REVIEW AND EVALUATION OF SHAPE EXPANSION METHODS

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ABSTRACT

Correlation criteria and modeshape expansion techniques deal with the spatial incompatibility linked to the measurement of modeshapes through a limited set of physical sensors and their analytical prediction at a (larger) number of finite element (FE) degrees of freedom (DOFs). Expansion techniques have two conflicting objectives : estimate the motion at all FE DOFs and smooth test errors. The paper first proposes a unified perspective that covers most existing expansion methods. Examples, based on a model of the GARTEUR SM-AG-19 testbed, are used then to analyze the performance of major expansion methods (modal, static, dynamic, minimum residual, and minimum residual with test error). The impact of three usual sources of errors is then considered : inaccuracies in the way true sensor locations are taken into account; error in the test data; errors in the FE model used for the expansion.

1 INTRODUCTION

Expansion methods seek to estimate the motion at all DOFs of a finite element model based on measured information (modeshapes or frequency response functions) and prior, but not necessarily accurate, information about the structure under test in the form of a reference finite element model.

The paper first seeks to give a unified perspective on expansion methods allowing their classification. The proposed classification is based on how various methods combine information about **test** and **modeling** errors. Section 2.1 addresses subspace methods, which use the model to define a subspace of possible FE deformation, and define a projector giving a direct mapping between test and expanded shapes based on a minimization of test errors. These expansion methods include modal/SEREP [1,2], static (based on Guyan reduction [3]), dynamic [4] and hybrid [5,6]. Section 2.2 addresses model based methods that combine test and modeling error measures [7,8]. Finally, section 2.3 addresses the distinction between sensors and DOFs, and model reduction techniques which are needed for a general and numerically acceptable implementation of expansion methods.

The second part of the paper uses a model of the GARTEUR SM-AG-19 testbed [9] to analyze standard sources of error and evaluate the qualities of major expansion methods: modal, static, dynamic, minimum residual, and minimum residual with test error. Three major sources of errors are successively considered : the effect of inaccuracies in the way true sensor locations are taken into account; the impact of error in the test data; the impact of errors in the FE model used for the expansion.

2 UNIFIED PERSPECTIVE ON EXISTING METHODS

2.1 Subspace expansion methods : interpolation, modal/SEREP, static

A large class of methods, called **subspace methods** here, only use modeling information to select a subspace of possible displacements with dimensions inferior or equal to the number of sensors. If $[T]_{N \times NR}$ is a basis of this subspace, one assumes that the full displacement is of the form $\{q_{Ex}\} = [T]\{q_R\}$ (modeling error is minimized for responses within the subspace). An estimate of the full response is then simply obtained by minimizing test error (distance between the test data and the associated response for the expanded shape). The minimum is generally obtained by solving the least-squares problem

$$\{q_R\} = \arg\min ||\{y_{Test}\} - [c][T]\{q_R\}||_2^2 \qquad (1)$$

where the observation matrix c is defined in section 2.3 for readers not familiar with it.

Wire-frame representations are the most trivial form of subspace expansion method : they assume that on the line between two connected test nodes the motion varies linearly between the values of motion taken at each end. The considered subspace corresponds to linear responses along each line to a unit displacements of each sensor. In this case the subspace dimension is equal to the number of sensors so that $\{y_{Test}\} = [c]\{q_{Ex}\}$.

Spline interpolations are a way to extend a geometrical construction of the subspace but they are ill suited for complex geometries.

If one has a FEM model of the structure under test (even a poor one), the easiest approach to select a subspace is to use this model. The two natural subspaces in modal analysis are the low frequency modeshapes and the static responses to loads or displacements applied at sensors.

Expansion based on the subspace of low frequency modes is known as **modal** [1] or **SEREP** [2] expansion. The subtle difference between the two approaches is the fact that, in the original paper, modal expansion preserved test results on test DOFs (DOFs and sensors were assumed to coincide) and interpolated motion on other DOFs (when SEREP uses (1)).

An advantage of the modal methods is the fact that you can select less target modes that you have sensors which induces a smoothing of the results. Since test errors are always present (due to inaccurate measurements or identification procedures), this smoothing is often beneficial.

Expansion based on the subspace of static responses to unit displacements at sensors is known as **static** expansion or Guyan reduction [3]. For tests described by observation matrices [7], the unit displacement problem can be replaced by a unit load problem $[T] = [K]^{-1}[c]^T$. For structures without rigid body modes this generates the same subspace as the unit displacement problem. In other cases $[K]$ is singular and can be simply mass-shifted (replaced by $K + \alpha M$ with α usually taken small when compared to the square of the first flexible frequency).

When expanding modeshapes or FRFs, each deformation is associated to a frequency. It thus seems reasonable to replace the static responses by dynamic responses to loads/displacements at that frequency. This leads to **dynamic** expansion [4]. In general, building a subspace for each modeshape frequency is computationally prohibitive. The alternative of using a single "representative" frequency for all modes was proposed in [10] but suffers from the same limitations as choosing this frequency to be zero (Guyan reduction). Reduced basis dynamic expansion [7] gives a solution to the computational cost problem.

With dynamic expansion, the underlying assumption on modeling error is that inaccuracies in the model can be represented by an arbitrary distribution of harmonic forces applied at the sensor locations. For static expansion, one adds the assumption that inertia forces can be neglected for all the expanded shapes. Test errors are assumed to be zero which is not desirable.

The ideal formulation would combine the ability to account for test errors found in modal expansion, and the use of a frequency dependent projector found in dynamic expansion. In formulating such approaches, one considers subspaces that are larger than the number of sensors (potentially the full model subspace) and the difficulty is to define a proper projector (since (1) is underdetermined). So called hybrid methods have been proposed in Refs. [5,6] but leave open questions of the relative weight given to each basic method and of target mode selection for the modal part.

2.2 Model based minimization methods

Subspace methods only use prior information to build the subspace. A more general class of methods formulates expansion as a minimization problem combining modeling and test errors.

Test errors are typically taken into account using a quadratic norm

$$\epsilon_j = \|\{y_{j,Test}\} - [c]\{q_{j,ex}\}\|^2_{Q_j} \tag{2}$$

where the choice of the Q norm is an important issue. Using a norm that takes into account an estimated variance of the various components of y_{Test} seems most appropriate. Various energy based metrics have also been considered in [8] although the motivation for using a energy norm on test results is unclear.

Modeling errors are taken into account using the norm of a dynamic residual. Natural dynamic residuals are $R_j = Z(\omega_j)\phi_j$ for modeshapes and $R_j = Z(\omega)q - F$ for frequency response to the harmonic load F. Since the residuals generally correspond to generalized loads, they should be associated to displacement residuals and an energy norm. A standard solution [11] is to compute the static response to the residual and use the associated strain energy

$$\|R_j(q_{ex})\|^2_K = \{R_j\}^T \left[\hat{K}\right]^{-1} \{R_j\} \tag{3}$$

where $\hat{K}$ is the stiffness of a reference FEM model and can be a mass-shifted stiffness in the presence of rigid body modes. Variants of this energy norm of the dynamic residual can be found in [8].

Given metrics on test and modeling error, one uses a weighted sum of the two types of errors to introduce a generalized least-squares problem

$$min_{q_{j,ex}} \|R(q_{j,ex})\|^2_K + \gamma_j \epsilon_j \tag{4}$$

or, as proposed in [12], only set a bound on test errors and minimize the modeling error metric (solve a least-squares problem with a quadratic inequality LSQI). Note that a way to solve the LSQI problem is to change the relative weighting (γ_j coefficient) iteratively until the desired bound on test error is reached.

While this class of formulations is more general, gives a general mechanism to create so called "oblique projectors" [13], and has received significant attention from many research groups (see [14,12,8] to only cite a few), it has only rarely been applied to large industrial models. The reduced basis version of these methods proposed in [7] allows simple implementations and minor computational costs so more applications should be made in the near future.

This class of methods will be illustrated in section 3 using two approaches. Minimum Residual Dynamic Expansion (**MDRE**) where ϵ_j is constrained to be zero and the norm of the dynamic residual (3) is minimized. MDRE with test error (called **MDRE-WE**) based on the solution of (4) with γ_j set iteratively so that the relative error on tests data is less than 10%.

2.3 Sensors, DOFs, model reduction

This section presents the notions of observability equations and model reduction, that are needed for general implementations of the proposed methods but are rarely presented in the literature on shape expansion methods.

Models of structural dynamics typically are second order differential equations, which in their discretized form

$$[M_{FE}]_{N\times N}\{\ddot{q}\} + [K_{FE}]\{q\} = [b]_{N\times NA}\{u(t)\} \tag{5}$$

are described by a number of DOFs $\{q\}$. When building such models using the finite element methods, DOFs usually correspond to translations or rotations at nodes. The physical meaning of DOFs $\{q\}$ depends on a particular finite element model. If you model the same structure with a different mesh, the DOFs won't be the same and yet the two models are comparable. If you model shells using special thin solid elements, you will not have rotational DOFs.

Tests give estimates of physical quantities. In experimental modal analysis one typically measures translations at points. In special applications rotations and deformations are also considered.

Measured physical quantities can, and should, be defined independently from the DOFs by an observation equation. This is a standard procedure in control theory but not in the modal analysis community. In all applications (measured translations, rotations, deformations, ...), the physical quantity $\{y\}$ is a linear function of the DOFs $\{q\}$

$$\{y(t)\}_{NS\times 1} = [c]_{NS\times N}\,\{q(t)\}_{N\times 1} \tag{6}$$

The observation matrix $[c]$ is Boolean if the measurements are made at FEM nodes using sensors placed using the FEM coordinate system. In most industrial applications, measurements do not verify this condition so that a procedure to approximate $[c]$ is needed.

Since measurements of quantities other than translation are rare, the main problem is the non-coincidence of test and FEM nodes. Solutions found in various modal analysis packages are

- find the closest FEM node and ignore the distance
- move the closest FEM node to the test node position
- add the test node to the FEM model and put a rigid link between it and the closest FEM node
- provide tools to build an observation equation without modifying the FEM

Section 3.1 will analyze the impact of ignoring test/FEM node distance (first approach) for a single sensor at a time while using an observation equation based on rigid link with interpolated rotations (last methods) for the other sensors. Ref. [7] also discusses these choices.

The details of how to implement expansion methods are always presented using the assumption that measurements corresponds to DOFs. The **equivalence** property for classes of dynamic models states that the u, y relationship in eqs. (5)-(6) is identical when using DOFs q or a non-singular transformation $\{q\} = [T]\{\tilde{q}\}$ of the model

$$[cT]\left[T^T Z_{FE}(s) T\right]\left[T^T b\right] = [c][Z_{FE}(s)][b] \tag{7}$$

Provided that the observations are independent (c is full rank), it is thus always possible to numerically build a coordinate transformation such that $[cT] = \left[[I]_{NS\times NS}\ \ [0]_{NS\times(N-NS)}\right]$. This transformation must be built carefully to optimize computational times and avoid numerical conditioning problems but the author's experience is that it is possible to do so [15].

The minimization of modeling error (3) is totally out of reach for full order models of industrial size. As proposed in [7], the solution is thus found by projecting the model on a reduction basis combining static modes associated with the sensors and low frequency modes of the model. For model updating the modeshape sensitivities would typically be added.

3 A TRIP THROUGH COMMON PROBLEMS

3.1 About the example

During 1995 and 1996, 12 members of the GARTEUR Structures and Materials Action Group 19 tested a representative structure shown in figure 1. The results of this Round-Robin exercise have been publicized in different papers [9] and some of the test data is publicly available (contact the author for more information).

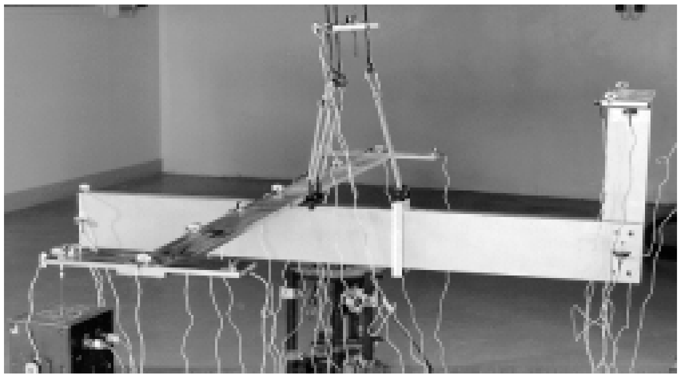

Figure 1: General view of the GARTEUR Structures and Materials -Action Group -19 testbed

This study uses a simple 1032 DOF/124 element model of the testbed. The nominal 24 sensor configuration (figure 2), and the first 14 flexible modeshapes. Computations are performed using the finite element code, correlation and expansion tools of the *Structural Dynamics Toolbox* [15].

Since the objective is to evaluate accuracy, one only uses analysis data for which the exact result is known. The criteria used for the comparison are the mean MAC for the first 14 flexible modes (this should be close to 1) and the mean

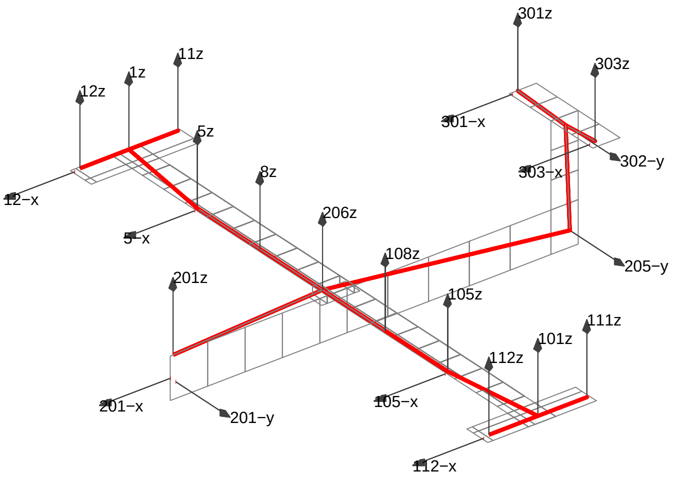

Figure 2: Sensor configuration, wire-frame display and FEM mesh.

deviation from 1 of the Pseudo Orthogonality Check (POC)

$$\text{Mean}\,|\text{POC-1}| = \frac{1}{14}\sum_{j=1}^{14}\left|\{\phi_j\}[M]\{\tilde{\phi}_{ex}\} - 1\right| \qquad (8)$$

where $\tilde{\phi}_{ex}$ is either the expansion of the observed vector $[c]\{\phi_j\}$ or the expanded vector mass-normalized to 1 (as $\{\phi_j\}$). The deviation from 1 of the POC should be small (much below 0.1 if possible).

The mean MAC gives an indication of differences between test data and the observation of the expanded vector (test error). The mean deviation of the POC gives an indication of modeling error and is a better indication ability to use the expanded shapes to analyze model properties.

Note that to emphasize methods that work well, ordinate scales in the following figures are often selected to truncate high error values.

3.2 Errors linked sensor locations

The various methods used to manage the non coincidence of test and FEM nodes (see section 2.3) have a very significant impact on expansion methods.

The problems encountered are illustrated here by building reference "measurements" through observation of exact modeshapes using an elaborate technique (rigid link shown as thick lines in figure 3 with interpolated rotations using nodes shown with circles). One then corrupts the expansion by replacing, sequentially for each sensor, the true test node position by the nearest FEM node. This is really a test set-up verification tool but it illustrates quite well the impact of sensor location errors on expansion methods.

Figure 4 illustrates problems linked to *modal expansion* methods by comparing three target mode selections. Modes 7:20 are the first 14 flexible modes (those used to evaluate accuracy). Modes 1:20 add the rigid body modes to the target

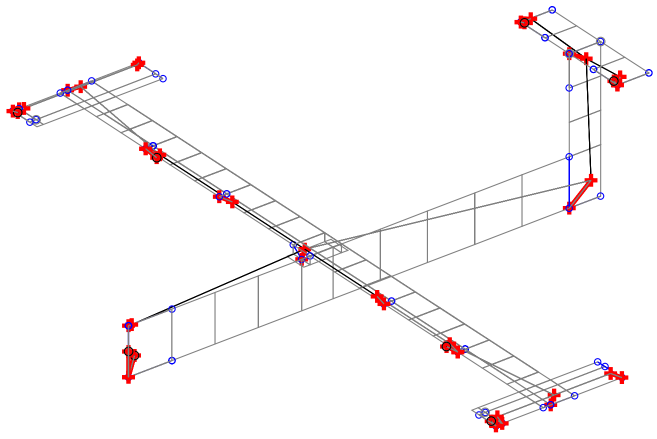

Figure 3: Links used to observe motion at sensor locations

mode set. Modes 7:30 use 24 flexible modes (as many as sensors). The MAC results are only affected (and not much) for mid-wing sensors 5z and 8z. The POC results indicate very high errors for some modes when non-target modes are retained (rigid body or higher frequency).

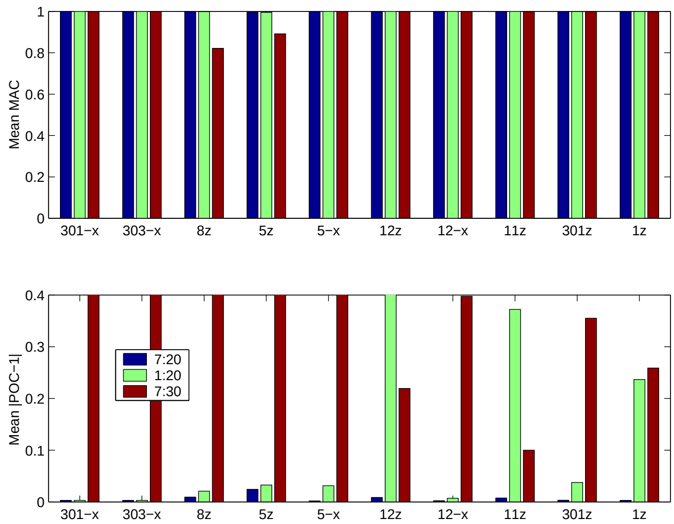

Figure 4: Impact of neglecting sensor/FEM node distance on various sensors when using modal expansion and different target mode selections

The high POC errors indicate that modal expansion has no good mechanism to guarantee the physical signification of the expansion result. This is illustrated in the expanded deformation of figure 5. For mode 2 using modes 7:30 as targets and ignoring test/FEM node distance, the exact verification of the motion at sensor locations with a slight offset leads to strong level of wing bending near the wing/fuselage connection. This effect is clearly not physical at such a low frequency. But all target modes are treated the same by modal expansion which leads to this poor result.

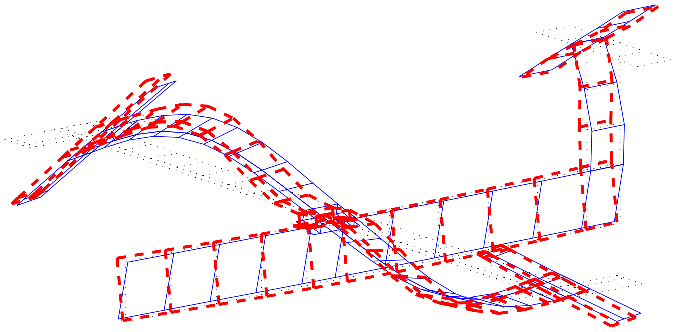

Figure 5: Problem with modal expansion : the strong bending near the wing/fuselage connection is not physical

Figure 6 illustrates the performance of static, dynamic and minimum residual expansion methods (in this case one imposes the exact match of expanded results at sensors positions $\epsilon_j = 0$). Errors on sensor positions have minimal effects on MAC comparisons since these methods preserve the measured response.

The POC errors are relatively poor for static expansion because of high frequency modes. This is well illustrated in figure 7 where one sees that the two horizontal sensors cannot capture fuselage bending for static expansion (a third sensor located at the middle of the fuselage would solve the problem). A simple way to determine the frequency limit for static expansion is to fix motion at sensors and to compute the lowest fixed interface frequency (see [16]). In the present case, this frequency corresponds to a fuselage bending mode at 44 Hz.

Figure 6 shows the errors found for the dynamic and minimum residual expansions to be quite small except for dynamic expansion with an error on the position of sensor $201-y$. This major error is mostly due to mode 5 where the sensor is very close to a node line so that the position error leads to a sign inversion and the very poor result found.

3.3 Expansion in presence of test data errors

The next difficulty is linked to the fact that test results are never exact. For an exact model, one expands corrupted modeshapes $\{c\phi_j\} + \{\Delta_j\}$ where $\{\Delta_j\}$ was taken to be a vector with components following a normal distribution of variance the $1/10^{th}$ of the mean amplitude of $\{c\phi_j\}$ over the 24 sensors and 14 first flexible modes. This variance leads to errors on modeshape observations that would typically be considered as negligible (worst MAC at .98).

Figure 8 shows the POC deviations obtained for a series of 20 random corruptions on the observations of the first 14 flexible modes.

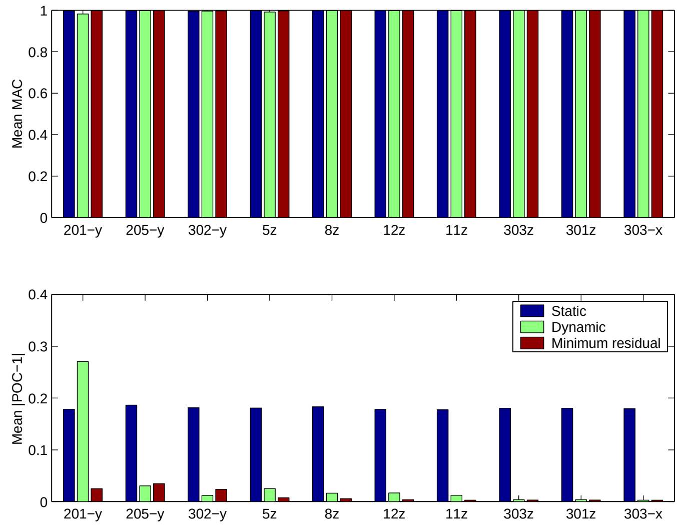

Figure 6: Impact of neglecting sensor/FEM node distance on various sensors when using static, dynamic and minimum residual expansion.

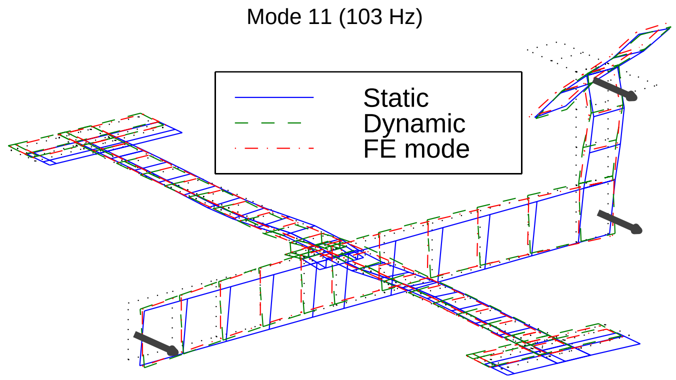

Figure 7: Problems with static : the two horizontal sensors cannot capture fuselage bending for static expansion.

The modal expansion is considered in the favorable case where only the 14 target modes are retained. The smoothing effect linked to using less target modes than sensors is clearly beneficial and modal expansion is, in this case, a very good method with respect to noise sensitivity.

The frequency limit of 44 Hz for static expansion corresponds to the first 5 modes. One indeed sees that static and dynamic expansion give similar results up to this frequency and that static expansion rapidly deteriorates afterwards. Note that static expansion results are poor, they are not significantly worse in the presence of noise. This is really just a bad case for static expansion of anything but the first 6 modes (mode 6 is even above the frequency limit).

Renormalization of the expanded vector deteriorates correlation for both static and dynamic while this effect is less visible

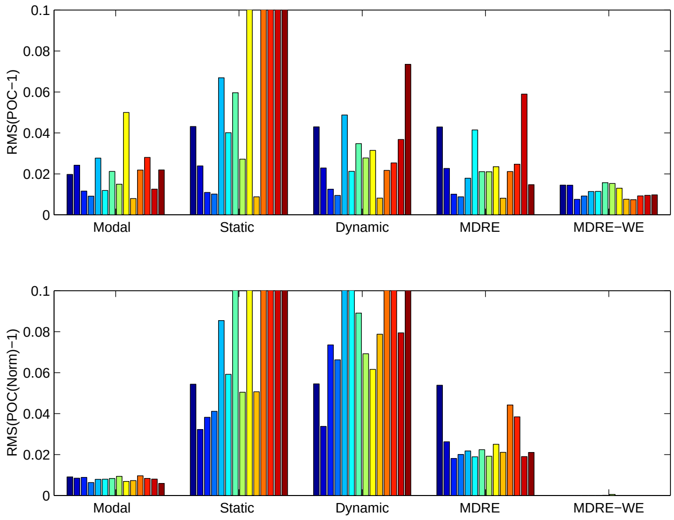

Figure 8: Impact on expansion results of random noise added to modeshapes

for MDRE. These three methods do not account for test errors and the sensitivity to this effect is clear. Between the three MDRE seems the best suited.

Renormalization improves results for modal and MDRE-WE. This is easily explained by the fact that

$$\min_{\alpha} \|\alpha\{c\phi_j\} - \{\{c\phi_j\} + \{\Delta_j\}\}\| \tag{9}$$

has no reason to be found for $\alpha = 1$ since $\{c\phi_j\}$ and $\{\Delta_j\}$ are not uncorrelated. It must be noted however that renormalization can be performed for modeshape expansion but is not applicable for measured frequency responses which are thus systematically biased.

The MDRE-WE result leads to a new and very interesting conclusion: the shape resulting from the expansion is exact but its amplitude is biased. This would however not hold if the true errors on modeshape observations were larger than the assumed ones and if modeling errors were present (see section 3.4).

3.4 Impact of model errors

To illustrate the impact of modeling errors, a deteriorated model with no concentrated masses, the viscoelastic constraining layer removed, and a 50% decrease of the tail stiffness is considered. The comparison of modeshapes for the erroneous and exact models clearly indicate that these modifications significantly deteriorate the correlation. It was also verified that these modifications had a significant impact on both frequencies and modeshapes (including modal crossing).

Figure 8 indicates that modal expansion performs very well (again with a favorable selection of target modes). Static expansion is only acceptable for the first few modes (but again the sensor configuration is not favorable). Dynamic expansion is better than static but shows very significant problems on some modes. MDRE results are very good and it is the best method in this case.

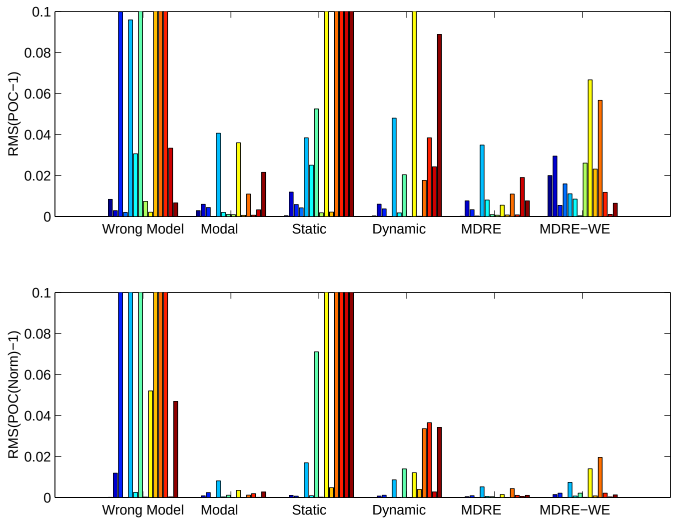

Figure 9: Impact of modeling errors

Depending on the relative weight given to test and model errors, MDRE-WE results go from the wrong model (test data not used) to MDRE (test data assumed exact). Here, γ_j was adjusted in (4) so that ϵ_j is less than 10% of $|y_{j,Test}|$. This clearly illustrates the fact that MDRE-WE is trade-off between an unavoidable level of test data errors and the use of FE model that is always incorrect to some degree. If both types of errors are high, the algorithm cannot be expected to give good results.

4 CONCLUSION

While all methods work on many cases, some are more robust than others. It is of course abusive to draw conclusions from a single example but the results shown here are coherent with the author's experience with other applications, so that it seems appropriate to conclude on the following trends summarized in table 1 and detailed below.
Modal based methods perform very well when an appropriate set of target modes is selected. The only but essential limitation seems to be the absence of design/verification methodologies. Furthermore is unclear whether a good selection always exists.

Static and dynamic expansion really give similar results when the frequency limit of the static condensation is much above the last target frequency. This was not illustrated in this paper, but can be obtained by an a priori design of the sensor configuration. The main problem of these methods is that they don't have a mechanism to account for test errors (bias in identification, test node location errors, ...).

Minimum residual dynamic expansion (MDRE) is clearly superior to static and dynamic because it emphasizes the physical meaning of the expanded response (close verification of the FEM equations expected to hold for each type of measured

TABLE 1: Possible impact of various sources of error on expansion accuracy

Method	Modal	Static	Dynamic	MDRE	MDRE-WE
Sensor position errors	low to high	low	medium	low	low
Error on test	low to high	high	high	medium	low
Model errors	low to high	medium	medium	low	medium
User choices	high	No	No	No	medium

response). The associated numerical cost is however only acceptable when using reduced bases. When using the typical basis combining low frequency modes and static responses, MDRE appears as a dynamic combination of modal and static expansion. The main limitation of MDRE is that it does not account for test error.

MDRE with bounds on test data error seems a near ideal method. It is the only method that clearly shows the trade-off between test and modeling error which is a fundamental limitation of all estimation techniques and thus of expansion methods. Remaining questions are linked to model reduction (since this method can only be used on reduced models), the selection of norms used to evaluate modeling and test errors, and the incorporation of statistical information about measurements (the development of such information is another research topic).

The author hopes that these results will encourage people to implement other methods than the widespread modal and static expansion methods which work great very often but show poor robustness to very usual problems.

Acknowledgment This work was partially funded by Aérospatiale and the Eureka/Sinopsys project.

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