SDTools
VIBRATION SOFTWARE & CONSULTING

CENTRALE PARIS

Modes and regular shapes.
A perspective on CMS theory.

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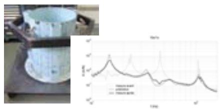
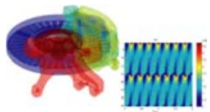
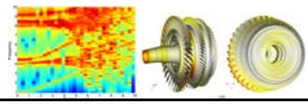
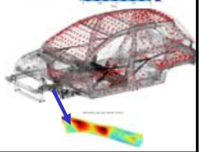
IMAC 26, Orlando

Sample problems beyond coupling subsystems

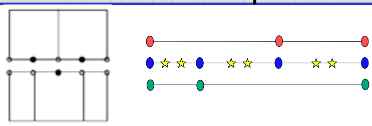
1. Structural Dynamics Modification SDM with added damping
2. Fluid structure interaction (in particular with heavy fluids)
3. Reduce a brake model while keeping
   - all elements of NL contact area
   - exact modes of linear model
4. Design of damping treatment for structure borne transfer
5. Predict response of full shaft as a function of rotation speed, mistuning, temperature

Outline

- Coupling components (substructures)
- Reducing component models
  - Assumptions on spatial and frequency content that underlie a valid reduction
  - Discuss extensions on these assumptions
- Fixed reduction for parametric problems
- Error control on reduced models

Traditional : displacement continuity



Solve with zero relative interface motion

$$\{y_{1\,int} - y_{2\,int}\} = [c_1 \quad -c_2]\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = 0$$

- Classical : Eliminate constraint
- Lagrange multiplier solution
- Penalize

$$\begin{bmatrix} Z(s) & c_{int}^T \\ c_{int} & 0 \end{bmatrix}\begin{Bmatrix} q \\ \lambda \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix}$$

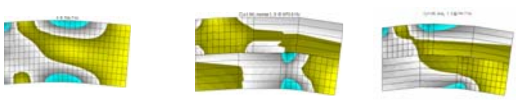
$$\left(\begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix} + \begin{bmatrix} c_1^T \\ -c_2^T \end{bmatrix}\left[\frac{I}{\epsilon}\right][c_1 \quad -c_2]\right)\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = [b]\{u(s)\}$$

R. R. Craig : Review of time domain and frequency domain CMS methods (1987)

A measure of compatibility

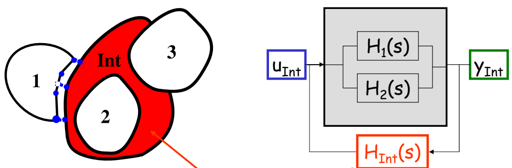
- Compatibility of a displacement $C_i(u_i) = \frac{\|\Pi_2^1 u_i\|}{\|u_i\|} \in [0,1]$
- Projection characterized by scalar product $(q_1, q_2)_V$ given by matrices A_{11}, A_{21}, A_{22}
- 1-ε compatibility found by solving Rayleigh Quotient

$$(1-\epsilon(v_1))^2 = \frac{\{q_1\}^T[A_{21}]^T[A_{22}]^{-1}[A_{21}]\{q_1\}}{\{q_1\}^T[A_{11}]\{q_1\}}$$



- H. Ben Dhia, E. Balmès, Giens 2003
- Continued work volume continuity (Arlequin, partition of unity)

CMS principles



1. Reduced component model
2. Coupled by
   - Continuity constraint
   - Physical interface

$$\begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix}\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}\{u(s)\}$$

$$\{y\} = [c_1 \quad c_2]\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}$$

Stiffness coupling

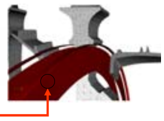
- Interface motion $\{y_j(X,s)\} = [c_{j_{int}}(X)]\{q_j(s)\}$

- Interface stiffness

$$\begin{bmatrix} Z_{jj_{int}} & \cdots & Z_{cj_{int}} \\ \vdots & \ddots & \vdots \\ Z_{jc_{int}} & \cdots & Z_{cc_{int}} \end{bmatrix} \left[\left\{ \begin{matrix} [c_{int}]\{q_j\} \\ \vdots \\ \{q_{int}\} \end{matrix} \right\}\right] = \left\{ \begin{matrix} F_{int} \\ \vdots \\ \{0\} \end{matrix} \right\}$$

- Coupled equations

$$\left(\begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix} + \begin{bmatrix} c_1^T & 0 \\ 0 & c_2^T \end{bmatrix} Z_{int} \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \right) \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}$$



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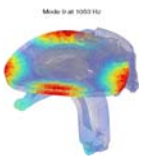
Models of structures

σ(x,t)
f(x,t)



- Geometry
- Discretization element choice
- Constitutive properties

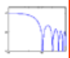
u(x,t)
σ(x,t)

Reduction based on restrictions :
- Excitation (space & freq)
- Responses
- Coupling …

- CMS, Optimization
- Coupling (fluid, control), variability, non linearity

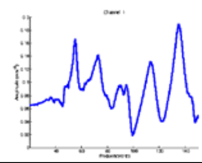
Dynamic system models

When



Where

Dynamic system transfer



Sensors

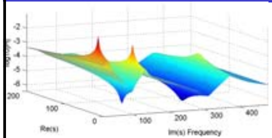
Reduction based on restrictions :
- Excitation (space & freq)
- Responses
- Coupling …

- Coupling (structure, fluid, control)
- Optimization, variability, damping, non linearity, …

Classical CMS

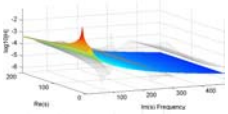
Extensions

MS 1 : Modes and residual terms



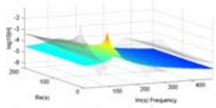
$[c]\,[Z(s)]^{-1}\,[b]\,\{u\} \approx$

$\dfrac{c\phi_1\phi_1^T bu}{s-\lambda_1}$



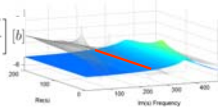
+

$\dfrac{c\phi_2\phi_2^T bu}{s-\lambda_2}$



+

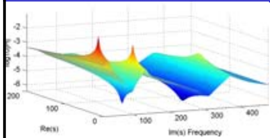
$[c]\left[Z(0)^{-1} - \sum_i \dfrac{\phi_i\phi_i^T}{\lambda_i}\right][b]$



Response ≈
modal contributions
+ residual flexibility

$\{q\} = [T]\{q_R\} = \left[\phi_1\ \phi_2 ... Z(0)^{-1}b\right]\{q_r\}$

MS 2 : Modes and residual terms



$[c]\,[Z(s)]^{-1}\,[b]\,\{u\} \approx$

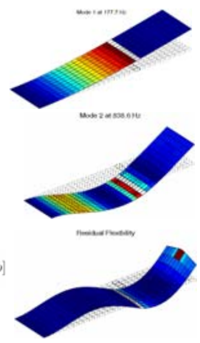
$\dfrac{c\phi_1\phi_1^T bu}{s-\lambda_1}$

+

$\dfrac{c\phi_2\phi_2^T bu}{s-\lambda_2}$

+

$[c]\left[Z(0)^{-1} - \sum_i \dfrac{\phi_i\phi_i^T}{\lambda_i}\right][b]$

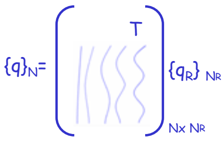


Response ≈
modal contributions
+ residual flexibility

$\{q\} = [T]\{q_R\} = \left[\phi_1\ \phi_2 ... Z(0)^{-1}b\right]\{q_r\}$

MS 3 : Ritz analysis

•Hypothesis : Approximation of solution in subspace generated by T (size N x N_R):



•Ritz-Galerkin approximation :

$$[T^TMTs^2+T^TCTs+T^TKT]_{NR\times NR}\{q_R(s)\}=[T^Tb]_{NR\times NA}\{u(s)\}_{NA\times 1}$$

$$\{y(s)\}_{NS\times 1}=[cT]_{NS\times NR}\{q_R(s)\}_{NR\times 1}$$

MS 4 : reduction bases

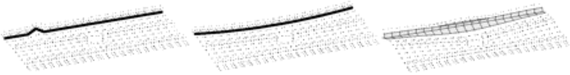
Reduction based on restrictions :
- Excitation
  - space
  - frequency
- Responses
- Coupling …

- Classical bases in CMS :
  - Free modes
  - Flexibility : static response to unit load
  - Constraint modes (Guyan) static response to unit displacement
  - Fixed interface modes

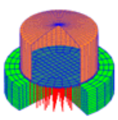
Proposed perspective :
- Any representative response to unit load/displacement will give space validity
- Any representative mode will give frequency validity
- Residual can be used to evaluate error
- Basis valid for parametric range

Redefining unit displacement

Guyan : static response to unit displacements on the interface
- 1 vector/DOF : often to costly
- Polynomial functions : not general
- Modes of an interface model



Interface model
- Condensation on the interface (Craig 72)
- Elements connected to the interface
- Any representative model (mechanical SVD)



Application : soil/structure interaction MSSMat

Redefining interface

Sensors can be considered as interface

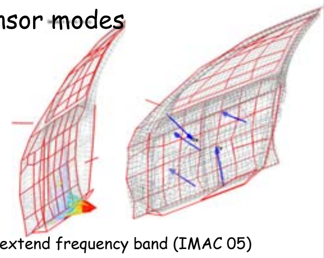
$\{y\} = [c_{int}]\{q\}$

Sample use fixed sensor modes

$$[K-\omega_j^2M]\{\phi_j\}=\{0\}$$

With

$$[c_m]_{m\leq k}\{\phi_j\}=0$$



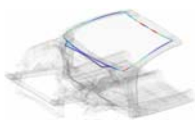
Use : place additional sensors to extend frequency band (IMAC 05)

Redefining unit load

Damped viscoelastic resp.

$[Z(E(s),s)]\{q\} = \{F\}$ rewritten as

$[Z(E_o,s)]\{q\} = \{F\}-[\Sigma\,(E(s)-E_o)/E_o[Im(Z-Z_o)]]\{q\}$

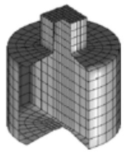


Basis contains
- •Modes to represent nominal resonances
- •Flexibility to viscoelastic loads associated with nominal modes

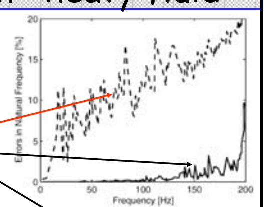
$T = [\phi_{1:NM}\;\; K_o^{-1}[Im(Z-Z_o)]\,\phi_{1:NM}]$

Modes — static response to unit load

Enhanced reduction : heavy fluid



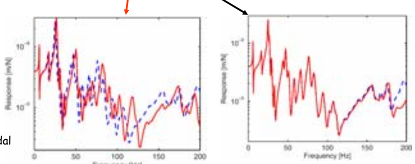
300 Hz : 152 solid, 32 fluid modes



$$\left[\begin{bmatrix} M & 0 \\ C^T & K_p \end{bmatrix}s^2+\begin{bmatrix} K(s) & -C \\ 0 & F \end{bmatrix}\right]\begin{Bmatrix} q \\ p \end{Bmatrix}=\begin{Bmatrix} F^{ex} \\ 0 \end{Bmatrix}$$

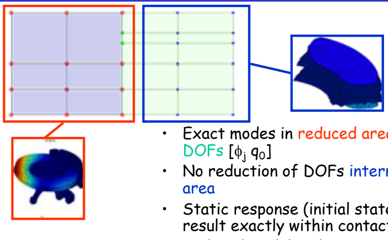
$$\left[\hat{T}^S\right]=\left[T^S\;\;[K_0]^{-1}[C][T^F]\right]$$

$$\left[\hat{T}^F\right]=\left[T^F\;\;[F_0]^{-1}[C]^T[T^S]\right]$$

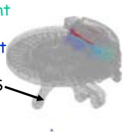


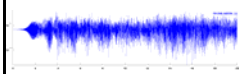
Thanks : DGA/CTSN
NASTRAN DMAP by N. Roy TopModal

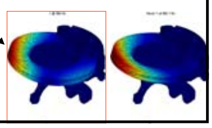
Redefining modes



- Exact modes in reduced area & dependent DOFs $[\phi_j \; q_0]$
- No reduction of DOFs internal to contact area
- Static response (initial state) : ABAQUS result exactly within contact area
- Reduced model with exact system modes



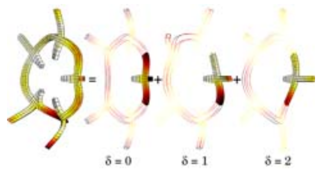


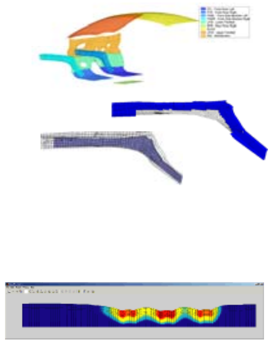


SDTools/Bosch Time domain squeal simulation
800e3 DOFs, 500e3 time steps

Redefining modes

- Nominal modes & component redesign
- Use cyclic/periodic solutions



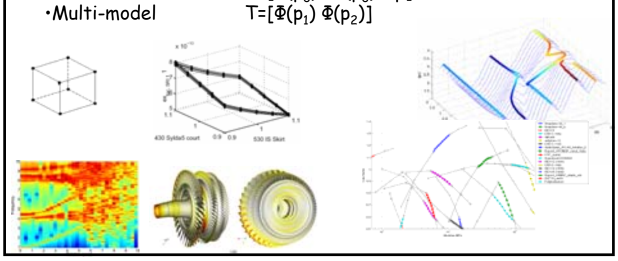


Parametric families

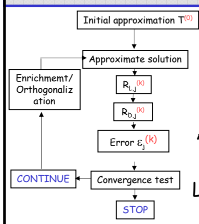
Reanalysis $\left[\left[T^T K(p) T \right] - \omega_{jR}^2(p) \left[T^T M(p) T \right] \right] \{\phi_{jR}(p)\} = \{0\}$

Basis can be fixed for range of parameters

- Modes T=[Φ(p$_0$)]
- Modes + sensitivities T=[Φ(p$_0$) ∂Φ(p$_0$)/∂p]
- Multi-model T=[Φ(p$_1$) Φ(p$_2$)]



Error control



Example : compute modes

$$\left[K - \omega_j^2 M\right]\{\phi_j\} = \{0\}$$

Approx solution $T^{(k)^T}\left[K - \omega_{j,R}^2 M\right] T^{(k)} \{\phi_{j,R}\} = 0$

Load residual $R_{L,j}^{(k)} = \left[K - \omega_{j,R}^2 M\right] T^{(k)} \{\phi_{j,R}\} \neq 0$

Displacement residual $\left\{R_{D,j}^{(k)}\right\} = \left[\hat{K}\right]^{-1}\left\{R_{L,j}^{(k)}\right\}$

Error evaluation $\epsilon_j^{(k)} = \frac{\|\left\{R_{D,j}^{(k)}\right\}\|_K^2}{\|T^{(k)}\{\phi_{j,R}\}\|_K^2}$

Enrichment $\epsilon_j^{(k)} > Tol \Rightarrow T^{(k+1)} = \left[T^{(k)}, \left\{R_{D,j}^{(k)}\right\}\right]$

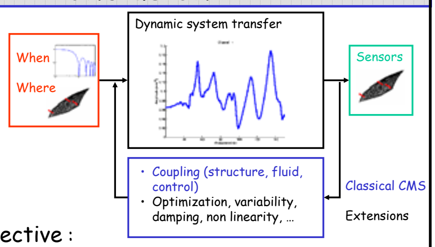
Residue iteration (Bobillot 2002)

Error control

- Motivates existing reduction methods
- Shows strong relation with pre-conditioned conjugate gradient methods
- Provides straightforward extension mechanism

Conclusion

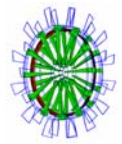
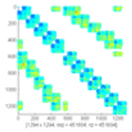
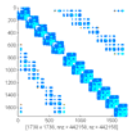
Coupling
- Continuity
- Stiffness



Reduction perspective :
- Any representative response to unit load/displacement will give space validity
- Any representative mode will give frequency validity
- Residual can be used to evaluate error
- Basis valid for parametric range

Current trends

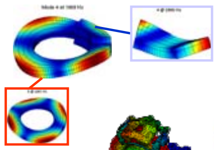
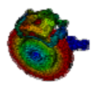
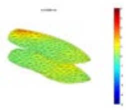
- Non-linear contact/friction simulations
- Parametric studies associated with
  - Design and robustness (stochastic models)
  - Temp. & freq. viscoelastic dependence
  - Rotation speed
- Reduction & sparsity

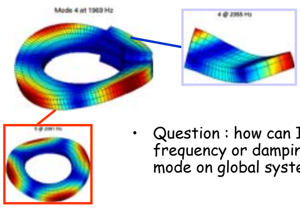
Current trends

- Coupled physics (Acoustics, MEMS, thermoelastic, …)
- Component redesign
  - Specification on component frequencies
  - Design & restitution



Model reduction to analyze component effects



- Question : how can I see the effect of frequency or damping change of component mode on global system mode