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FROM TRANSIENT SIMULATIONS TO EXTENDED COMPLEX MODE ANALYSIS. SQUEAL SIMULATION OF AN INDUSTRIAL AUTOMOTIVE BRAKE

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ABSTRACT - This paper presents advances in non-linear simulations for systems with contact-friction. Presented applications deal with squeal analysis of industrial (600,000 DOF) brake models. First a reduction method allowing to keep the exact system real modes is proposed for all simulations. For transient simulations a modified non-linear Newmark scheme is used to evaluate the apparition of limit cycles. A space-time limit cycle decomposition is then performed to correlate complex modes and the limit cycle. Although the shapes are close, the comprehension in the system behaviour is much finer in the time domain; in particular, since saturation patterns are then available. Since transient computations remain costly (12h), the end of the paper focuses on quick continuation methods to evaluate saturation threshold based on complex modes trajectories. Encouraging results are obtained, making the concept viable for industrial systems.

1. INTRODUCTION

Automotive brake design is nowadays oriented towards an optimized weight/performance ratio which tends to generate noisy systems. High friction coupling happening at the pad/disc interface is responsible for self-sustained instabilities in the audible frequency range. The noise can attain 120dB in the brake vicinity and is known as squeal between 1 and 16 kHz or moan under 1kHz. Squeal, unlike low frequency vibrations, does not alter the brake performance and happens mostly in low pressure, low speed conditions. The perceived quality is however altered, as the driver's feeling is disturbed, and the environmental nuisance is not welcome.

Silent brake design methods are mainly empirical and difficult to control, due to modelling issues (e.g. contact complexity) or due to implementation difficulties. Classical design methods for brake vibrations are set in the frequency domain and widely spread in industry. This approach is coherent with experimental results on unstable mode lock-in patterns, see for example Massi *et al*. [1]. The system is linearized around a working point, function of global parameters such as the friction coefficient or the braking pressure, to apply Lyapounov theorem to compute complex modes. The system stability is here related to the damping of its poles. This method shows great limitations as it provides growth ratios at a given deformation state. The resulting ranking of unstable mode by the real part of their poles is biased as no information is obtained about vibration levels once limit cycles are attained. Such observation have been made for example by Sinou *et al.* [2,3], or Lorang [4].

Working in the time domain allows simulating the system with its full non linearities and then gives a clear view of the brake stability. Such implementation raises many issues - a direct simulation on a full industrial model would require prohibitive computational costs. Contact handling requires relatively small time steps around 10^{-6}s which makes long (100ms) simulations difficult to handle. For the industrial brake application, illustrated in figure 1, using a non linear implicit Newmark scheme on a 600,000 DOF system would actually generate over 1TB of data in over 700 hours. These issues are dealt with model reduction techniques and an adaptation of the Newmark scheme to non linear penalized contact vibrations, briefly presented in section 2, and in [7]. The simulation cost becomes then affordable, yielding from 500MB to 5GB of data in 12 hours.

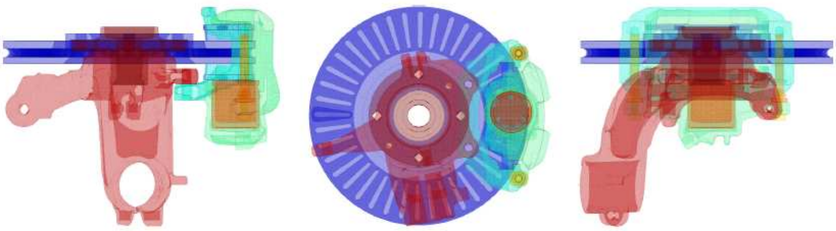

Figure 1: Presentation of the application case, a full brake system provided by Bosch

From the time simulations obtained, the analysis presented in section 3 focuses on the evaluation of changes from the nominal complex modes using shape correlation and a dynamic stability analysis. The focus is set on the saturation pattern responsible for the limit cycle.

Since transient simulations are costly and static complex modes are easier to work with in the scope of design oriented analysis, a methodology is investigated in section 4 to enhance the complex mode information at a cheaper computational cost. The suggestion aims at evaluating complex mode evolution with the amplitude of their trajectory, which has a clear relation with continuation methods commonly used in the framework of the non-linear normal modes [5]. Pseudo dynamic stability diagrams are computed over complex mode trajectories showing interesting results in section 4.2. Section 4.3 eventually explores finer complex mode continuation ideas.

2. EFFICIENT METHODS FOR INDUSTRIAL SQUEAL SIMULATIONS

Simulation of large industrial models in the design and validation processes has a number of implications which are not met by current computational performance. Transient simulation of such systems is not directly available in reasonable time. Two levers are proposed to tackle the problem; a reduction method to decrease the system size is presented in section 2.1.Contact formulation choices are presented in section 2.2. An efficient time integration scheme based on an implicit non linear Newmark scheme has been used. It is not detailed here, but all details can be found in [7].

2.1 Reduction method adapted to large models with local non-linearities

Computational power improvement and algorithmic advances like Automated Multi Level Solvers allow solving systems over a few million DOF on usual workstations. The full real modes of large system are thus accessible, at least in nominal configuration, which opens the way to new reduction methods more adapted in several applications than the traditional Component Mode Synthesis (CMS) method [6].

CMS was indeed based on the assumption of component independence, static solution capability and explicit boundary coordinates. Interaction information is thus a priori ignored, such that the full finite element basis of their interface is kept. The target application is however different from the aim of this study, which is to reproduce dynamic vibrations of an automotive brake working near a static steady state.

Brake squeal models rely on the quality of the pad/brake interface, as it is the location of the main instability. The idea for time simulations is thus in a first approach to consider the rest of the system as linear, based on the pseudo-periodic initial state. All DOF in the vicinity of this contact area are thus kept explicitly, noted as q_c. The remaining of the system DOF are noted q_i.

To achieve the accuracy objective of exact dynamic behaviour, the trace of the exact real modes of the assembly is used as Rayleigh-Ritz vectors. As only the q_i DOF will be reduced, the trace (or restriction) of the Rayleigh-Ritz basis on this part is only considered. The reduction basis, illustrated in figure 2, is then expressed as

$$\begin{Bmatrix} q_i \\ q_c \end{Bmatrix} = \begin{bmatrix} 0 & \begin{bmatrix} \Phi_{|i} & q_{0|i} \end{bmatrix} \\ I_c & 0 \end{bmatrix} \begin{Bmatrix} q_R \\ q_c \end{Bmatrix} \quad (1)$$

The pad/disc section is kept unreduced, while all remaining parts are reduced in a super-element. In the process, the interface DOF are implicitly reduced on the system real modes.

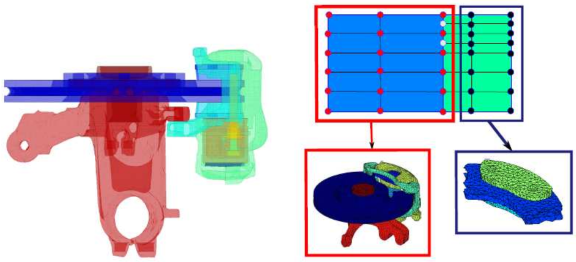

Figure 2: Industrial brake and model reduction strategy for non linear simulations

To generate a cinematically coherent coupling between the finite element part and the superelement, the reduction basis must in addition contain the stationary condition q_{0i}. The final time model features then 30,700 DOF, with a reasonable sparsity. More details, on this reduction method can be found in [7].

2.2 Contact-friction modelling

Contact/friction modelling is commonly split into two formulation strategies contact, giving normal forces f_n, and friction giving tangential forces f_t depending on the friction coefficient μ. The exact Signorini/Coulomb laws represent contact for ideally smooth surfaces, while functional representations take into account a level of asperity compression through a controlled interpenetration.

A functional representation has been chosen here, using an exponential stiffness, as illustrated in figure 3. Practically, a relationship between the gap and contact pressure is established to account for an approached contact constraint. Implementation details and numerical aspects can be found in [7]. The exponential contact law retained is thus defined at each contact point by

$$p(g) = p_0 e^{-\lambda g} \quad (2)$$

where p is the contact pressure, g is the gap, p_0 and λ are parameters to define depending on the interface properties. It can be noted that the use of an exponential formulation still allows rather brutal non linear events in case of local over-constraints and opening.

Friction implementation follows the definition of the Coulomb law, which for two solids relates the sliding velocity and the friction forces. A basic regularization is shown in figure 3 and considered in the study. Low sliding velocities (noted w_s) are penalized through the introduction of a parameter k_t, such that

$$\begin{cases} f_t = k_t w_S \text{ if } \mu \cdot w_S \leq k_t \cdot f_n & (3) \\ f_t = \mu \cdot f_n \text{ else} \end{cases}$$

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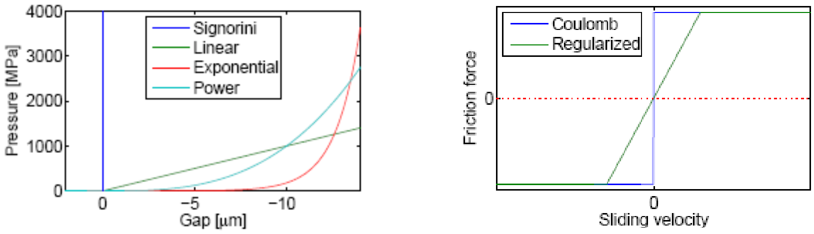

Figure 3: Sample contact (left) and friction (right) laws.

3. TIME/FREQUENCY ANALYSIS

Stability and transient analyses are performed in sections 3.1 and 3.2. A correlation is proposed using the Singular Value Decomposition (SVD) in section 3.2.1. In depth analysis of the limit cycle is given in section 3.2.2.

3.1 Sliding stability of the periodic solution

At 12 Bar, the brake system provided features several unstable modes, following the stability diagram of figure 5.

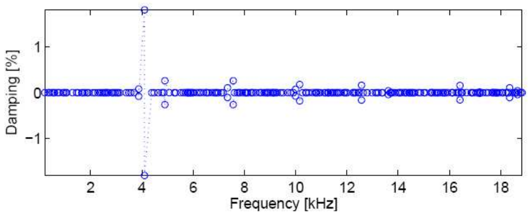

Figure 5: Brake stability diagram at 12 Bar

Some unstable modes can be highlighted, in particular complex modes C44 and C51, whose shapes are shown in figure 6. The illustration is derived from the Component Mode Tuning (CMT) method presented in [7,8], which allows integrating explicit component information at the system level.

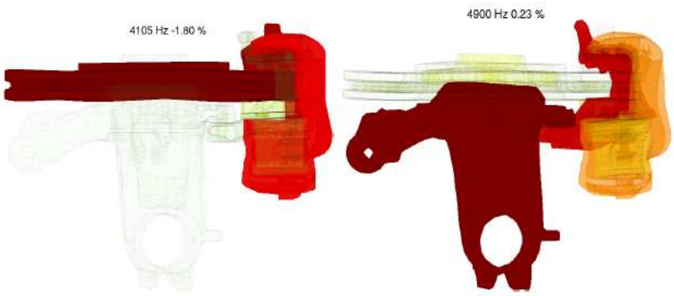

Figure 6. Shapes of unstable complex modes C44 (left) and C51 (right). Colors from blue to red ranking in ascending elastic strain energy per component relative to the total strain energy.

Complex mode 44 shows a component interaction between the disc, outer pad and caliper. Complex mode C51 shows a knuckle/anchor interaction with little participation of the disc and pads. Structural effects thus seem important in this brake system, as components outside the pad/disc interface are significantly involved in the deformations. This is here a great justification of the modeling choices that allows using models with very refined geometries.

3.2 Transient simulations and correlation to complex modes

A 100ms transient simulation is obtained in 12h using the methods presented in section 2; the resulting braking torque, and vibration levels are presented in figure 7. After a modulation period, the signal becomes very stable, this result is called a limit cycle although no further characterization is attempted. This purely practical definition is deemed sufficient for industrial applications.

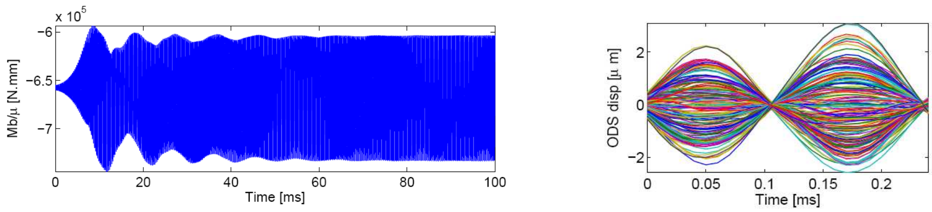

Figure 7: Installation of a limit cycle over a 100ms transient simulation performed on the industrial brake. Displacement result on the disc surface in the end of the time response (μm).

3.2.1 Time/frequency correlation using SVD

The limit cycle extracted from the end of the transient simulation in figure 7 should be correlated to the complex modes of the initial deformation. This limit cycle constitutes a non-linear normal mode, following the definition given by Kerschen *et al.* [5]. The transient simulation output is indeed a family of shapes and a frequency defining a periodic cycle. Several shape identification methods exist, as proposed by the author in [7] or for example by Lorang in [4].

The method presented here performs a space-time decomposition of the limit cycle through an SVD. From a response vector, the SVD extracts a deformation basis ranked by amplitude in the provided cycle, and their transient participation evolution.

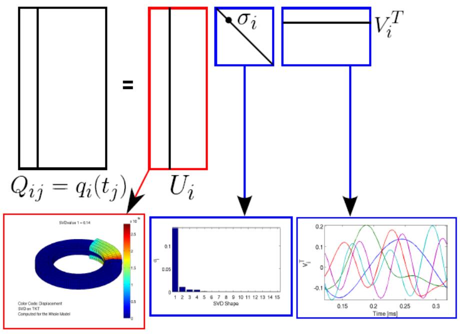

Figure 8: Space time decomposition of a limit cycle with the SVD

The concept is illustrated in figure 8, and can be related to what is performed in *a posteriori* Proper Orhogonal Decomposition (POD) methods [5]. A limitation of the direct SVD application is the absence of a mechanical norm. This can be improved by performing the SVD on the strain energy, as presented in [7].

The limit cycle application result is provided in figure 9. It can be seen that the cycle is basically of dimension 2. With a main instability at 4kHz and an harmonic at 8kHz. The first shape, presented in figure 9 (left) shows a contact opening pattern at the rear side of the outer pad.

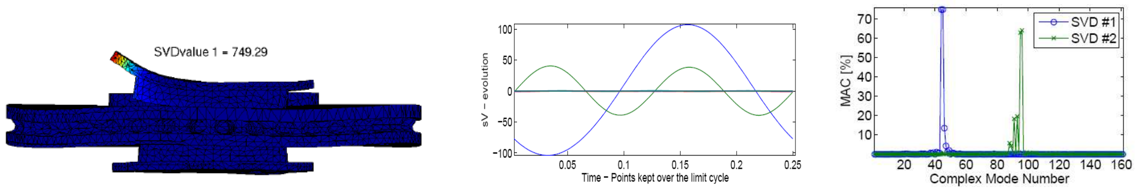

Figure 9: SVD applied to the computed limit cycle. Left: main deformation shape. Middle: amplitude in the response. Right: correlation by MAC with the nominal complex modes.

These shapes are well correlated to complex modes, as shown in figure 9. These modes are however fully unstable and cannot explain the apparition of a limit cycle over a full divergence. A more in depth analysis must thus be performed.

3.2.2 Dynamic stability

A specific post-treatment is performed from the transient simulation presented in figure 7. The system state in displacement/velocity/non linear forces for a time sampling over the limit cycle is exploited to evaluate the stability as function of time. The resulting diagrams are consequently called *dynamic stability diagrams*. Two representations of these diagrams are presented in figure 10 for the frequency band of interest (mode C44, 4kHz).

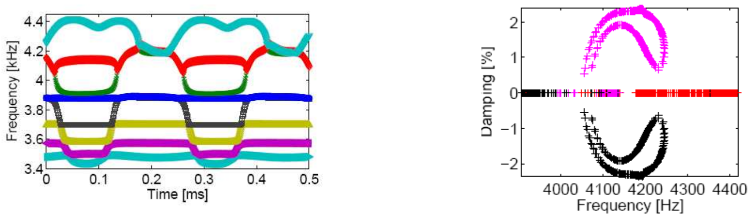

Figure 10: Dynamic stability diagram in the 4kHz frequency range in the limit cycle

Clear mode coalescence patterns are observed, which can be interpreted as the effect of a contact opening pattern observed in figure 9. The frequency diagram shows in particular the variation of a complex mode in the squeal frequency range. The frequency/damping diagram in the same frequency range shows in addition a stable/unstable transition.

By tracking the complex mode showing large frequency variations in figure 10, and computing component wise strain energies, shape evolution can be assessed as presented in figure 11.

Figure 11: Evolution of the component strain energy repartition as function of time in the limit cycle.

As a first observation, the shape is linked to the one of complex mode C44. Significant changes however occur as the strain energy distribution evolves. The indistinguishable pad/disc/caliper interaction given by mode C44 shape can here be decomposed in a specific sequence.

At the beginning of the cycle, the pad/disc interaction is preponderant and is the cause of the instability. As the pads follows the trajectory given by mode C44, the outer pad rear side opens contact, and increases its coupling with the caliper, which consequently comes into high strain energy. At this state, the system becomes stable and the caliper pushes back the pad onto the disc. Once the outer pad rear contact closes again the instability restarts.

4. EXTENDED COMPLEX MODE ANALYSIS

4.1 Introduction

The observations obtained in section 3 match common concepts of the non-linear mechanics literature fields. The centre manifold theory [2,5] starts from the idea that at a certain threshold, a fully stable system will have a mode coming into an area of controlled instability.

The classical formalism exploited in the centre manifold theory is however limited, as it assumes a fully stable system for a starting point. This conducts most authors to consider a set of control parameters including the friction coefficient as a cause of instability trigger, it was however demonstrated here that a constant friction coefficient is sufficient to evolve to a limit cycle. A second limitation comes from the consideration of a single complex mode for limit cycle evaluations, which tends to over-simplify the instability mechanisms in such system. In the cycle found here, two shapes were found to dominate the response but others were involved.

Developments, such as the CNLMA proposed by Sinou et al. in [2], seek to find the limit cycle amplitude associated to the unstable complex mode detected at the Hopf bifurcation point. The complex mode amplitude is then used as a control parameter to find the limit cycle. Indeed, from the Hopf bifurcation point to the limit cycle point, the system will show a divergence phase, characterized by an unstable mode with a strictly positive real which will come to zero once the limit cycle is attained (no further divergence).

The approach proposed here is, given a set of unstable complex modes, to analyze their stability by processing responses associated with their trajectory at variable amplitudes.

4.2 Simulating complex mode cycles

The free decay trajectory of a complex mode is normally of the form

$$\left\{\Psi_j(t)\right\} = \mathrm{Re}\left(\alpha_j \left\{\psi_j\right\} \cdot e^{\lambda_j t}\right) \quad (5)$$

which has for an unstable mode an increasing amplitude associated with the real part of the pole. In practice, since there is a limit cycle, this representation does not correspond to the physical response. To analyze the complex mode, one proposes to use the trajectory given by

$$\left\{\Psi_j(t)\right\} = \mathrm{Re}\left(\alpha_j \left\{\psi_j\right\} \cdot e^{i\omega_j t}\right) \quad (6)$$

associated with period $2\pi / \omega_j$ and trajectory of amplitude α_j.

Figure 12 illustrates the pseudo-trajectory based on mode C44, plotted on a point of the disc surface. The range of vibration amplitude is from 0 to 4 μm as function of the coefficient amplitude, which corresponds to the displacement values that can be experimented with brake systems.

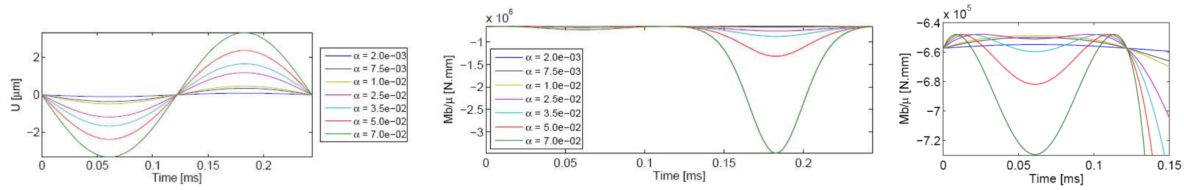

Figure 12: Complex mode C44 trajectory as function of the amplitude coefficient, left: displacement on the disc surface over the trajectory. Middle: associated braking torque. Right: Braking torque zoom in (first half).

The braking torque associated with the trajectory is also shown in figure 12. The cycle starts with the brake unloading as the outer pad rear side separates from the disc. In the second half of the cycle, the outer pad comes back into contact with an over-penetration peak. For all these trajectories, the pseudo instantaneous tangent states can be evaluated and new complex modes can be computed. The evolution of the stabilities and instabilities can therefore be assessed.

The following results focus on one sample amplitude, α=3.5.10^{-2}. This gives maximum displacement amplitude of 2.3µm. The patterns of figure 10 and figure 13 are directly comparable which clearly shows the pertinence of using complex mode trajectories. The absence of mechanical equilibrium is however a great limitation.

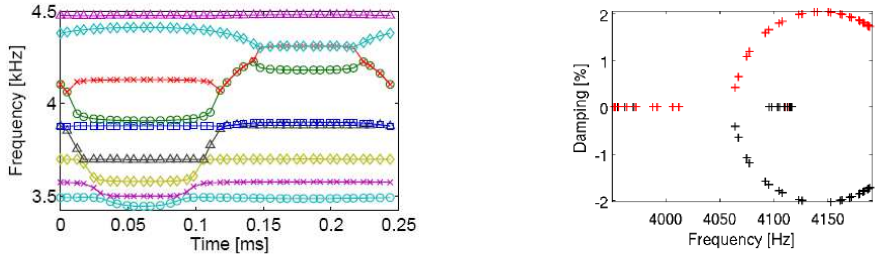

Figure 13: Pseudo dynamic stability diagram over mode C44 trajectory

4.3 Non-linear analysis of complex mode cycles

To improve the results of section 4.2, it is proposed to simulate complex mode cycles but respecting the mechanical equilibrium at each step. The chosen strategy considers the complex mode trajectory driven by its velocity and acceleration, while the displacement must comply with a mechanical equilibrium. At each step of the pseudo time, a non-linear static resolution is thus performed, with the inertial and damping forces taken as a constant load input, while the new displacement and contact-friction forces are updated. Using assumed acceleration $\ddot{\psi}_j$ and velocity $\dot{\psi}_j$, one resolves $\left(\tilde{\psi},\, f_{nl}\right)$ such that

$$\tilde{\psi}_j(t_i) = K^{-1}\left(f_{ext} + f_{nl}(\tilde{\psi}_i(t_i)) - M\ddot{\psi}_i(t_i) - C\dot{\psi}_j(t_i)\right) \quad (7)$$

This resolution produces a succession of quasi-static solutions which can be considered as a cycle, continuity conditions are however not verified between time steps. The braking torque, shown in figure 14 for low vibration amplitudes, is very different from the previous ones. The contact opening in the first phase is associated with a decrease of the braking torque and closing in the second phase with a torque increase. The disc displacement is however very low (0.01 µm) which suggests that the proposed method to enforce the field needs to be refined.

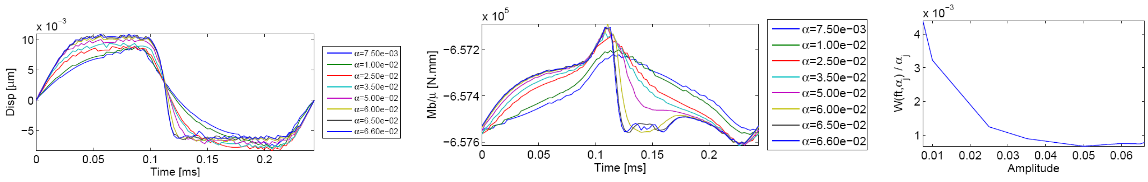

Figure 14: Complex mode trajectory with static displacement equilibrium for low amplitude vibrations. Left: disc displacement. Middle: braking torque. Right: Friction work relative to the amplitude.

The friction work obtained is positive, which is coherent with an instability where friction forces introduce energy into the system. The decrease of this work, relative to the amplitude, is coherent with the fact that to achieve a limit cycle the work of friction forces must converge to zero.

At higher amplitudes the contact closing becomes more brutal. One thus observes a threshold at which the pad compression becomes so high that the displacement pattern is altered on the full system during the peak of contact forces. Figure 15 thus shows that at higher amplitudes transitions effects that still need to be explained. The threshold pattern, which occurs for α=6.7, is confirmed by the computation of the friction work over the cycle, in figure 14, and 15, using the velocity computed by finite differences of the displacement.

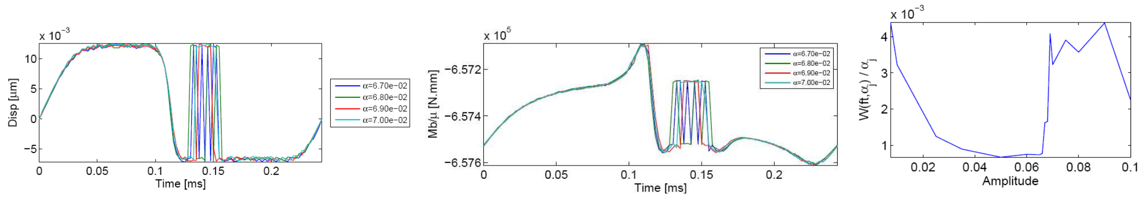

Figure 15: Complex mode trajectory with static displacement equilibrium for low amplitude vibrations. Left: disc displacement. Middle: braking torque. Right: Friction work relative to the amplitude.

5. CONCLUSION

The simulation of systems with contact-friction becomes a stake for more and more applications. In particular, brake squeal appears to be a critical application for the automotive sector.

To the difference of most applications the simulation method proposed here aims at keeping fine geometrical details for all computation. This was attained through an *ad hoc* reduction method, which contained the system size and matrix sparsity, two distinct indicators of the performance achievable for a given finite element model.

Long transient simulations have been performed and correlated to complex modes of the periodic state through the use of SVD. Although comparable, great differences exist in terms of system behaviour between the shapes from the transient and the complex modes.

To improve computation times, it was suggested here to use continuation methods adapted to the problem. For complex modes trajectories, a mechanical equilibrium must be found to get to the observation of saturation patterns. Such simulations, briefly tested in this paper, appear very encouraging to develop for design studies. Computations are indeed much quicker than transient simulations, which could be used for validation only.

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