Analysis and Design Tools for Structures Damped by Viscoelastic Materials

Etienne Balmès, Adrien Bobillot

SDTools, 52 rue Vergniaud, 75013 Paris, France
ECP, MSSMat, 92295 Châtenay-Malabry, France
balmes@sdtools.com

ABSTRACT

Constrained viscoelastic layers have traditionally been considered as damping enhancement mechanisms. More recently bonding has appeared as an alternative to traditional welding strategies and sandwich shells with thick plastic cores have been considered to provide high flexural stiffness for a low overall weight. The present study focuses on computational strategies that can be used to analyze and design the vibroacoustic behavior of such assemblies. Difficulties of the task are the use of experimentally derived complex moduli for the viscoelastic materials, studies in the frequency and temperature domains, integration of the bonded shells into large finite element models, ability to modify the geometry of a particular assembly in an optimization phase or the ability to account for changes in the viscoelastic material properties during the part forming process. The study is illustrated by applications to an automotive oil pan.

1 INTRODUCTION

Constrained viscoelastic layers have traditionally been considered as damping enhancement mechanisms. More recently bonding has appeared as an alternative to traditional welding strategies and sandwich shells with thick plastic cores have been considered to provide high flexural stiffness for a low overall weight.

When studying the vibroacoustic behaviour of such assemblies, using elastic representations of the viscoelastic materials is often not appropriate. Difficulties in creating proper models are the use of experimentally derived complex moduli for the viscoelastic materials, studies in the frequency and temperature domains, integration of the bonded shells into large finite element models, ability to modify the geometry of a particular assembly in an optimization phase or the ability to account for changes in the viscoelastic material properties during the part forming process.

The present study reviews important issues for the modeling of damped structures and illustrates the typical computations performed in a design phase. Section 2 addresses issues linked to element selection and material representation. Section 3 discusses practical solvers for frequency response, eigenvalue and time domain problems.

Finally section 4 illustrates typical computations for the model of an automotive oil pan. An aluminum and two sandwich designs with thin and thick cores are first compared. The robustness of a particular design to temperature variations is considered. Finally, the need to properly account for the relative motion of skins in sandwich designs is illustrated.

2 MODELING STRATEGIES

2.1 FEM models of sandwiches

The easiest approach to represent a sandwich structure is a shell/volume/shell model as shown in figure 1. The shell offsets result in a correct representation of displacement continuity at the bonded shell surface.

Shells are preferred over volumes because volume element formulations are sensitive to shear locking when considering high aspect ratio (dimensions of the element large compared to thickness).

The use of a volume element for the viscoelastic core is acceptable in most applications, because the core is significantly softer than the shells so that almost all its energy is associated with shear [1]. This same fact makes the use of shell elements inappropriate for soft cores. Note finally that shear corrections used in some FEM codes to allow bending representation with volumes must be turned off to obtain appropriate results.

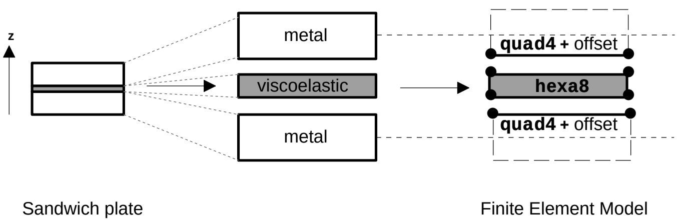

Figure 1: Shell/volume/shell model for sandwiches

Alternatives to this direct use of basic elements is the development of sandwich elements with higher order section kinematics. The problem with this approach is that developing good shell elements is very difficult so that most developments for sandwiches will not perform as well as state of the art shell elements.

Dealing with curved sandwiches, particularly when they are press formed, pauses a number of additional difficulties. Discontinuity in the surface normals between shell elements leads to various possible choices in meshing the core and sharp edges can be difficult to mesh properly when starting with a surface description of the part.

There are furthermore many unknowns in how the forming process affects the core thickness and material properties. In particular, most materials used for their high damping properties are also very sensitive to pre-stress. To the author's knowledge no significant work has been done on characterizing the actual effect of the forming process on the final properties of a curved sandwich.

The final difficulty is to deal properly with boundary conditions of the skin layers. Since differential motion of the skins plays a major role in the effectiveness of the core, the boundary conditions of each skin has to be considered separately. This effect will be illustrated in section 4.3.

2.2 Handling viscoelastic materials

The basic assumption of linear viscoelasticity is the existence of a relaxation function $h(t)$ such that the stress is obtained as a convolution with the strain history

$$\sigma(t) = \int_{-\infty}^{0} \varepsilon(t-\tau) h(\tau, T, \sigma_0) d\tau \tag{1}$$

Using the Laplace transform, one obtains an equivalent representation where the material is now characterized by the *Complex Modulus* E (transform of the relaxation function)

$$\sigma(s) = E(s, T, \sigma_0)\varepsilon(s) = (E' + iE'')\varepsilon(s) \tag{2}$$

For all practical purposes, one can thus, in the frequency domain, deal with viscoleasticity as a special case of elasticity where the material properties are complex and depend on frequency, temperature, pre-stress and other environmental factors.

In practice, the complex modulus is determined experimentally using dynamic excitation. For a given set of material test results, analysis requires knowledge of $E(s)$ for arbitrary values of s or at least of the frequency on the Fourier axis ($s = i\omega$). Three approaches are typically used :

- $E(i\omega)$ is interpolated from tabulated material test data.
- $E(s)$ is represented by a rational fraction

$$E(s) = E_0 \frac{1 + \alpha_1 s + ... + \alpha^{nn} s^{nn}}{1 + \beta_1 s + ... + \beta^{nd} s^{nd}} \tag{3}$$

  Some particular reduced forms of a rational fraction may be used in practice (see section 3.1).

- $E(s)$ is represented using another analytical representation, in particular fractional derivatives [2].

When proper care is taken, all three approaches are capable of closely approximating material test data. They thus have the same "physical" validity. The differences are really seen in how each representation can be integrated in FEM solvers and in the validity of extrapolations outside the tested material behaviour range. On the later point, the actual process used to obtain the parameters has a strong influence, it may thus be easier to obtain a good model with a particular representation even if that representation is not inherently better.

Dependence on environmental factors (temperature, pre-stress, ...) should `a priori` be arbitrary. In practice however, one generally assumes that environmental factors only act as shifts on the frequency [3]. Tests thus seek to characterize a master curve $E_m(s)$ and a shift function $\alpha(T, \sigma_0)$ describing the modulus as

$$E(s, T, \sigma_0) = E_m(\alpha(T, \sigma_0)s) \tag{4}$$

3 PRACTICAL SOLVERS FOR FEM MODELS

3.1 Dynamic stiffness representations

Given a constitutive law described by parameters $E_i(s, T, \sigma_0)$, one can use the fact that element stiffness matrices depend linearly on those parameters to build a representation of the dynamic stiffness a linear combination of constant matrices

$$[Z(E_i, s)] = \left[Ms^2 + K_e + \sum_i E_i(s, T, \sigma_0)\frac{K_{vi}(E_0)}{E_0}\right] \tag{5}$$

This representation is the basis used to develop practical solvers for viscoelastic vibration problems.

Of particular interest are the cases where the E_i have a rational fraction expression. An arbitrary rational fraction (3) that is proper and has distinct poles, can be represented as a sum of first order rational fractions

$$E(s) = E_\infty - \left(\sum_{j=1}^{n} \frac{E_j}{s + \omega_j}\right) \tag{6}$$

By introducing an intermediate field $q_{vj} = -\frac{E_j}{(s+\omega_j)}q$, one can rewrite (5) as a higher order first order problem, which for a single q_{vj} takes the form

$$\left[\begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix} s + \begin{bmatrix} 0 & -M & 0 \\ K_e + E_\infty K_v & 0 & K_v \\ E_j M & 0 & \omega_j M \end{bmatrix}\right] \begin{Bmatrix} q \\ sq \\ q_v \end{Bmatrix} = \begin{Bmatrix} 0 \\ F \\ 0 \end{Bmatrix} \tag{7}$$

Depending on the operators available in the FEM code, one may want to use a second order form. The *Anelastic Displacement field* method [4] thus writes the model as

$$\left[s^2 \begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix} + s \begin{bmatrix} 0 & 0 \\ 0 & \frac{K_v}{E_j} \end{bmatrix} + \begin{bmatrix} K_e - E_\infty K_v & K_v \\ K_v & \frac{\omega_j}{E_j} K_v \end{bmatrix} \right] \left\{ \begin{matrix} q \\ q_{vj} \end{matrix} \right\} = \left\{ \begin{matrix} F \\ 0 \end{matrix} \right\} \tag{8}$$

with possibly multiple q_{vj} for each pole in (6).

But this form has no mass associated with q_{vi} which may be a problem for some solvers. An alternative is the GHM method [5], which represents E in the form

$$E(s) = E_\infty \left(1 + \sum_{j=1}^{n} \frac{\alpha_j}{s^2 + 2\zeta_j \omega_j s + \omega_j^2} \right) \tag{9}$$

and defines fields $q_{vj} = \frac{\alpha_j}{s^2+2\zeta_j\omega_j s+\omega_j^2} q$. Note however that not all rational functions can be represented in form (9).

3.2 Frequency response

The frequency response of a FEM model is obtained by solving a problem of the form

$$[Z(E_i, s)]\{q\} = \{F(s)\} \tag{10}$$

at various operating points (values of frequency, T and/or σ_0).

While most FEM codes will handle one instance of problem (10) easily, typical design studies require computation of a few thousand frequency points at tens of design points thus making direct frequency resolution totally impractical.

Computing the dynamic stiffness using a formulation similar to (5) is a first improvement over the reassembly performed in certain codes.

The next step is to avoid direct resolution (10) since the factorization of Z is expensive. For a given Z, iterative solvers already outperform direct ones [6].

But one can make further use of the fact that one solves many similar problems. Ref. [7] discusses the adaptation to multiple shifted problems of restarted GMRES and similar algorithms which are used classically for the iterative resolution of linear systems of equations. Ref. [8] describes an automated substructuring strategy which might be extended to damped cases. For section 4.1, one will use the approach described in Ref. [9] and summarized below.

Since one solves a class of problems, one can consider preconditioners that are too expensive for standard iterative methods. The tangent elastic stiffness

$$K_0 = Real\left(Z(E_i, 0)\right) \tag{11}$$

is a good candidate with the significant advantage of being real and thus faster to invert.

Spending time to get a good starting guess also makes sense since it will be used many times. As proposed in [10], a basis composed of normal modes associated with K_0 and a correction for the viscoelastic loads generated by these shapes. For the subspace

$$T = \left[\phi_{1:NM}(K_0) \;\; K_0^{-1} \text{imag}(Z(\omega_j, T_0, \sigma_0(0)))\phi_j \right] \tag{12}$$

one estimates the response using a simple model reduction

$$\{\hat{q}\} \approx [T] \left[T^T Z(\omega_j, T, \sigma_0) T \right]^{-1} \left[T^T \right] \{F(s)\} \tag{13}$$

The error associated with this approximation is obtained by computing the strain energy error of the displacement residual

$$R_d = [K_0]^{-1} [Z(\omega_j, T, \sigma_0)\hat{q} - F(s)] \tag{14}$$

If this residual is large, it can be used to enrich subspace T until convergence to the exact solution is obtained [9]. Note that similar residual iterations can be used for complex eigenvalue computations [11].

3.3 Eigenvalue extraction

Poles and modes are non-zero solutions of the generalized eigenvalue problem

$$[Z(E_i, \lambda_j)]\{\psi_j\} = \{0\} \tag{15}$$

Given a material representation, one can distinguish two main strategies to solving (15): algebraic and non-linear solvers.

Algebraic formulations introduce additional fields, as shown in section 3.1, to obtain a standard eigenvalue problem with constant matrices. This approach is also feasible for fractional derivative models [2].

While transformation to a standard constant matrix form allows the use of eigenvalue solvers present in FEM codes, the increase in the number of degrees of freedom can be significant and the high connectivity between elements of the sandwhich means that the sparsity pattern of the considered matrices is rather full. Working on improved solvers that are not too sensitive to the order augmentation can thus be important (see the computational time comparisons in section 4).

Non-linear eigenvalue solvers, search a direct resolution of (15). A full search of the complex plane being impractical for large models, such solvers take into account the expected pattern of solutions. Since the considered damping is still relatively low, one can have meaningful estimates of the complex modes by defining a reference eigenvalue problem where the stiffness is constant.

From this initial estimate, continuation techniques [12] or estimation using specific transfer functions [10] can be used to converge to the true solution. The later solution is the only one applicable for interpolated tabular material data which is only known on the Fourier axis ($s = i\omega$).

Another difficulty for non-linear eigenvalue solvers is the determination of the modeshape scaling condition needed to create a representation of transfer functions as a sum of modal contributions.

3.4 Time domain models

Rational fraction representations of the modulus lead to equations similar to (7)-(8) which are directly related to time domain representations. Model reduction techniques motivated by control or FEM considerations can then be used to cope with large models.

With other representations of the complex modulus using the inverse Fourier transform is often not practical. The solution considered in [13] is to build an equivalent model with viscous damping properties. The underlying principle of this approach is to define a set of transfer functions, that is representative of the dynamics one seeks to approximate, and to use identification techniques, developed for experimental modal analysis, to build a model that has a time domain representation.

4 TYPICAL DESIGN STUDIES

This section illustrates typical computations needed to validate the design of a particular sandwich structure. The example retained is an aluminum oil pan shown in figure 2. The objective of the paper is to illustrate computational strategies and not the technological advantage of a particular solution. The constitutive laws used for the viscoelastic are thus not associated to a particular material on the market.

4.1 FEM model and CPU times

The nominal model has 5561 elements with 8507 nodes (33003 DOF). Sandwich models have 14227 elements and for cases without cuts 12931 nodes (57457 DOFs). The nominal mesh was generated with I-DEAS and the additional elements needed to model the sandwich designs are generated using the MATLAB based *Structural Dynamics Toolbox* [14].

Computations shown in this paper are performed using a 3 parameter model for the viscoelastic cores

$$E(s,T) = E_{max} \frac{\alpha_T s + \omega_{min}}{\alpha_T s + \frac{E_{max}\omega_min}{E_{min}}} \quad (16)$$

$$log_{10}(\alpha_T) = -c_1 \frac{T - T_{ref}}{T - (Tref - c2)} \quad (17)$$

with for the thin core $E_{max} = 10GPa$, $E_{min} = 3GPa$, $\omega_{min} = 300Hz$, $c_1 = 2$, $T_{ref} = 70^oC$, $c_2 = 100^oC$. For the thick core $E_{min} = 8GPa$.

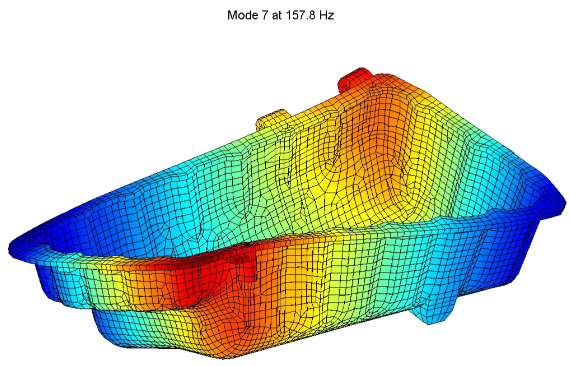

Figure 2: First flexible mode of the oil pan model.

Computational times are obtained on a 64 processor Origin 2000 computer running MSC-NASTRAN [15] version 70.7.0 and SDT 5.0 beta1 [14]. While the solvers are partially parallel, total CPU times are shown in table 1.

TABLE 1: CPU times in seconds of some key steps (design B with 57457 DOFs, N.A. : not applicable)

	NASTRAN	SDT
Assembly of design B	41	N.A.
Factorization of K	50	90
Forward/back. substitution	5.2	2.7
20 normal modes	273	281
Factorization of Z	162	249
Forward/back. substitution	7.9	5.6
20 cpx. modes Hysteric	2042	370
20 cpx. modes visco.	N.A.	477 to 1016
Iterative $Z^{-1}F$ per freq.	N.A.	1 to 20

Both codes use similar multifrontal sparse factorization routines and the differences in factorization time can be attributed to different settings in the approach used to create the elimination trees. For both real K and complex Z matrices, the SDT spends more time optimizing the factorization which results in faster forward/backward substitution. Thus times for normal mode solutions and direct frequency solution are similar.

For the solution of the complex eigenvalue problem with a constant loss factor (hysteretic damping), NASTRAN uses a complex Lanczos algorithm, while residual iterations [11] are used in the SDT. The speedup is significant and it is interesting to note that extensions to a viscoelastically damped model only marginally affects convergence speed for the SDT. The range of times shown in this case corresponds to different temperatures with convergence being slower in cases with higher damping.

Finally the iterative direct frequency solver [9] gives for 1000 frequency points very interesting times per point from 20 to below 1 second depending on tolerances and damping level (quality of initial guess).

4.2 Basic designs

The first step in designing a sandwich is to select the core material and thickness. While vibroacoustic properties are important, other factors such as adhesion, behaviour during press forming, fatigue, ... are also essential but not considered here.

To illustrate, the effect of possible design changes one will consider the following designs of an aluminum oil pan

- A Nominal design : 3.9 mm shell. Weight 2.58 kg.
- B Sandwich with 1.95 mm shells and 0.1 mm highly damped core. Weight 2.73 kg.
- C Design B with transverse cut of the inner skin cut along the width of the pan. Weight 2.73 kg.
- D Sandwich with 0.5mm shells and 3 mm core. Weight 2.04 kg.

In reality, manufacturing designs B,C,D would imply a number of changes in the attachments so that keeping the same model is only interesting for design purposes.

Figures 3 and 4 illustrate the main motivations for sandwich designs in terms of vibration behaviour.

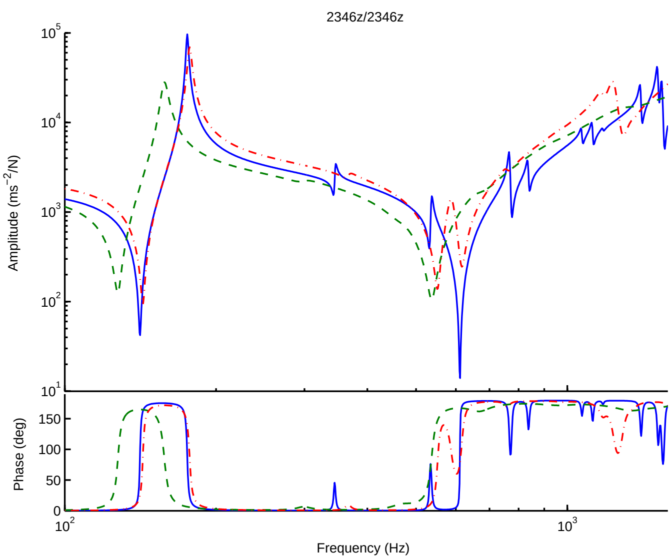

Figure 3: Impedance at center point for designs (—) A , (– –) B (0.3 loss factor in core),(– - –) D (0.05 loss factor in core), 0.01 loss factor in aluminum

Thin viscoelastic cores (design B) can be soft and highly damped. As they are thin, the overall stiffness is not very affected and a significant amount of strain energy is transmitted through the core leading to high overall damping of the modes. Here the damping is more effective at higher frequencies with a major decrease in peak levels of the cross transfer above 800 Hz.

Thick cores (design D) require stiffer materials which are typically less damped. This design is thus interesting because of the mass gain. The improved weight of design D is not great here because the densities of aluminum (2700 kg/m^3) and stiff plastic cores (1500 kg/m^3) are not very different. The ratio would be more interesting for steel sandwiches.

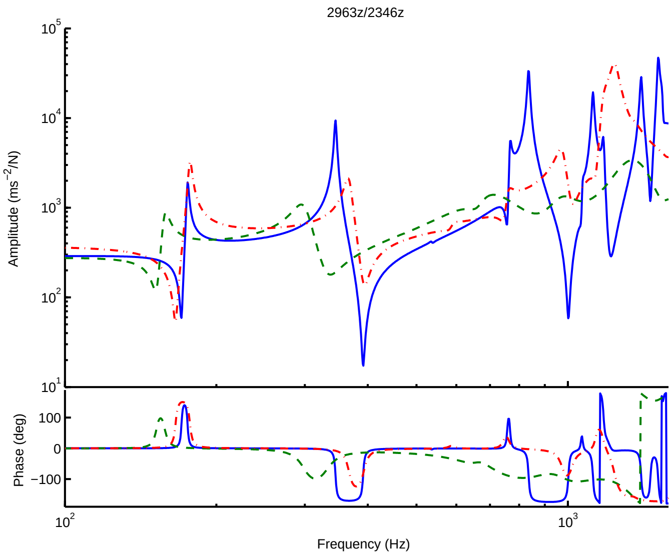

Figure 4: Cross transfer between center and edge for designs A,B,D

4.3 Environmental factors

Metal/viscoelastic/metal sandwiches are very sensitive to environmental parameters, in particular temperature and prestress, which must thus be studied with care to validate the robustness of a particular design.

Accounting for pre-stress requires both a characterization of the influence of pre-stress on material properties and the simulation of the part forming process. Both information are hard to obtain and were not considered in this study although the overall effect is known to be significant.

Designing for a temperature range is fairly straight forward but is a major computational challenge since direct frequency responses or non-linear eigenvalue analyses need to be performed at a set of temperatures.

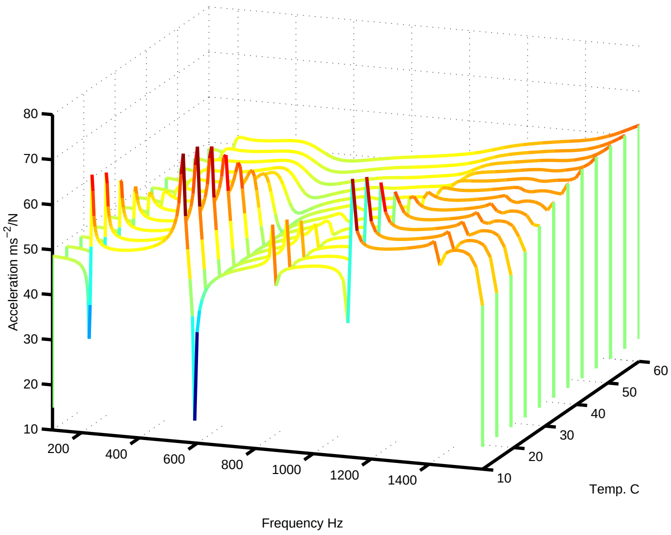

Figure 5: Cross transfer in the $10-60^oC$ range, design B

The FRFs shown in figure 5 clearly illustrate the sensitivity design B to temperature with an optimum located around 50 ^oC.

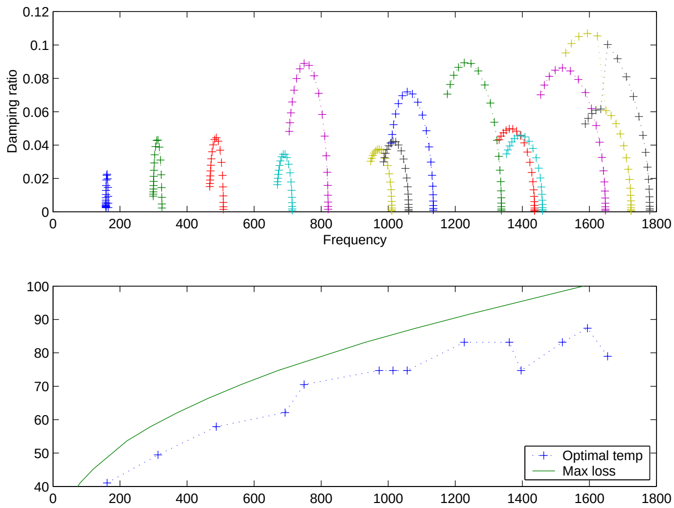

Figure 6: a. Pole locations in the the $10-80^oC$ range for design B b. Optimal temperature for each mode compared to frequency of maximum loss factor.

The tracking of poles in figure 6 gives other interesting information. The bell shape of pole locations corresponds to the shifting of the highest material loss factor from below the resonance to above it. The second curve shows that the optimal temperature for each pole is systematically lower than the temperature at which the material dissipates most. This shows that the core must not be too soft or it will not transmit enough strain energy. The optimum does however depend on the mode shape.

4.4 Layer cuts & Boundary conditions

A consequence of the fact that a major part of the strain energy is due to shear in the core is that weld spots, or on the contrary, selective cuts in the constraining layers significantly affect the dynamic response and should thus be properly accounted for.

Figure 7 illustrates the effect of a transverse cut of the inner skin at the mid length of the pan. Here the cut significantly lowers the energy fraction in the core and thus the overall damping in the model. For other structures the effect might be the opposite.

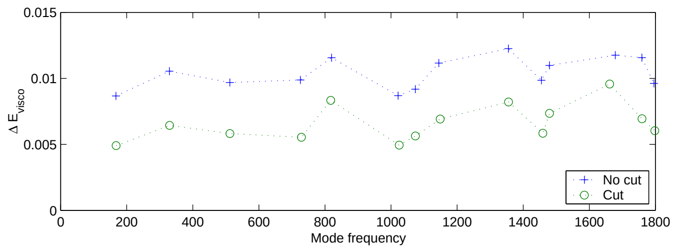

Figure 7: Fraction of strain energy in core for design B with and without cut

5 CONCLUSION

The present study showed that all the tools needed to properly handle the design of viscoelastically damped structures are currently available. While not widely available and still likely to improve, state of the art solvers are practical for direct frequency response and eigenvalue solutions. This only leaves open the question of building time domain models for transient simulation.

By analyzing the example of an oil pan, it clearly appears that major difficulties remain to design press formed sandwich structures. Among important issues, one can cite

- trade-offs between manufacturing and vibroacoustic objectives;
- automatic remeshing of a model to allow variable core thickness;
- experimental and analytical tools to determine the effect of the forming process on the core material properties and thickness;
- systematic strategies for material selection and design robustness verification;
- practical management of separate boundary conditions for the skins;
- shape optimization if the sandwich is formed by gluing a constraining layer which could present cuts.

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