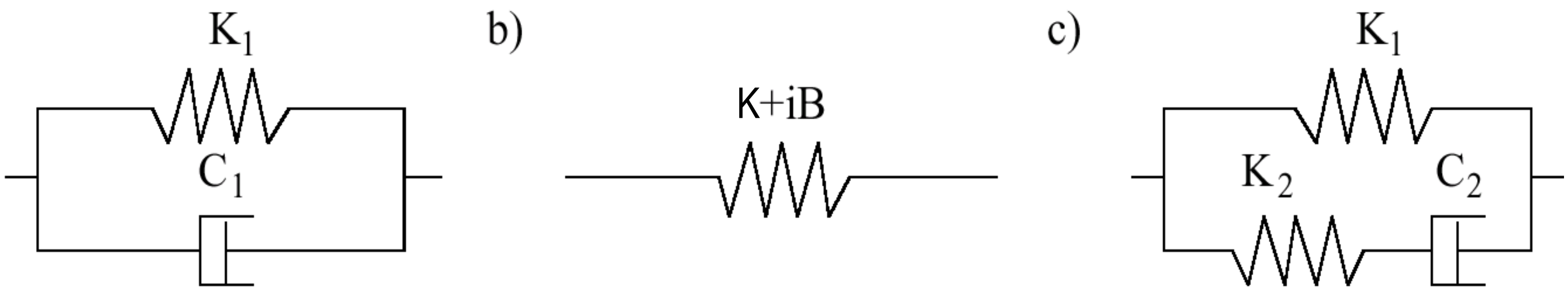


Reduced frequency nomograms



Viscoelastic constitutive laws

Viscous Histeretic Standard viscoelastic



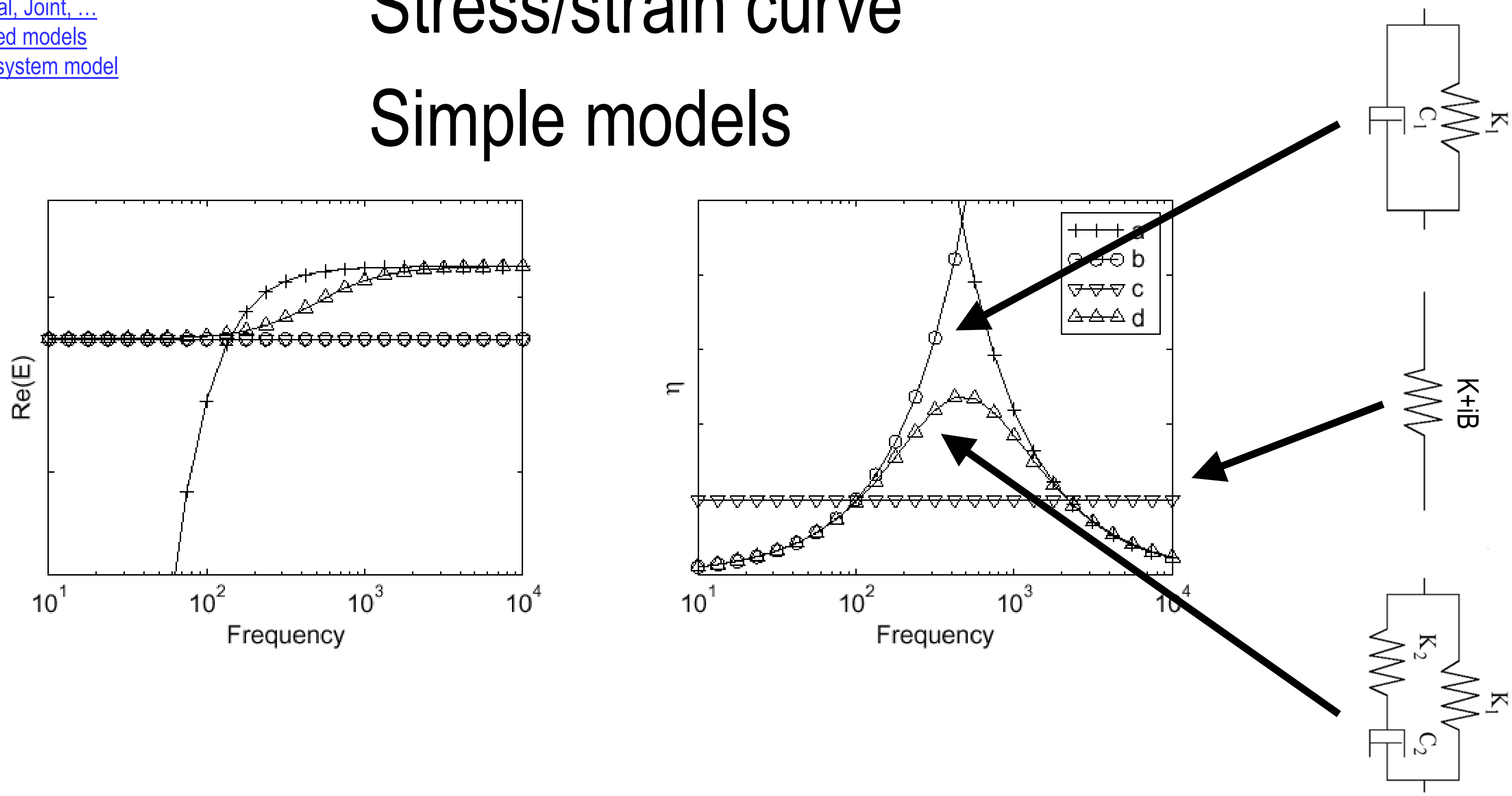
Alternatives :

- More relaxation constants
- Fractional derivatives
- Direct use of experimental master curve

Material damping models

Stress/strain curve

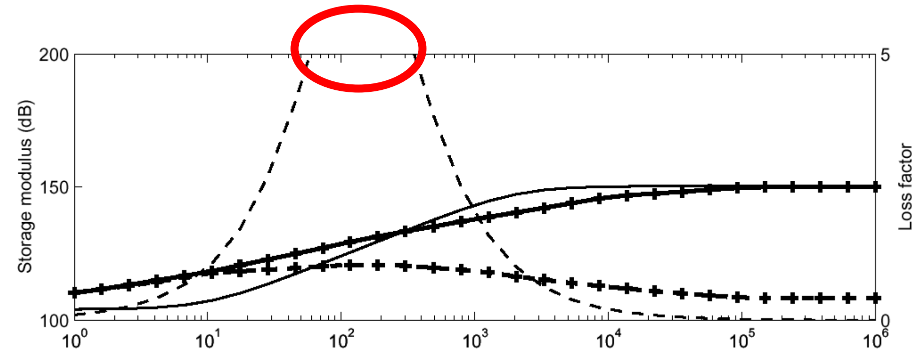
Simple models



Consitutive model order

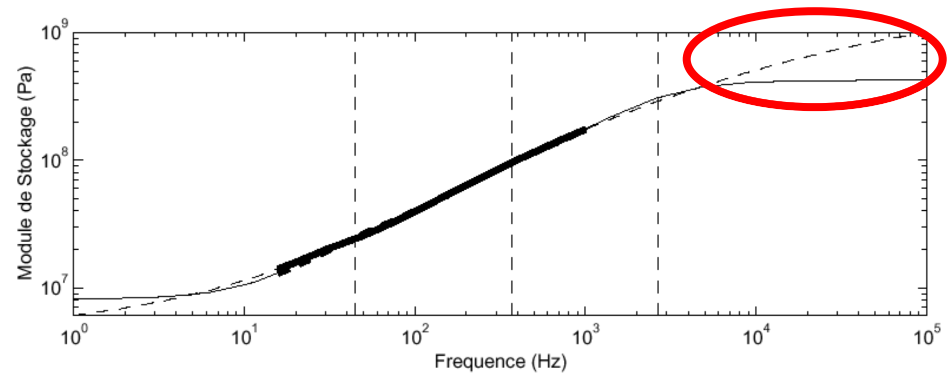
1 pole model (3 parameter)

loss factor is wrong

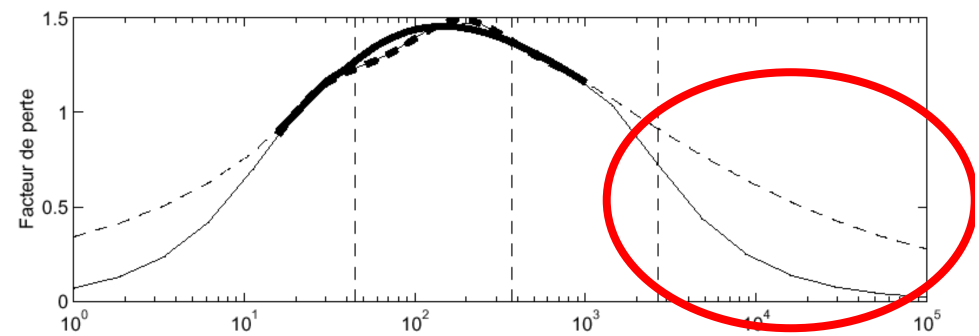


3 pole model : better match in band but

not very good outside



Good models require high order



Other sources of dissipation

- Non linearities
  - Material (plasticity, …), joint
  - Contact (friction dampers, joint damping, micro-slip, …)
- Coupling with other media
  - Radiation in air, water, soil, etc.
  - Gyroscopic damping
  - Electrical systems (active control)
  - Particle filled cavities
  - Lubrication, …

Frequency dependent models

- Dynamic stiffness : linear combination of fixed matrices

$$[Z(E_i,s)] = \left[Ms^2 + K_e + \sum_i E_i(s,T,\sigma_0)\frac{K_{vi}(E_0)}{E_0}\right]$$

- Direct frequency response

$$[Z(E_i,s)]\{q\} = \{F(s)\}$$

- Non-linear eigenvalue extraction

$$[Z(E_i,\lambda_j)]\{\psi_j\} = \{0\}$$

Frequency independent models

Trick increase model order
to gain frequency idependence

- Material formulation with internal fields (rational and fractional derivates)

$$E(s) = E_\infty - \left(\sum_{j=1}^{n} \frac{E_j}{s+\omega_j}\right) \Rightarrow q_{vj} = -\frac{E_j}{(s+\omega_j)} q$$

$$\left[\begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix} s + \begin{bmatrix} 0 & -M & 0 \\ K_e + E_\infty K_v & 0 & K_v \\ E_j M & 0 & \omega_j M \end{bmatrix}\right] \begin{Bmatrix} q \\ sq \\ q_v \end{Bmatrix} = \begin{Bmatrix} 0 \\ F \\ 0 \end{Bmatrix}$$

Frequency independent models

- Proved methodologies ADF (Lesieutre), GHM (Gola, …) , Prony series (abaqus)

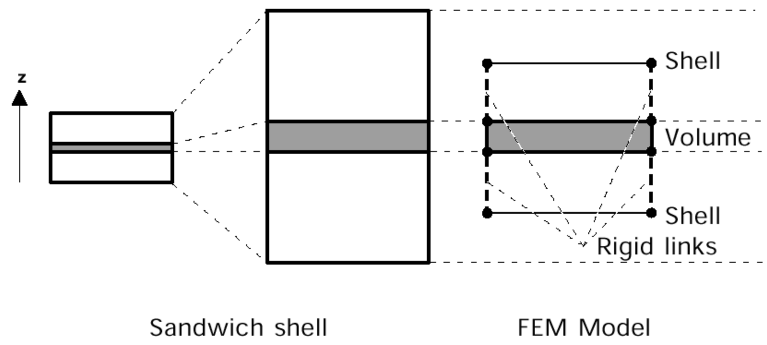
+ Integrates into standard solvers

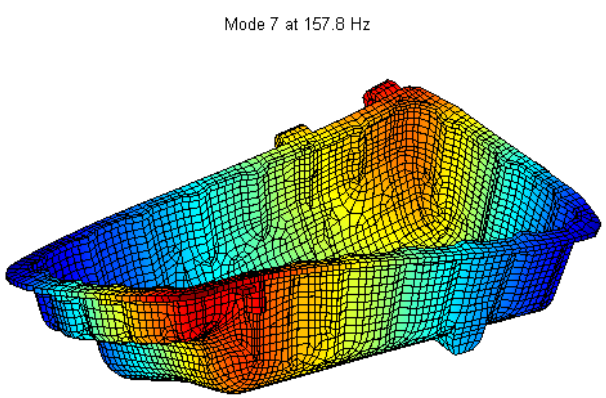
+ Time equivalent

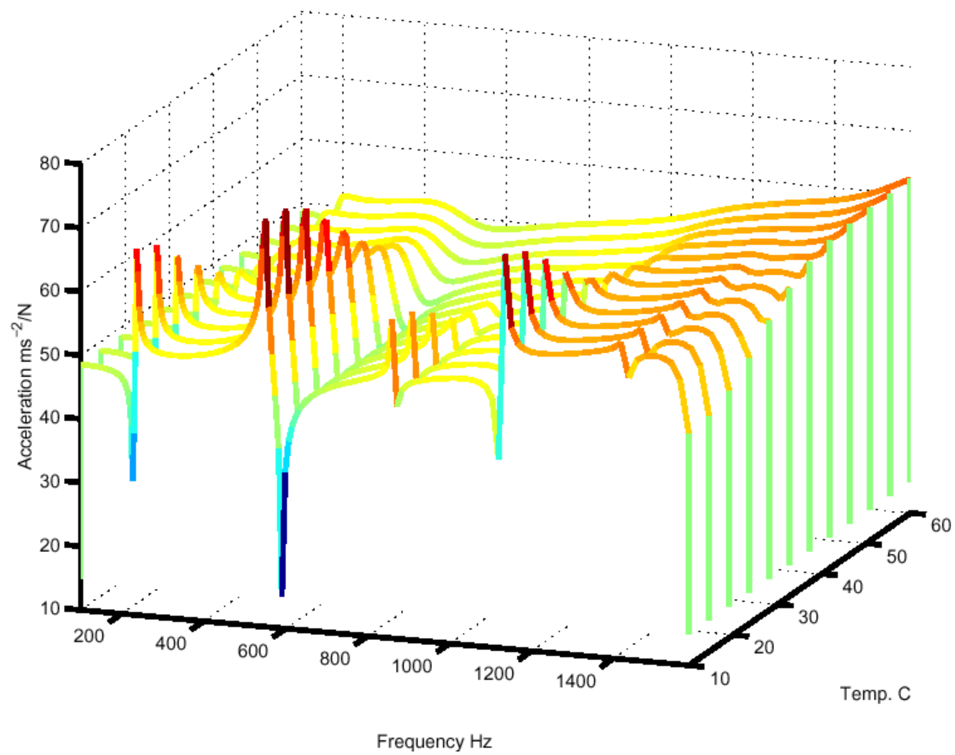
– High order for good material (solvers need to account for block structure)

A major computational challenge

- Sandwich oil pan
- 58766 DOFs







Modes
Spectral decomposition
When real ?
Proportional damping
Real from complex
Complex non modes
Damping
Material, Joint, …
Damped models
Local/system model

A major computational challenge

- NASTRAN 70.7 & SDT 5
- 58766 DOFs

10 temperatures
1000 frequency

NASTRAN direct : 9 days
SDT Iterative : 612 s
Speedup : 1300

	NAST.	SDT
$M-K$ Assembly	45	N.A.
Factorization of K	20	67
F/B substitution (7 vect)	1.85	4.02
75 normal modes	166	730
Z reassembly	N.A.	6
Projection $T^T ZT$	N.A.	25
Factorization of Z	77	153
F/B substitution (1 vect)	2.7	4.1

Still work to be done on accuracy/performance trade-off

More examples here

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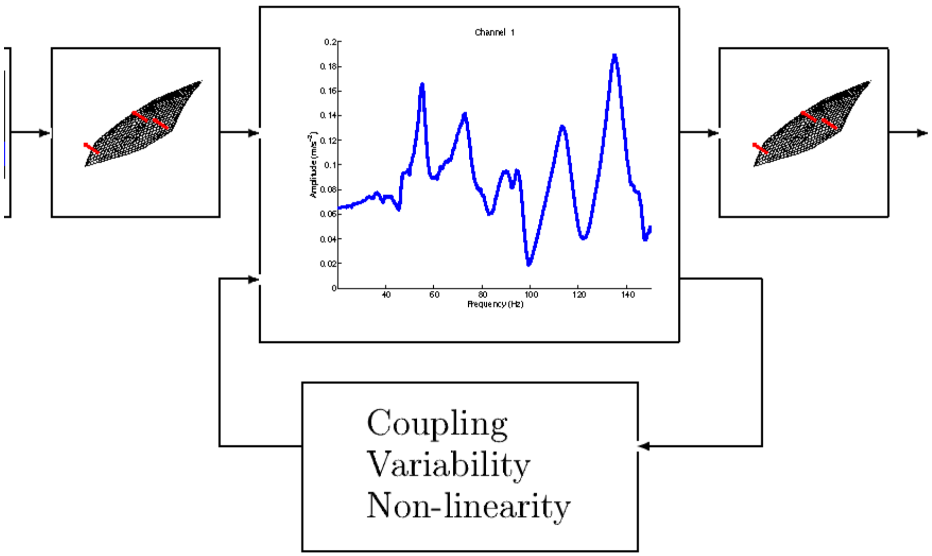
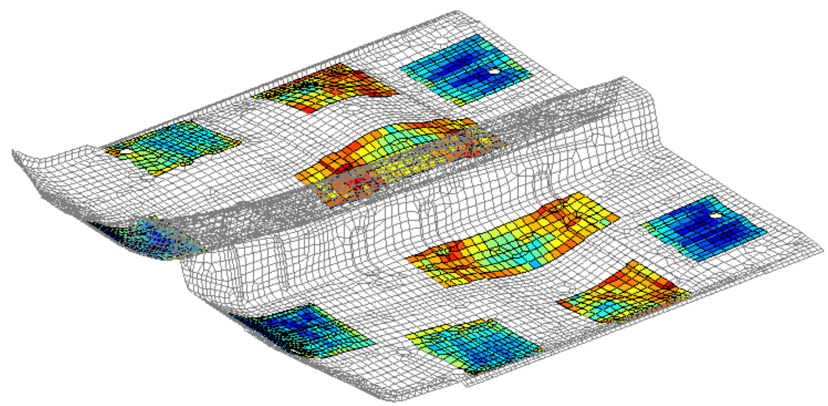
System vs. local

- System models (dynamic behaviour)

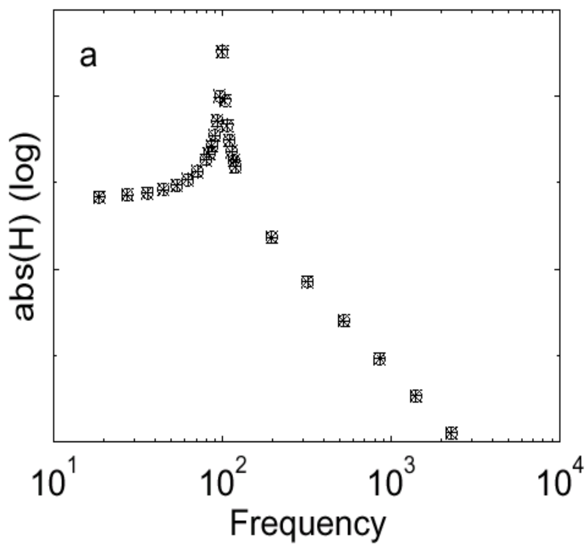


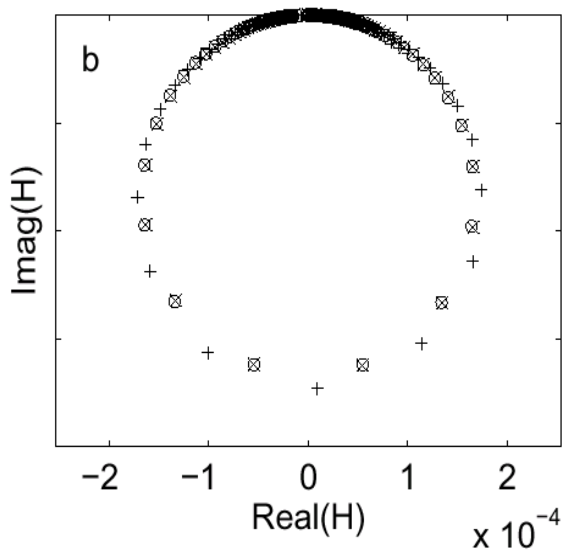
- FEM models (geometry/material knowlege)

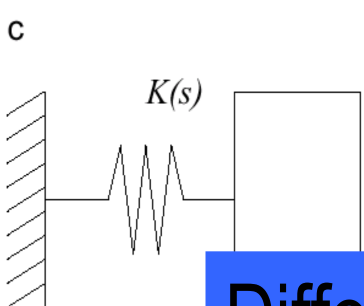


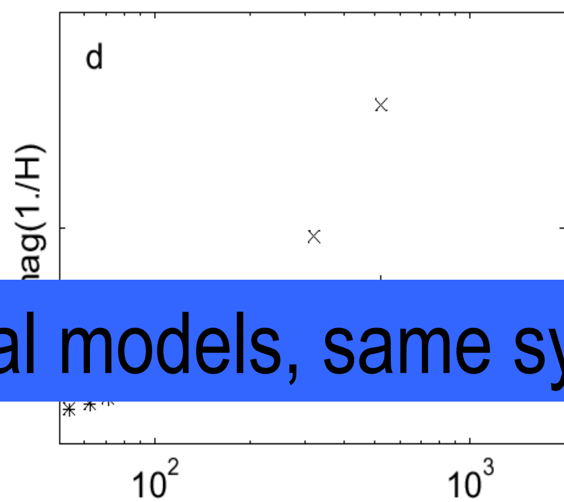
- Each objective requires different assumptions

System damping. 1 DOF example.









a) Bode
b) Nyquist
c) 1DOF model
d) Dynamic stiffness

dynamic stiffness equal at resonance

$\Rightarrow$

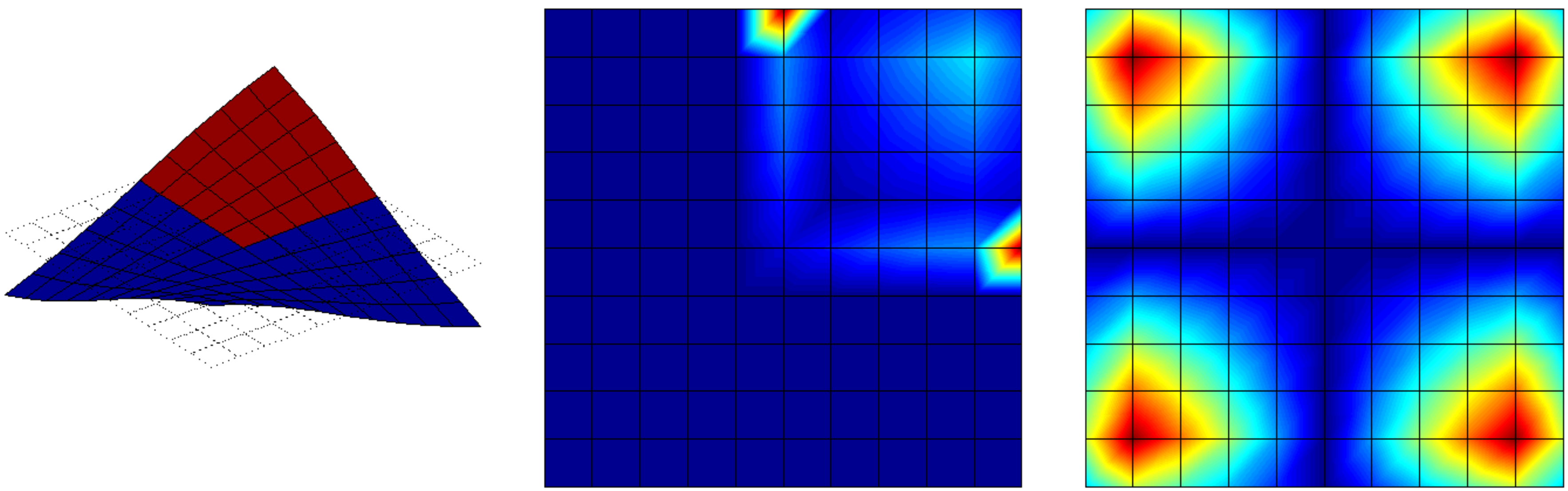
Response is the same

Different local models, same system response

Proportional damping model

- Modal damping is a good for system not for local
- Forced response along first mode

$\{q\} = \{\phi_j\} \cos(\omega_j t)$ $\quad$ $\mathrm{Im}\,(K)\,\{\phi_j\}\cos(\omega_j t)$ $\quad$ $(M\phi_j)\, 2\zeta_j\omega_j \cos(\omega_j t)$



True $\quad$ modal

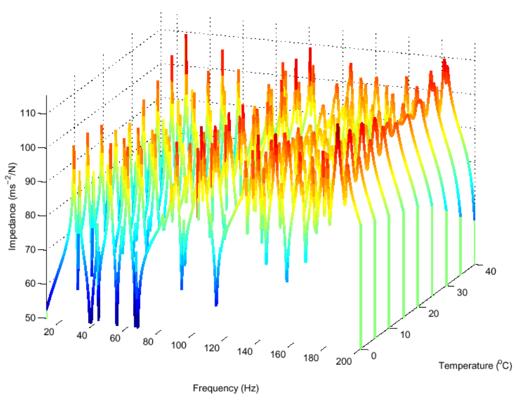
Modal looses localization of damping

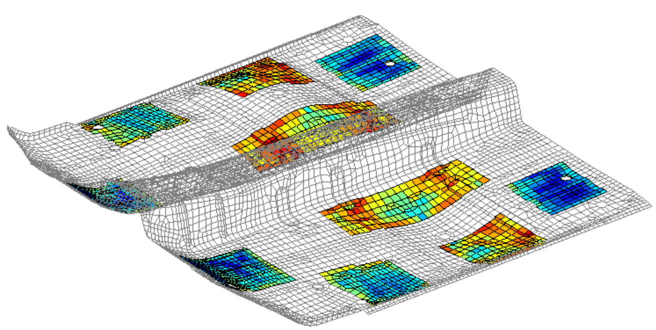
Systems and material level tests

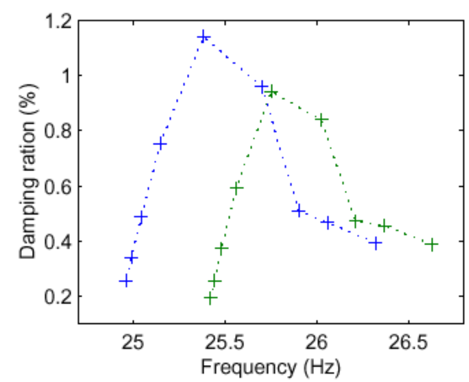
- Based on system level test, you get test derived damping ratio for system model they
  - are difficult/impossible to extrapolate to other system configurations
  - require matching of test/FEM modes
  - can rarely be translated in local damping information
- Based on materials/components tests, you get
  - damped FEM models, which are still difficult to solve
  - are only valid if almost all damping comes from well characterized parts
- Most damping models are arbitrary design parameters

Conclusion

- A lot of complex shapes are just not modes
- Some of them do come from damping
- Damping is becoming part of design
- We will see more of complex modes







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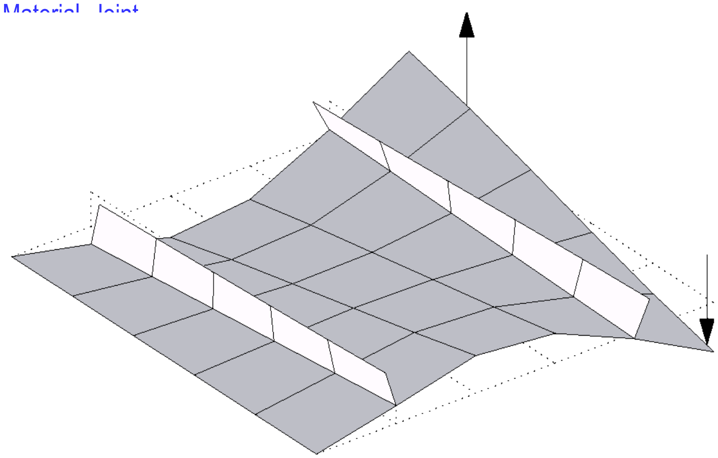
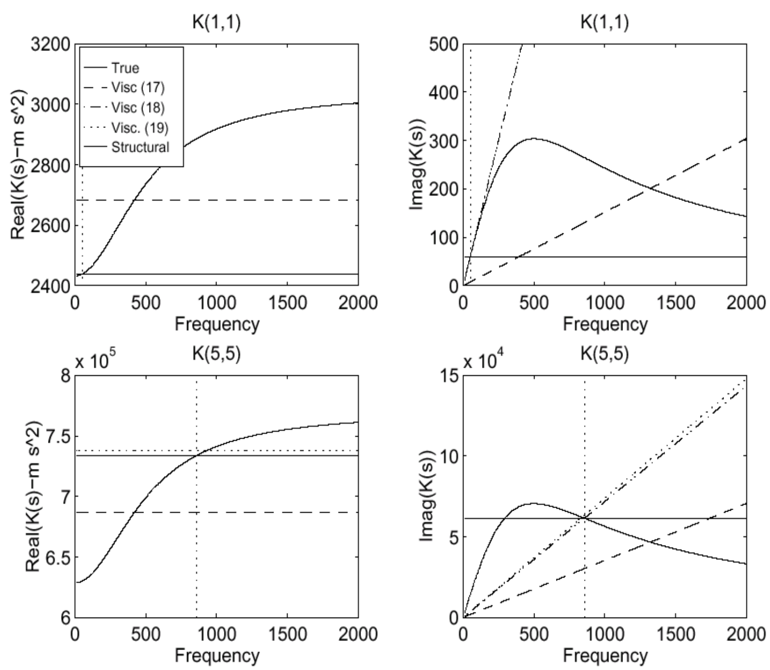
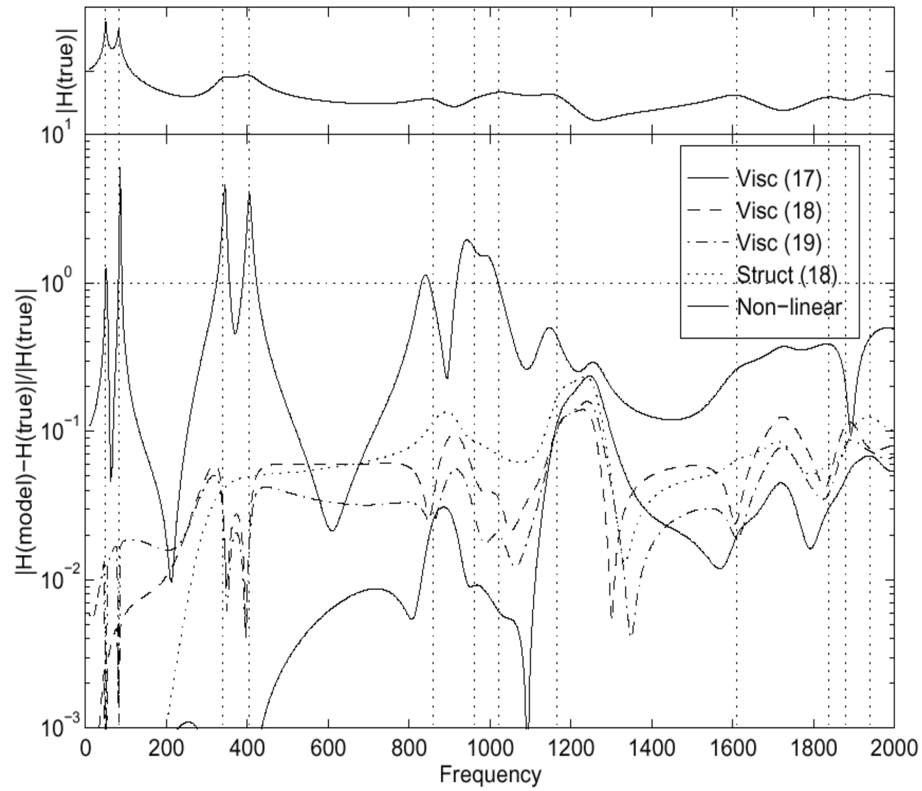
System damping model

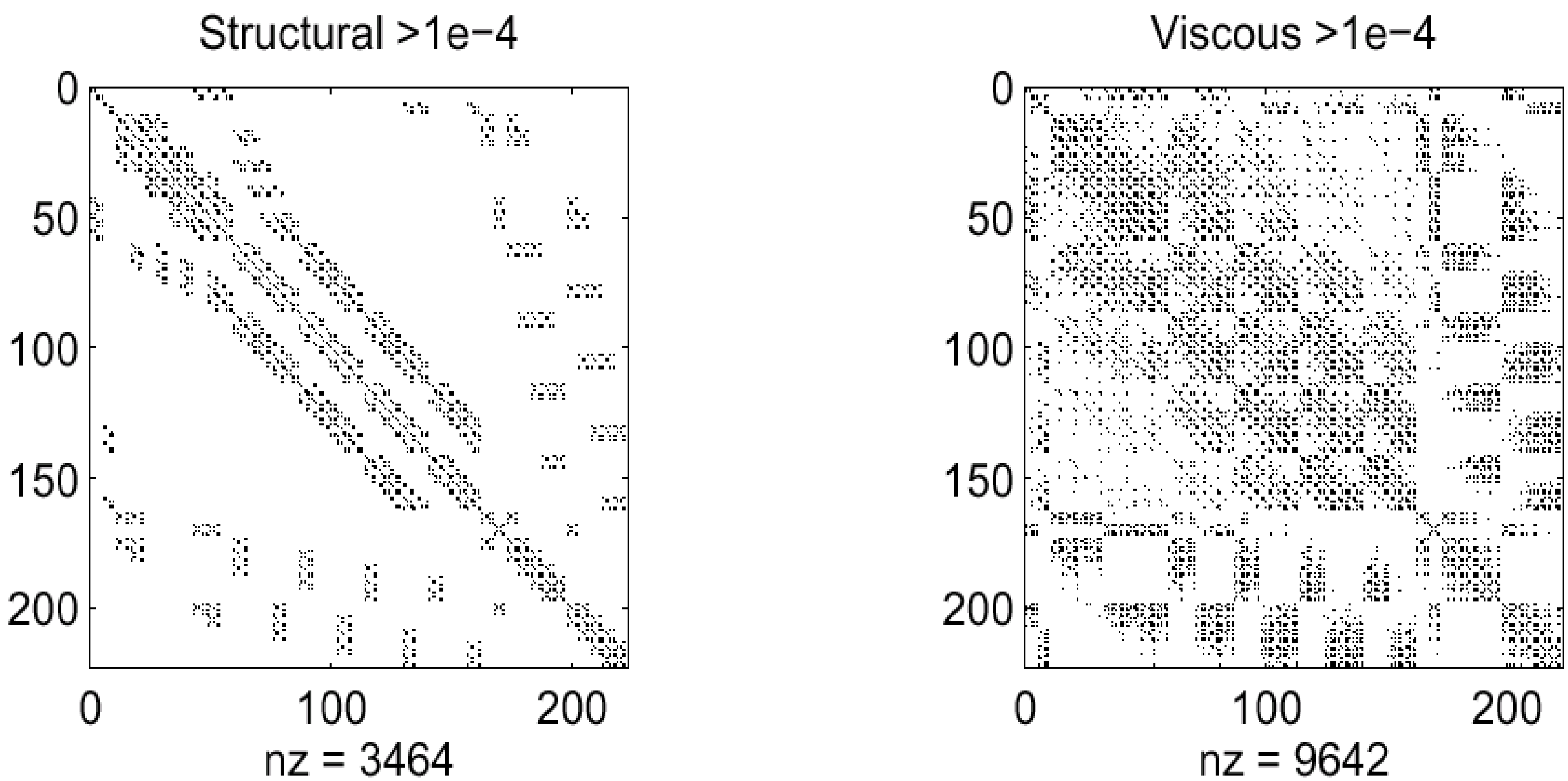






Balmès IMAC 97 : you can build equivalent system models in modal coordinates

1 % damping : viscous & structural



- structural material damping leads to sparse matrix
- Viscous material damping leads to full matrix

Complex modes history

Just a few authors who talked about complex modes at IMAC

Balmes 94, Chung 87, Debao 87, Ewins 93, Gladwell 95, Hamidi 89, Ibrahim 83, 93 Imregun 91, 93, Inman 86, 95, Jun 84, Kirshenboin 87, Lallement 84, 87, Mitchell 90, 92, Montgomery 93, Niedbal 84, Ozguven 82, 86, Sas 92, Sestieri 93, Wei 87, Wicks 96, Zhang 84 …

Floor pannel design

Floor panel (7998 nodes, 7813 elements)

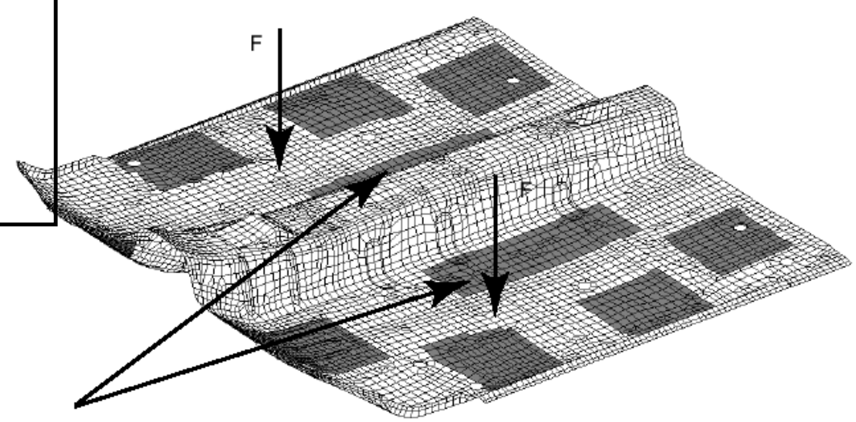
+ free layers 2195 nodes, 1908 elements or

+ constrained layer 6595 n, 3816 elts)

NASTRAN element formulations

Objective : propose design steps

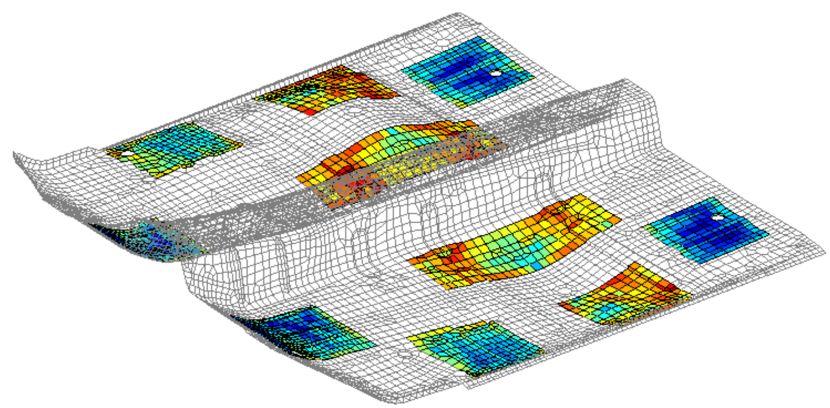
Validation addressed elsewhere



Additional patches

Design problems

- Basic design : selection of relevant materials
- Thickness optimization
- Treatment nature and position
  - A1 free layer 2.47 mm viscoelastic
  - B1 constrained layer : 50 mm visco, 0.3 mm steel



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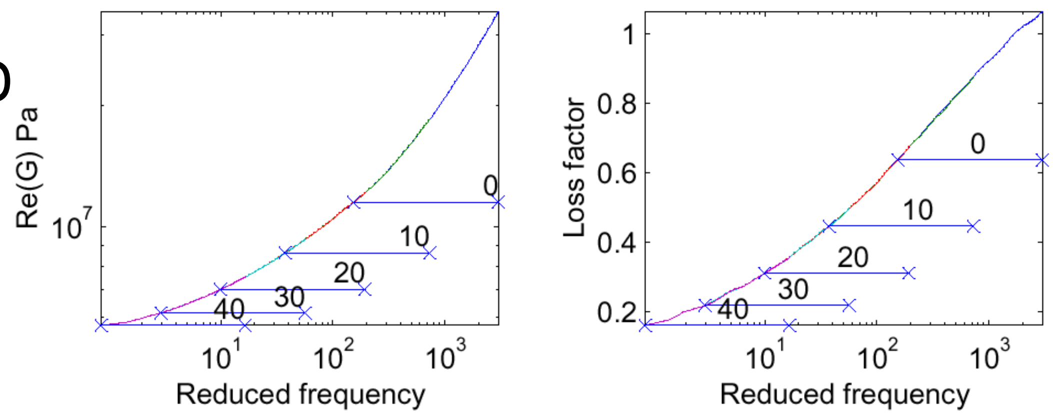
Material Selection

1. Select frequency range
2. Select temp range
3. Validate relevance o nomogram

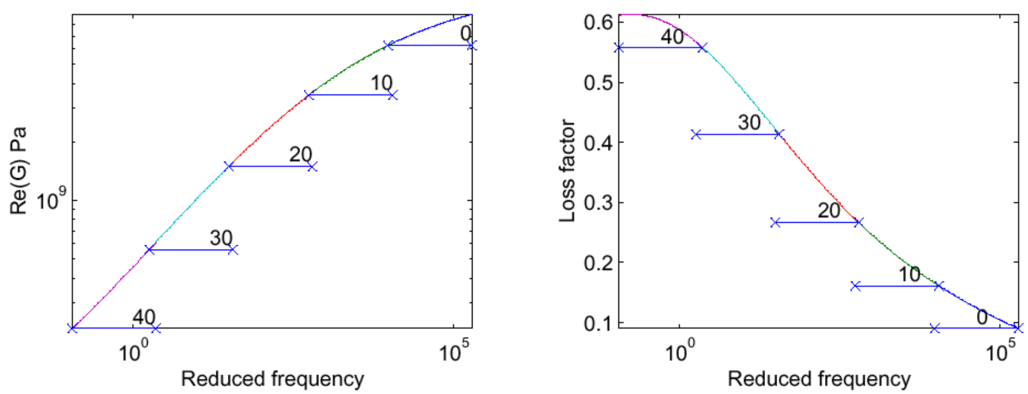
SM50e



Other considerations

Manufacturing, price, outgazing, aging, oil, …

PL3023

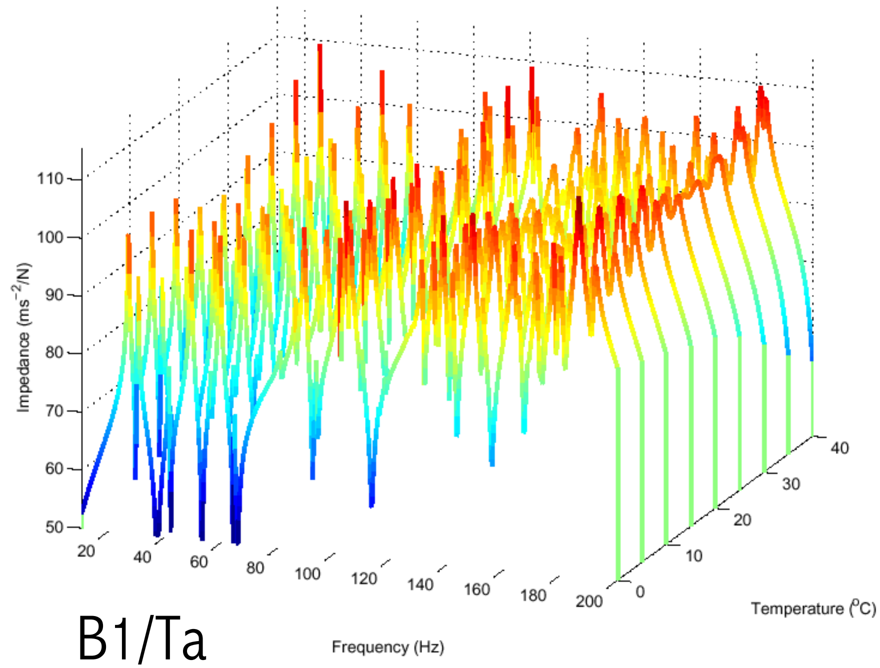


Temperature robustness validation

For a selected design performance is judged by

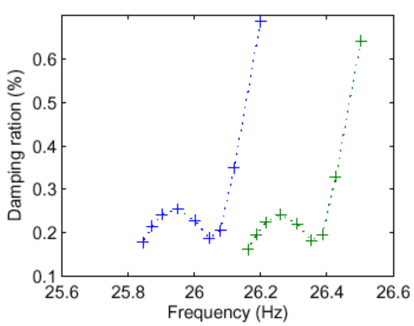
FRFs and Poles

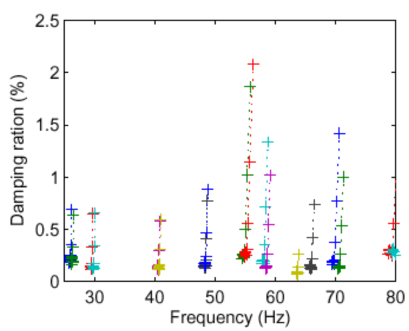
Sensitivity to temperature must be evaluated

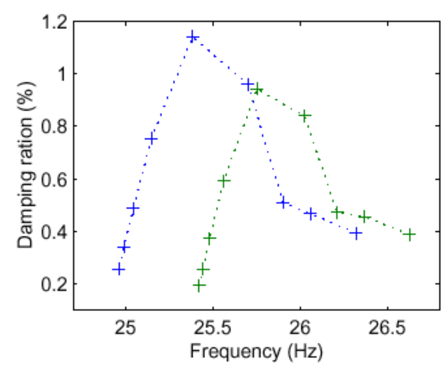


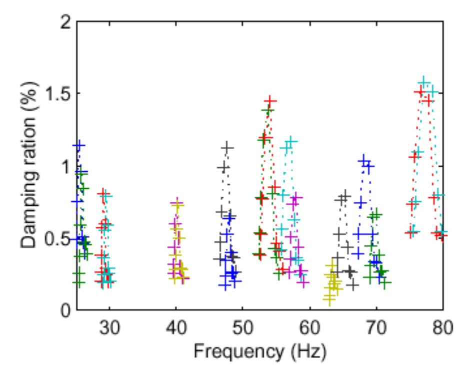
B1/Ta

B1/SM50e

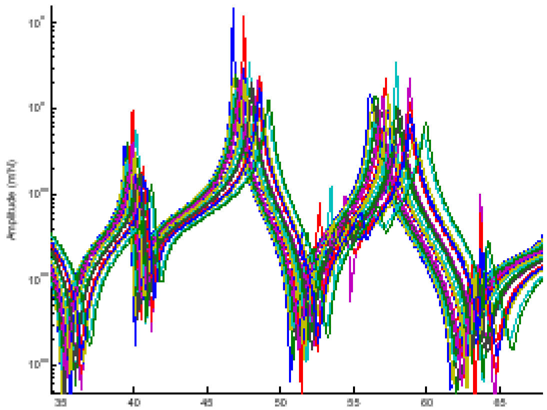


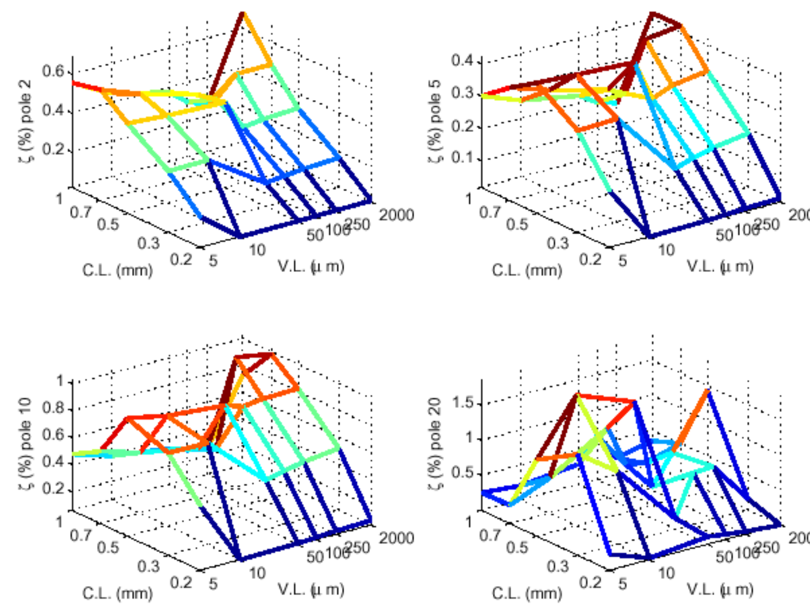






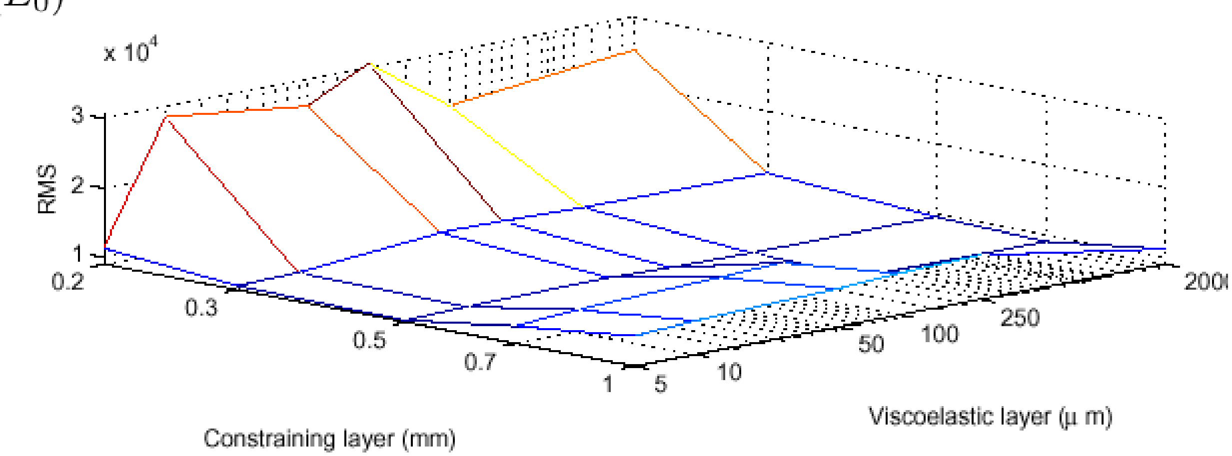
Thickness optimization



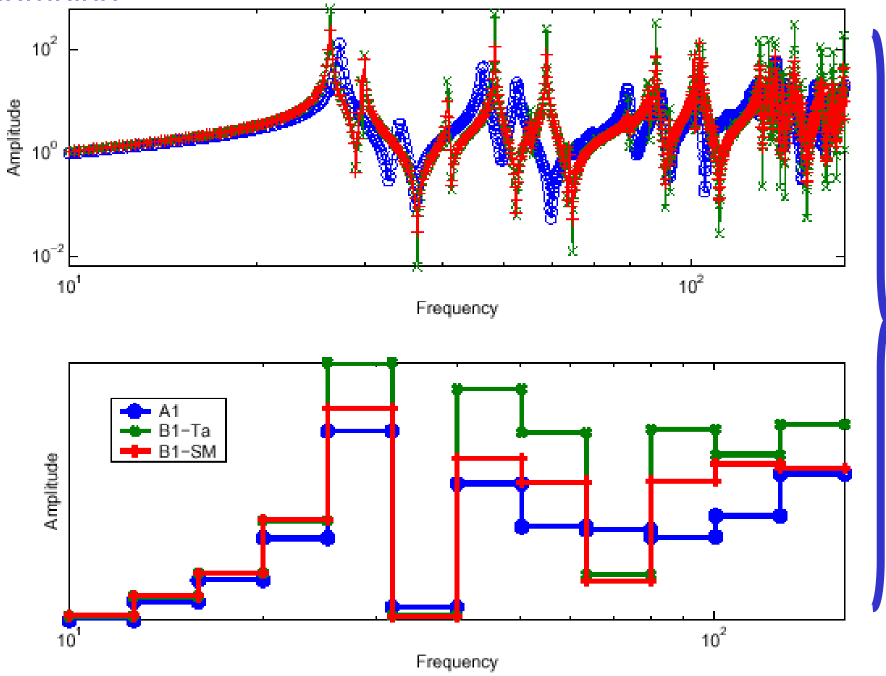


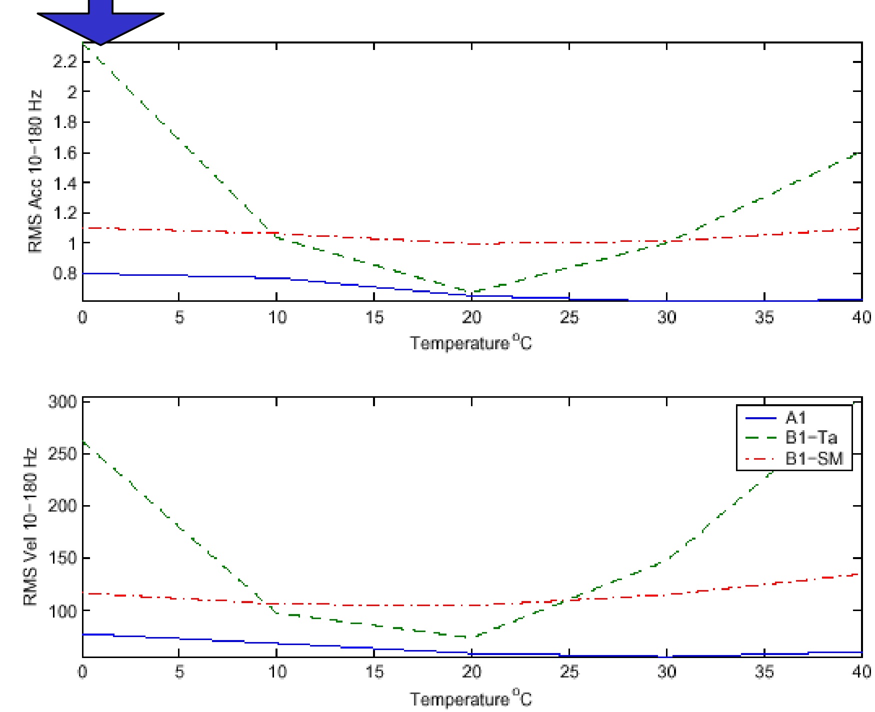
$$K_{vi}(h_v, E_i) \approx \frac{h_{v0}}{h_v} \frac{E_i(s, T, \sigma_0)}{E_0} K_{vi}(E_0)$$

$$K_{ci}(h_c) \approx \frac{h_c}{h_{c0}} K_{ci}(E_i)$$



Design comparisons





- A1 (Free layer) mass = 2 x B1 mass
- B1-Ta very as efficient as A1 at 20°C
- B1-SM very robust but not very good